

WAVELET APPROXIMATION OF ADOMIAN'S DECOMPOSITION APPLIED  
TO THE NONLINEAR PROBLEM OF A DOUBLE-BEAM RESPONSE  
SUBJECT TO A SERIES OF MOVING LOADS

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The dynamic response of a double-beam resting on a nonlinear viscoelastic foundation and subjected to a finite series of moving loads is analysed. The beams are connected by a viscoelastic layer and the load moving along the upper beam represents motion of a train on the rail track. The mathematical model is described by a coupled system of fourth order partial differential equations with homogeneous boundary conditions. The nonlinearity is included in the foundation stiffness of medium supporting a lower beam. The coiflet based approximation combined with Adomian's decomposition is adopted for the displacements derivation. The developed approach allows one to overcome difficulties related to direct calculation of Fourier integrals as well as the small parameter method. The conditions for correctness of the approximate solution are defined. The influence of some factors on the system sensitivity is discussed, with special focus on the distance between the separated loads. Numerical examples are presented for a certain system of physical parameters.

*Keywords:* infinite double-beam, nonlinear problem, Adomian's decomposition, coiflet approximation, moving load

## 1. Introduction

The subject of moving load problems, especially the analysis of beam vibrations arising from dynamic excitations, is a very important direction of studies associated with modern railway engineering. New conditions, never met in the past, such as high speed rails or intensive freight transport, lead to necessity of better modelling and prediction of possible scenario for operational transport systems (Bogacz and Frischmuth, 2009; Bogacz and Krzyzynski, 1991; Krylov, 2001; Thompson, 2009). In order to achieve this aim, new computational approaches are needed. The new techniques should allow effective parametrical analysis and show phenomena that can appear in certain situations. One can find a number of published results showing closed form solutions for special cases of linear models representing the rail track on a rigid foundation. The presented analyses usually involve classical approaches such as Fourier and Laplace transforms or the Green function technique (Fryba, 1999; Hussein and Hunt, 2006; Oniszczyk, 2003).

It was shown that the nonlinear model reflects behaviour of the beam deflection better than the linear model when compared with measurements (Dahlberg, 2002). In the case of a nonlinear foundation, classical procedures such as the finite element method or the perturbation approach are often employed, and they usually give results that are insufficiently exact, making unable an efficient parametrical study (Kargarnovin *et al.*, 2005; Kuo and Lee, 1994; Sapountzakis and Kampitsis, 2011; Wu and Thompson, 2004). The coiflet based approximation combined with Adomian's decomposition (Adomian, 1989; Koziol, 2010; Wang *et al.*, 2003), adapted in this paper to the double beam problem solution, appeared more efficient and leading to reliable results enabling parametrical analysis of nonlinear systems of the beam-foundation type (Hryniewicz and Kozioł, 2013; Kozioł and Hryniewicz, 2012). The method has been validated in a number

of studies, e.g. by FEM and wavelet FEM in the case of the Euler-Bernoulli beam resting on a linear viscoelastic foundation (Musuva *et al.*, 2014) and by the Lindstedt–Poincaré regular perturbation method in the case of the Euler-Bernoulli beam resting on a nonlinear foundation (Kargarnovin, 2005; Koziol, 2013a) both subjected to moving loads.

This paper introduces nonlinear assumptions to the linear model of a double-beam (Oniszczuk, 2003; Jang *et al.*, 2008) assuming that the rail and slabs are modelled by the Euler-Bernoulli beam and the bottom layer has nonlinear characteristics. Preliminary results for the proposed model solved by coifflet estimation were presented at the Railways 2012 conference (Hryniewicz and Koziol, 2012), and this paper is an essential extension of the past work. It is assumed that the load is represented by a finite series of harmonically varying loads distributed on separated intervals, this being a reliable representation of train movement. The study carried out shows that the developed method is efficient enough for the parametrical analysis. The undertaken effort is focused on the investigation of the influence of the distance between certain loads moving along the upper beam on the nonlinear response of the structure, and also on difficulties in the method application arising from nonlinear assumptions. It is shown that the conditions needed for a solution exact enough in the linear case (Koziol, 2010; Wang *et al.*, 2003) are insufficient for the nonlinear system.

## 2. Double-beam on a nonlinear foundation

The discussed problem of the dynamic response of the double-beam system supported by a viscoelastic nonlinear foundation subject to a series of loads moving along the upper beam is very important in railway engineering. There is a need for structural dynamics prediction for constructions associated with train transportation, and these can be modelled analytically or numerically. The analysed model is related to the rail track consisting of rails, railpads, the floating slab and slab bearings. The theoretical model is composed of an upper beam to account for the rails (with the mass  $m_1$  and the bending stiffness  $EI_1$ ), a lower beam representing the slab (with the mass  $m_2$  and the bending stiffness  $EI_2$ ) and two viscoelastic layers; the first one linear (with the stiffness  $k_1$  and the viscous damping factor  $c_1$ ) between the rail and the slab, and the second one nonlinear and supporting the structure (with the linear stiffness  $k_2$ , the nonlinear part of stiffness  $k_N$  and the viscous damping factor  $c_2$ ).

The two coupled nonlinear partial differential equations of motion describe vertical vibrations of the double-beam system

$$\begin{aligned} EI_1 \frac{\partial^4 U}{\partial \bar{x}^4} + m_1 \frac{\partial^2 U}{\partial t^2} + c_1 \left( \frac{\partial U}{\partial t} - \frac{\partial W}{\partial t} \right) + k_1(U - W) - k_N W^3 &= \bar{P}(\bar{x}, t) \\ EI_2 \frac{\partial^4 W}{\partial \bar{x}^4} + m_2 \frac{\partial^2 W}{\partial t^2} + c_2 \frac{\partial W}{\partial t} + k_2 W - c_1 \left( \frac{\partial U}{\partial t} - \frac{\partial W}{\partial t} \right) - k_1(U - W) + k_N W^3 &= 0 \end{aligned} \quad (2.1)$$

where  $U$  and  $W$  are the transverse displacements of the upper and lower beam, respectively, at position  $\bar{x}$  and time  $t$ . The load moving along the upper beam is modelled by a finite series of loads harmonically varying in time and distributed on separated intervals in the space domain

$$\bar{P}(\bar{x}, t) = \sum_{l=0}^{L-1} \frac{P_0}{2a} \cos^2 \left( \frac{\pi[\bar{x} - Vt - (2a + s)l]}{2a} \right) H \left( a^2 - [\bar{x} - Vt - (2a + s)l]^2 \right) e^{i\Omega t} \quad (2.2)$$

where  $H(\cdot)$  is the Heaviside step function,  $2a$  is the span of the moving load,  $\Omega = 2\pi f_\Omega$  is the frequency of the load,  $V$  is the velocity of the moving excitation and  $s$  is the distance between the separated loads. In order to obtain a steady-state response, a moving coordinate system can be introduced

$$(\bar{x}, t) \rightarrow (x = \bar{x} - Vt, t) \quad (2.3)$$

The response in the following classical form is analysed in this paper

$$U(x, t) = u(x)e^{i\Omega t} \quad W(x, t) = w(x)e^{i\Omega t} \quad (2.4)$$

Applying representation (2.4) and new variables (2.3), one can rewrite equations (2.1) in form represented by differential operators

$$Q_4 u + R_1 w = k_N w^3 e^{2i\Omega t} + P(x) \quad Q_1 u + R_4 w = -k_N w^3 e^{2i\Omega t} \quad (2.5)$$

where

$$\begin{aligned} Q_1 &= \bar{\alpha}_1 \frac{d}{dx} + \bar{\alpha}_0 & Q_4 &= \alpha_4 \frac{d^4}{dx^4} + \alpha_2 \frac{d^2}{dx^2} + \alpha_1 \frac{d}{dx} + \alpha_0 \\ R_1 &= \bar{\beta}_1 \frac{d}{dx} + \bar{\beta}_0 & R_4 &= \beta_4 \frac{d^4}{dx^4} + \beta_2 \frac{d^2}{dx^2} + \beta_1 \frac{d}{dx} + \beta_0 \end{aligned} \quad (2.6)$$

and

$$P(x) = \sum_{l=0}^{L-1} \frac{P_0}{2a} \cos^2\left(\frac{\pi[x - (2a + s)l]}{2a}\right) H\left(a^2 - [x - (2a + s)l]^2\right) \quad (2.7)$$

with the new coefficients

$$\begin{aligned} \alpha_4 &= EI_1 & \alpha_2 &= m_1 V^2 & \alpha_1 &= -V(c_1 + 2im_1 \Omega) \\ \alpha_0 &= k_1 - m_1 \Omega^2 + ic_1 \Omega & \bar{\alpha}_1 &= c_1 V & \bar{\alpha}_0 &= -k_2 - ic_1 \Omega \\ \bar{\beta}_1 &= -c_1 V & \bar{\beta}_0 &= -k_1 + i\Omega(c_2 - c_1) & \beta_4 &= EI_2 \\ \beta_2 &= m_2 V^2 & \beta_1 &= -V(c_1 + c_2 + 2im_2 \Omega) & \beta_0 &= k_1 + k_2 - m_2 \Omega^2 + ic_1 \Omega \end{aligned} \quad (2.8)$$

The deflection, slope and curvature must tend to zero far from the excitation and, therefore, the boundary conditions can be assumed as follows

$$\begin{aligned} \lim_{x \rightarrow \pm\infty} u(x) &= \lim_{x \rightarrow \pm\infty} w(x) = 0 & \lim_{x \rightarrow \pm\infty} \frac{du}{dx} &= \lim_{x \rightarrow \pm\infty} \frac{dw}{dx} = 0 \\ \lim_{x \rightarrow \pm\infty} \frac{d^2 u}{dx^2} &= \lim_{x \rightarrow \pm\infty} \frac{d^2 w}{dx^2} = 0 \end{aligned} \quad (2.9)$$

### 3. Adomian's procedure

The method developed by Adomian assumes that the solution to any nonlinear problem described by differential equations can be represented by an infinite series of functions (Adomian, 1994; Koziol, 2010; Koziol and Hryniewicz, 2012). The first of these functions is a solution to the linear problem associated with the problem considered, and the other terms describe nonlinear factors influencing the response. The functions to be evaluated are represented by Adomian polynomials. One can find many procedures for the evaluation of these polynomials in the literature (Adomian, 1989; Adomian 1994; Hosseini and Nasabzadeh, 2006; Pourdarvish, 2006; Wazwaz, 1999; Wazwaz and El-Sayed, 2001). Some of them offer a bit better convergence in the sense that a smaller number of polynomials is needed for obtaining results with accuracy sufficient for the solution exact enough (Wazwaz, 1999; Wazwaz and El-Sayed, 2001). The performed simulations show that this feature is kept also for the problem considered, but computational difficulties grow due to complexity of the formulas. Therefore, a good balance between computational cost and effectiveness must be found when choosing the numerical procedure. In this paper, the following form of the polynomials is taken (Das, 2009; Pourdarvish, 2006)

$$A_j(x) = \frac{1}{j!} \left( \frac{d^j}{d\lambda^j} \sum_{k=0}^{\infty} \lambda^k w_k(x) \right) \Big|_{\lambda=0} \quad \text{for } j = 0, 1, 2, \dots \quad (3.1)$$

and for practical calculations, the first four polynomials are used

$$\begin{aligned} A_0 &= w_0^3 & A_1 &= 3w_0^2w_1 & A_2 &= 3(w_0w_1^2 + w_0^2w_2) \\ A_3 &= w_1^3 + 6w_0w_1w_2 + 3w_0^2w_3 & A_4 &= 3(w_1^2w_2 + w_0w_2^2 + 2w_0w_1w_3 + w_0^2w_4) \end{aligned} \quad (3.2)$$

For the effective solution of the problem considered (Eqs. (2.5)), one assumes that the beams vibrations are represented by the following series

$$u(x) = \sum_{j=0}^{\infty} u_j(x) \quad w(x) = \sum_{j=0}^{\infty} w_j(x) \quad (3.3)$$

and the nonlinear cubic term  $w^3(x)$  is characterized by the Adomian polynomials

$$w^3(x) = \sum_{j=0}^{\infty} A_j(x) \quad (3.4)$$

The problem of the series convergence (Eqs. (3.3)) was solved and discussed in the literature. In order to control the accuracy of the Adomian approximation, one can use the convergence condition defined by the following formula (Hosseini and Nasabzadeh, 2006)

$$0 < \frac{\|u_{j+1}\|}{\|u_j\|} < 1 \quad 0 < \frac{\|w_{j+1}\|}{\|w_j\|} < 1 \quad (3.5)$$

with the norm  $\|u_j\| = \max_x |\operatorname{Re}[u_j(x)]|$  and  $\|w_j\| = \max_x |\operatorname{Re}[w_j(x)]|$ .

A reliable representation of the response can be described by the  $n$ -th order approximation

$$S_n(u, x) = \sum_{j=0}^n u_j(x) \quad S_n(w, x) = \sum_{j=0}^n w_j(x) \quad (3.6)$$

depending on the accuracy assumed. Combining equations (2.5), (3.3) and (3.4) leads to an effective algorithm for the evaluation of the terms  $u_j(x)$  and  $w_j(x)$  ( $j = 1, 2, 3, \dots$ )

$$\begin{aligned} Q_4u_0 + R_1w_0 &= P(x) & Q_1u_0 + R_4w_0 &= 0 \\ Q_4u_j + R_1w_j &= k_N A_{j-1}(x)e^{2i\Omega t} & Q_1u_j + R_4w_j &= -k_N A_{j-1}(x)e^{2i\Omega t} \end{aligned} \quad (3.7)$$

The practical form of Adomian polynomials (3.2) needed for consecutive evaluation of approximate series terms (Eq. (3.3)) is computed in this paper by using a wavelet based approximation allowing derivation of the Fourier transform without numerical approach and, in the same time, keeping the assumed accuracy. Essential details of the adopted method are presented in the next section.

#### 4. The Fourier transform and the coiflet approximation

The above system of equations (3.7) can be solved by applying the Fourier transforms

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(x)e^{-i\omega x} dx \quad f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega)e^{i\omega x} d\omega \quad (4.1)$$

Thus, one can obtain the system of formulas that are needed for the Adomian series computation in the transform domain

$$\begin{aligned} \hat{u}_0(\omega) &= \frac{\hat{R}_4\hat{P}(\omega)}{\hat{Q}_4\hat{R}_4 - \hat{Q}_1\hat{R}_1} & \hat{u}_j(\omega) &= \frac{k_N\hat{A}_{j-1}(\omega)(\hat{R}_4 + \hat{R}_1)}{\hat{Q}_4\hat{R}_4 - \hat{Q}_1\hat{R}_1}e^{2i\Omega t} \\ \hat{w}_0(\omega) &= \frac{-\hat{Q}_1\hat{P}(\omega)}{\hat{Q}_4\hat{R}_4 - \hat{Q}_1\hat{R}_1} & \hat{w}_j(\omega) &= \frac{-k_N\hat{A}_{j-1}(\omega)(\hat{Q}_4 + \hat{Q}_1)}{\hat{Q}_4\hat{R}_4 - \hat{Q}_1\hat{R}_1}e^{2i\Omega t} \end{aligned} \quad (4.2)$$

where  $j = 1, 2, 3, \dots$  and

$$\hat{P}(\omega) = P_0 \frac{i\pi^2 [1 - \exp(2ia\omega)]}{4a\omega(\pi^2 - a^2\omega^2)} \sum_{l=0}^{L-1} \exp[-i(a + 2la + ls)\omega] \quad (4.3)$$

Formulas (4.2) and (4.3) must be retransformed to obtain the dynamic response of the system in the physical domain. Because of complexity of the integrands appearing in the Fourier integrals, classical or numerical methods of the inverse Fourier transform calculation become ineffective and might give wrong results. An alternative method of derivation based on the coiflet expansion of functions is adopted in this paper (Koziol, 2010; Wang *et al.*, 2003). It uses the filter family called coiflets, possessing the property of vanishing moments for both wavelet and scaling functions, which allows one to estimate multiresolution coefficients relatively easy (Monzon *et al.*, 1999; Wang *et al.*, 2003)

$$\Psi(x) = \sum_{k=0}^{K_C} (-1)^k p_{K_C-k} \Phi(2x - k) \quad \Phi(x) = \sum_{k=0}^{K_C} p_k \Phi(2x - k) \quad (4.4)$$

$\Psi_C$  and  $\Phi_C$  are the wavelet function and the scaling function, respectively and  $K_C$  is the number of filter coefficients belonging to the applied family of coiflets  $p_k$ . Using the properties of coiflets and relations between the wavelets, the Fourier analysis leads to the formulas allowing one to approximate analytically the Fourier transforms of every function belonging to the  $L^2$  space

$$\begin{aligned} \hat{f}(\omega) &= \lim_{n \rightarrow \infty} \hat{f}_n(\omega) = \lim_{n \rightarrow \infty} 2^{-n-1} \prod_{k=1}^{\tilde{K}} \left( \sum_{k=0}^{K_C} p_k e^{ik\omega/2^{n+k}} \right) \sum_{k=k_{min}(n)}^{k_{max}(n)} f((k+M)2^{-n}) e^{-i\omega k 2^{-n}} \\ f(x) &= \lim_{n \rightarrow \infty} f_n(x) = \lim_{n \rightarrow \infty} \frac{1}{2^{n+2\pi}} \prod_{k=1}^{\tilde{K}} \left( \sum_{k=0}^{K_C} p_k e^{-ikx/2^{n+k}} \right) \sum_{k=k_{min}(n)}^{k_{max}(n)} \hat{f}((k+M)2^{-n}) e^{ixk 2^{-n}} \end{aligned} \quad (4.5)$$

where  $M = \sum_{k=0}^{K_C} k p_k$  and the range of summation  $k_{min}(n) = \omega_{min} 2^n - K_C - 1$ ,  $k_{max}(n) = \omega_{max} 2^n - 1$  must be determined in such a way that at least the interval  $[\omega_{min}, \omega_{max}]$  covers the set of the variable  $\omega$  having strong influence on the behaviour of the original function that guaranties proper application in the linear case. In the next section, it will be shown that this criterion, given in previous publications for linear problems (Koziol, 2010; Wang *et al.*, 2003), becomes insufficient for the problem considered. In this case, mainly due to the nonlinear characteristics of the system and the applied excitation consisting of a number of loads, the analysis of the power spectrum does not give all information needed for the determination of coiflet approximation, as opposed to the mentioned criterion applied to much simpler problems, i.e. a single load or linear models. The proper condition appropriate for the model analysed is the stabilisation of the solution with an increasing order of the approximation combined with the analysis of the power spectrum proposed before and a deep knowledge about possible behaviour of the structure.

Combining Adomian's decomposition with the coiflet approximation (Eqs. (4.5)) gives a semi-analytical procedure for the displacements evaluation for  $j = 0, 1, 2, \dots$

$$\begin{aligned} u_j(x) &= \frac{1}{2^{n+1\pi}} \prod_{k=1}^{\tilde{K}} \sum_{j=0}^{K_C} p_j \exp(ijx/2^{n+k}) \sum_{k=k_{min}(n)}^{k_{max}(n)} \hat{u}_j\left(\frac{k+M}{2^n}\right) \exp(ikx/2^n) \\ w_j(x) &= \frac{1}{2^{n+1\pi}} \prod_{k=1}^{\tilde{K}} \sum_{j=0}^{K_C} p_j \exp(ijx/2^{n+k}) \sum_{k=k_{min}(n)}^{k_{max}(n)} \hat{w}_j\left(\frac{k+M}{2^n}\right) \exp(ikx/2^n) \\ \hat{A}_j(\omega) &= \frac{1}{2^n} \prod_{k=1}^{\tilde{K}} \sum_{j=0}^{K_C} p_j \exp(-ij\omega/2^{n+k}) \sum_{k=k_{min}(n)}^{k_{max}(n)} A_j\left(\frac{k+M}{2^n}\right) \exp(-ik\omega/2^n) \end{aligned} \quad (4.6)$$

An arbitrarily chosen point in the space can be taken for numerical simulations. Further analysis is carried out for  $\bar{x} = 0$  and the  $n$ -th order Adomian's approximation (Eqs. (3.6))

$$u(t) \approx S_n(u, t) = \sum_{j=0}^n u_j(-Vt) \quad w(t) \approx S_n(w, t) = \sum_{j=0}^n w_j(-Vt) \quad (4.7)$$

## 5. Simulations and discussion

The following parameters are considered for numerical examples (Kargarnovin *et al.*, 2005; Abu-Hilal, 2006; Hryniewicz and Koziol, 2013):  $K_C = 17$ ,  $\tilde{K} = 10$ ,  $n = 5$ ,  $P_0 = 5 \cdot 10^4$  N/m,  $EI_1 = 10^7$  Nm<sup>2</sup>,  $m_1 = 100$  kg/m,  $k_1 = 4 \cdot 10^7$  N/m<sup>2</sup>,  $c_1 = 6.3 \cdot 10^3$  Ns/m<sup>2</sup>,  $EI_2 = 1.43 \cdot 10^9$  Nm<sup>2</sup>,  $m_2 = 3.5 \cdot 10^3$  kg/m,  $k_2 = 5 \cdot 10^7$  N/m<sup>2</sup>,  $c_2 = 4.18 \cdot 10^4$  Ns/m<sup>2</sup>,  $k_N = 4 \cdot 10^{13}$  N/m<sup>4</sup>,  $f_\Omega = 5$  Hz,  $V = 20$  m/s.

One should note that the used system of parameters is adjusted in order to highlight the main features of the approach developed, and more realistic assumptions could be considered for real structures behaviour investigation. Nevertheless, the implemented values secure the convergence of the approximate solutions and show that the adopted method indeed enables parametrical analysis of the considered nonlinear dynamic system.

Answers to questions: how to describe the deflection of the beams after the load passed the observation point and how to characterize the behaviour of extreme values of the vibrations amplitude are of importance in railway engineering. For this study, the complex modulus of the solution can be examined, called in the literature "the maximal response" (Kim and Cho, 2006; Koziol and Hryniewicz, 2012; Sun, 2002). The maximal response describes changes of the system dynamic sensitivity in time. This kind of approach is helpful in investigation of the nonlinear system because for such cases, the real and imaginary part of the solution cannot be so easily interpreted as it is possible for linear modelling.

Numerical simulations show that the 5-th order Adomian approximation can be taken for the analysis as a reliable representation of the solution for the considered model and system of parameters (Eq. (3.6)). The order of the coiflet estimation can be fixed as  $n = 5$ , and above this index the solution stabilizes without showing significant changes when  $n$  increases.

Figures 1 and 2 present how the beams deflection changes in time, depending on the distance between the separated loads being parts of the moving excitations (Eq. (2.2)). One can observe that the nonlinear solution differs from the linear one much stronger when the distance  $s$  becomes smaller. The nonlinear response accumulates in such cases, especially for the upper beam representing the rail. The response of the upper beam is stronger than vibrations of the lower beam in each case. The more noticeable nonlinear effects can also be observed for the upper beam, which means that the influence of the nonlinear factor included in the layer supporting the lower beam propagates through the system to the surface where it causes the strongest variations. One can see that for the lower beam, the difference between the linear and the nonlinear response is hardly visible (Figs. 1d-f). It was shown previously that the increasing number of loads makes the nonlinear effects stronger in the systems similar to the model considered (Koziol, 2013b).

There are many factors with an important influence on the developed approximate solution. A good balance between the efficiency and cost effectiveness of the algorithm must be found in order to achieve a good procedure allowing parametrical analysis. The most important features of this approach were discussed in past publications, e.g. (Koziol, 2010; Koziol 2013b). However, in the case of a nonlinear problem, new questions concerning the wavelet approximation appear.

Figures 3 and 4 show that the criterion regarding the range of summation in the coiflet approximation formula (Eqs. (4.6)) formulated previously is insufficient for the double-beam system. It was shown that for the linear modelling it is enough to determine the range of

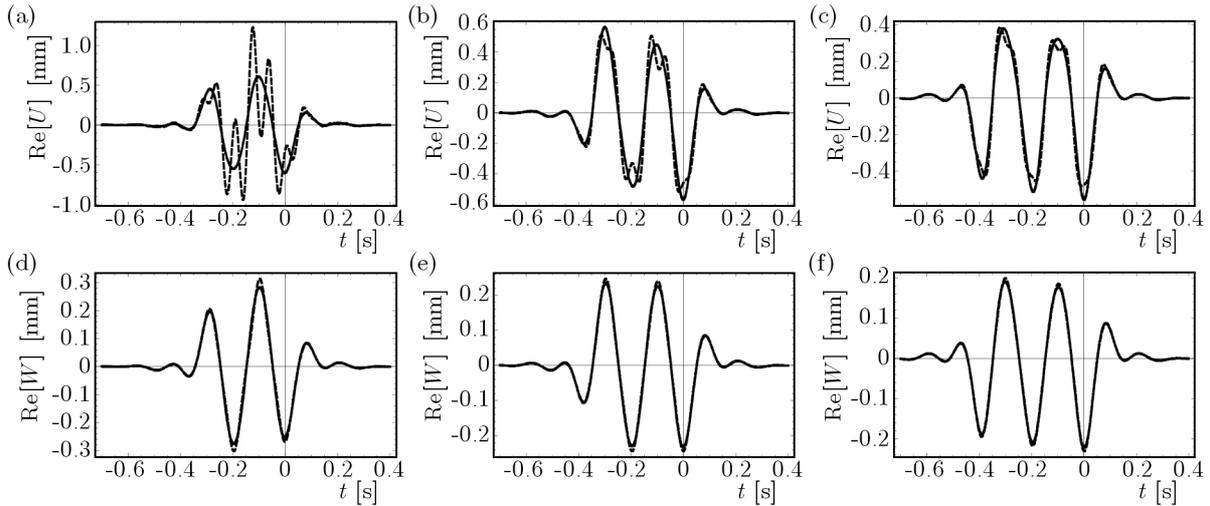


Fig. 1. Vibrations of beams in the case of linear (solid) and nonlinear (dashed) system ( $L = 2$ ; 3 loads): (a), (d)  $s = 2.5$  m; (b), (e)  $s = 3$  m; (c), (f)  $s = 3.5$  m

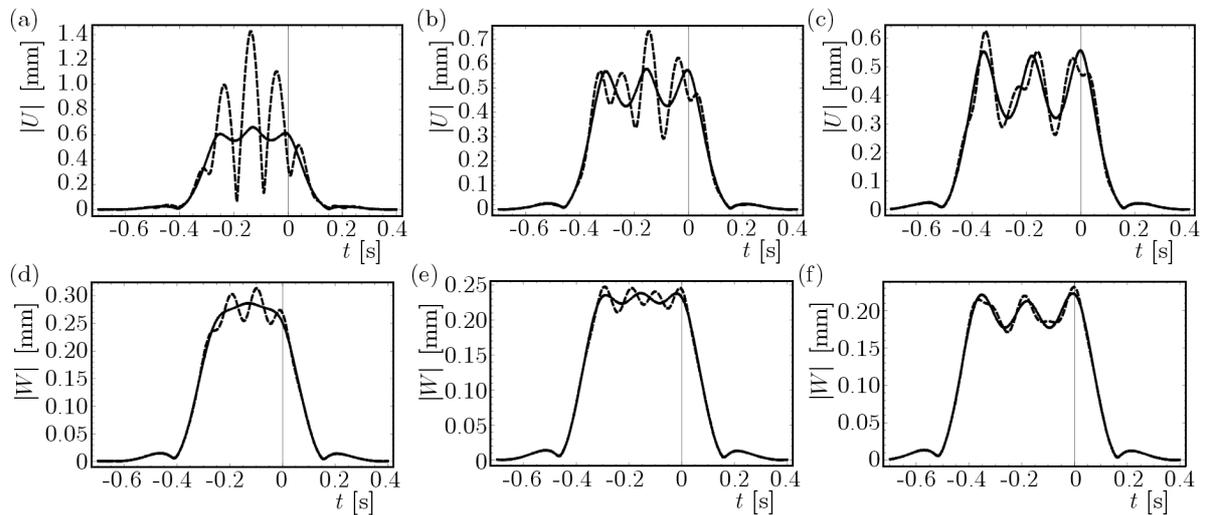


Fig. 2. The maximal response in the case of linear (solid) and nonlinear (dashed) model ( $L = 2$ ; 3 loads): (a), (d)  $s = 2.5$  m; (b), (e)  $s = 3$  m; (c), (f)  $s = 3.5$  m

summation  $k_{min}(n) = \omega_{min}2^n - K_C - 1$ ,  $k_{max}(n) = \omega_{max}2^n - 1$  on the basis of the behaviour of the transformed solution, i.e. by taking into account the fact that the interval  $[\omega_{min}, \omega_{max}]$  includes all points  $\omega$  important for the original solution. These can be recognized by the analysis of the support of the power spectrum for each term appearing consecutively in the approximating sequence (Eqs. (4.6)).

Figure 3 shows the solution for the double-beam system obtained by using the criterion based on the power spectrum analysis in the case of a relatively big distance of 15 m between 2 loads. One can see that the linear part of the solution clearly shows two separated excitations, whereas the nonlinear part does not reflect our expectations. One could presume that the second load also affects the nonlinear solution, especially when its distance from the first excitation is big enough to possibly neglect the combined accumulation of the loads. Figure 4 presents the solution for the same system of parameters but with an increased range of summation:  $-818 < k(n = 5) < 799$  (Eqs. (4.6)). Although the power spectrum analysis (Fig. 5) provides the information that the interval  $[\omega_{min}, \omega_{max}] = [-7, 7]$  should be enough for an appropriate choice of the approximate sum, the range is increased to  $[\omega_{min}, \omega_{max}] = [-25, 25]$  this time, leading to a higher number of functions that must be calculated for the approximation:  $-818 < k(n = 5) < 799$ . In both cases,

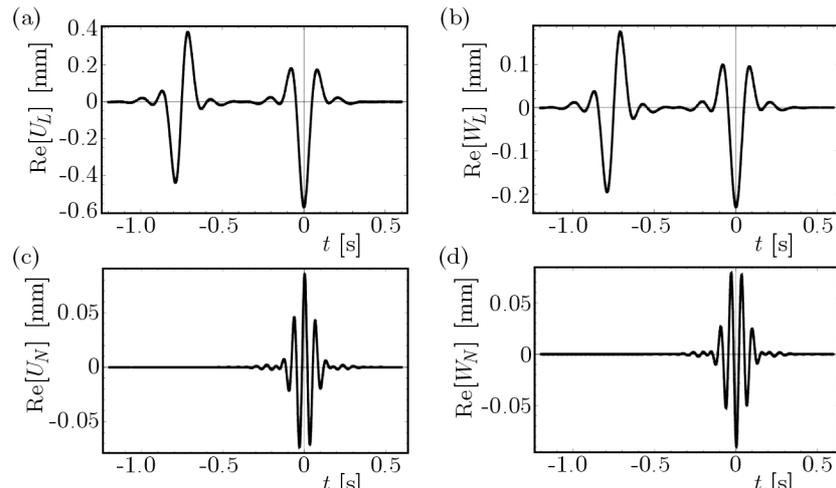


Fig. 3. The linear part  $W_L(0, t) = w_0(-Vt)e^{i\Omega t}$  and the nonlinear part  $W_N(0, t) = \sum_{j=1}^5 w_j(-Vt)e^{i\Omega t}$  of the vertical displacements:  $L = 1$  (2 loads),  $s = 15$  m and  $[\omega_{min}, \omega_{max}] = [-7, 7]$  ( $-242 < k(n = 5) < 223$ )

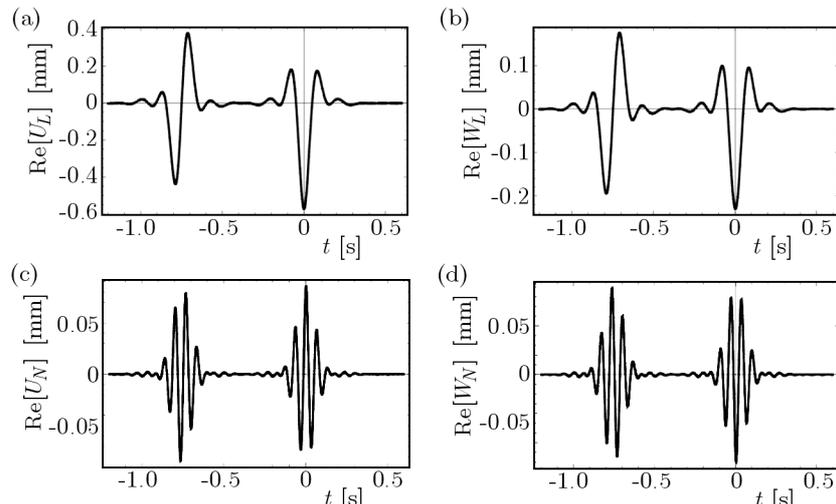


Fig. 4. The linear part  $W_L(0, t) = w_0(-Vt)e^{i\Omega t}$  and the nonlinear part  $W_N(0, t) = \sum_{j=1}^5 w_j(-Vt)e^{i\Omega t}$  of the vertical displacements:  $L = 1$  (2 loads),  $s = 15$  m and  $[\omega_{min}, \omega_{max}] = [-25, 25]$  ( $-818 < k(n = 5) < 799$ )

the solution stabilizes starting from the 5-th order of the coiflet procedure. This means that for the second case (Fig. 4) the solution stabilization is achieved for both factors: the approximation order for the coiflet approximation and the interval  $[\omega_{min}, \omega_{max}]$  taken for the sum evaluation on the basis of the power spectrum analysis.

Thus, the use of the criterion formulated for the linear case based on the power spectrum analysis only, cannot be used in the case of nonlinear modelling. The proper formulation of guidelines for the developed procedure allows one to analyse better the system vibrations arising from the moving load. Figure 6 shows that even for such a big distance between 2 loads as that one considered, the nonlinear effects are strong already for the first load passing by the observation point and changes visible in shape of vibration curves become noticeably different for the second load. This feature reflecting accumulation of nonlinear behaviour for a series of loads (Fig. 6b) could be overlooked when applying the previous condition sufficient for the linear case (Fig. 6a).

One should remember that this paper does not analyse all important aspects of the wave phenomena occurring in the considered double-beam nonlinear system and concentrates only

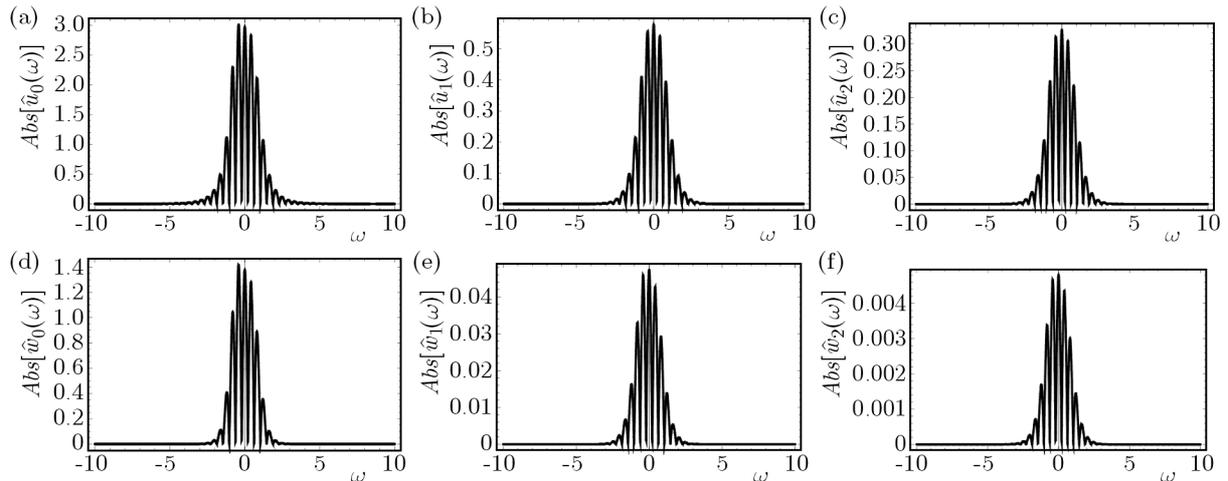


Fig. 5. The power spectrum  $Abs[\hat{u}]$ ,  $Abs[\hat{w}w]$  of the Adomian series terms  $u_0 = u_L$ ,  $u_1$ ,  $u_2$  and  $w_0 = w_L$ ,  $w_1$ ,  $w_2$  for  $L = 1$  (2 loads),  $s = 15$  m and  $[\omega_{min}, \omega_{max}] = [-25, 25]$  ( $-818 < k(n = 5) < 799$ )

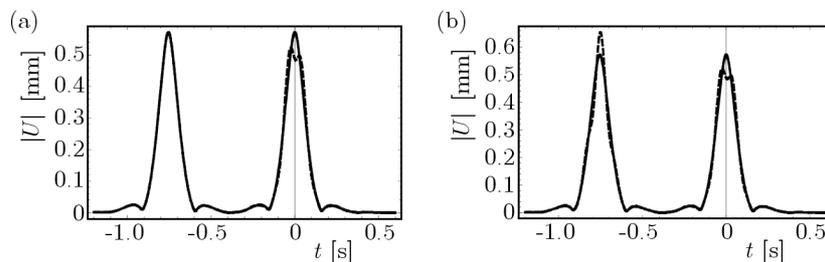


Fig. 6. Vibrations of the upper beam in the case of the linear (solid) and nonlinear (dashed) system for  $L = 1$  (2 loads),  $s = 15$  m: (a)  $[\omega_{min}, \omega_{max}] = [-7, 7]$ , (b)  $[\omega_{min}, \omega_{max}] = [-25, 25]$

on the development of the proper solution procedure. Other phenomena, e.g. stability domains, should be analysed for an appropriate description of the dynamic behaviour of the structure.

## 6. Conclusions

The reliable wavelet based approach to the solution of the double-beam nonlinear system subjected to a series of moving loads is developed. The method based on Adomian's decomposition combined with the coiflet estimation of the Fourier transform is modified by an introduction of a new criterion regarding an appropriate choice of parameters needed for selection of the approximation order and adjustment of the approximate formulas. The formulated guidelines lead to the procedure enabling effective parametrical analysis of the considered nonlinear model. The influence of the distance between separated moving loads acting on the upper beam on the system response is analysed. It is shown that the nonlinear effects propagate from the supporting medium to the upper beam and accumulate with the decreasing distance between the separated loads moving along the double-beam system. Numerical examples are presented for the considered system of parameters.

## References

1. ABU-HILAL M., 2006, Dynamic response of a double Euler-Bernoulli beam due to a moving constant load, *Journal of Sound and Vibration*, **297**, 477-491

2. ADOMIAN G., 1989, *Nonlinear Stochastic Systems Theory and Application to Physics*, Kluwer Academic Publishers, Dordrecht
3. ADOMIAN G., 1994, *Solving Frontier Problems of Physics: The Decomposition Method*, Kluwer, Boston, MA
4. BOGACZ R., FRISCHMUTH K., 2009, Analysis of contact forces between corrugated wheels and rails, *Machine Dynamics Problems*, **33**, 2, 19-28
5. BOGACZ R., KRZYZYNSKI T., 1991, On stability of motion of lumped system along continuous one modelled by Timoshenko beam, *Czasopismo Naukowe, Cracow University of Technology*, **2M/1991**, 3-53
6. DAHLBERG T., 2002, Dynamic interaction between train and nonlinear railway model, *Proceedings of Fifth International Conference on Structural Dynamics*, Munich
7. DAS S., 2009, A numerical solution of the vibration equation using modified decomposition method, *Journal of Sound and Vibration*, **320**, 576-583
8. FRYBA L., 1999, *Vibrations of Solids and Structures Under Moving Loads*, Thomas Telford Ltd., London
9. HOSSEINI M.M., NASABZADEH H., 2006, On the convergence of Adomian decomposition method, *Applied Mathematics and Computation*, **182**, 536-543
10. HRYNIEWICZ Z., KOZIOL P., 2012, The response of a double-beam on a nonlinear foundation arising from a moving load, [In:] J. Pombo (Editor), *Proceedings of the First International Conference on Railway Technology: Research, Development and Maintenance*, ISBN 978-1-905088-53-9, Civil-Comp Press, Stirlingshire, UK, Paper 112, 1-11 doi:10.4203/ccp.98.112
11. HRYNIEWICZ Z., KOZIOL P., 2013, Wavelet-based solution for vibrations of beam on nonlinear viscoelastic foundation due to moving load, *Journal of Theoretical and Applied Mechanics*, **51**, 1, 215-224
12. HUSSEIN M.F.M., HUNT H.E.M., 2006, Modelling of floating-slab tracks with continuous slabs under oscillating moving loads, *Journal of Sound and Vibration*, **297**, 37-54
13. JANG T.S., BAEK H.S., PAIK J.K., 2011, A new method for the non-linear deflection of an infinite beam resting on a non-linear elastic foundation, *International Journal of Non-Linear Mechanics*, **46**, 339-346
14. KARGARNOVIN M.H., YOUNESIAN D., THOMPSON D.J., JONES C.J.C., 2005, Response of beams on nonlinear viscoelastic foundations to harmonic moving loads, *Computers and Structures*, **83**, 1865-1877
15. KIM S.-M., CHO Y.-H., 2006, Vibration of dynamic buckling of shear beam-columns on elastic foundation under moving harmonic loads, *International Journal of Solids and Structures*, **43**, 393-412
16. KOZIOL P., 2010, *Wavelet Approach for the Vibratory Analysis of Beam-Soil Structures*, VDM Verlag Dr. Müller, Saarbrücken
17. KOZIOL P., 2013a, Nonlinear dynamics of an Euler-Bernoulli beam subjected to moving loads, [In:] B.H.V. Topping, P. Iványi (Editors), *Proceedings of the Fourteenth International Conference on Civil, Structural and Environmental Engineering Computing*, Civil-Comp Press, Stirlingshire, UK, Paper 54, doi:10.4203/ccp.102.54
18. KOZIOL P., 2013b, Vibrations of the Euler-Bernoulli beam resting on a nonlinear foundation, [In:] Maia N.M.M., Neves M.M., Sampaio R.P.C. (Editors), *Proceedings of the ICEDYN 2013 International Conference on Structural Engineering Dynamics*, Sesimbra, Portugal, paper K03
19. KOZIOL P., HRYNIEWICZ Z., 2012, Dynamic response of a beam resting on a nonlinear foundation to a moving load: coiffet-based solution, *Shock and Vibration*, **19**, 995-1007
20. KRYLOV V.V. (ED.), 2001, *Noise and Vibrations from High-Speed Trains*, Thomas Telford Ltd, London

21. KUO Y.H., LEE S.Y., 1994, Deflection of non-uniform beams resting on a nonlinear elastic foundation, *Computers and Structures*, **51**, 513-519
22. MONZON L., BEYLKIN G., HEREMAN W., 1999, Compactly supported wavelets based on almost interpolating and nearly linear phase filters (coiflets), *Applied and Computational Harmonic Analysis*, **7**, 184-210
23. MUSUVA M., KOZIOL, P., MARES C., NEVES M., 2014, Using coiflets, the wavelet finite element method and FEM to analyse the beam response to moving load, *Proceedings of the Second International Conference on Railway Technology: Research, Development and Maintenance, Railways*, in print
24. ONISZCZUK Z., 2003, Forced transverse vibrations of an elastically connected complex simply supported double beam system, *Journal of Sound and Vibration*, **264**, 273-286
25. POURDARVISH A., 2006, A reliable symbolic implementation of algorithm for calculating Adomian polynomials, *Applied Mathematics and Computation*, **172**, 545-550
26. SAPOUNTZAKIS E.J., KAMPITSIS A.E., 2011, Nonlinear response of shear deformable beams on tensionless nonlinear viscoelastic foundation under moving loads, *Journal of Sound and Vibration*, **330**, 5410-5426
27. SUN L., 2002, A closed-form solution of beam on viscoelastic subgrade subjected to moving loads, *Computers and Structures*, **80**, 1-8
28. THOMPSON D., 2009, *Railway Noise and Vibration: Mechanisms, Modelling and Means of Control*, Elsevier, Oxford
29. WANG J., ZHOU Y., GAO H., 2003, Computation of the Laplace inverse transform by application of the wavelet theory, *Communications in Numerical Methods in Engineering*, **19**, 959-975
30. WAZWAZ A.M., 1999, A reliable modification of Adomian decomposition method, *Applied Mathematics and Computation*, **102**, 77-86
31. WAZWAZ A.M., EL-SAYED S.M., 2001, A new modification of the Adomian decomposition method for linear and nonlinear operators, *Applied Mathematics and Computation*, **122**, 393-405
32. WU T.X., THOMPSON D.J., 2004, The effects of track nonlinearity on wheel/rail impact, *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, **218**, 1-12

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