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STABILITY OF A POROUS-CELLULAR CYLINDRICAL SHELL SUBJECTED TO COMBINED LOADS

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The subject of the paper is a metal foam circular cylindrical shell subjected to combined loads. Combinations of the external pressure and axial load are taken into account. The shell is simply supported on all outer edges. The mechanical properties of the metal foam vary continuously in the thickness direction. A non-linear hypothesis of deformation of a plane cross section of the shell is formulated. The field of displacements of any cross section and non-linear geometric relationships are assumed. The system of partial differential equations for the shell is derived on the basis of the principle of stationarity of the total potential energy. This system is approximately solved by the Bubnov-Galerkin method. The critical loads for shells are numerically determined. Results of the calculation are shown in figures.

Key words: stability, cylindrical shell, porous-cellular material

1. Introduction

Thin walled circular cylindrical shells are often used in many branches of industry. These structures are the building base of elementary structural parts of complex systems. The potential applications include lightweight structures made of homogeneous, sandwich and multilayer composites. The main assessment criterion of the practical application efficiency of these structures, except for the economic aspect, is a relatively low mass-strength or mass-rigidity ratio. However, these constructions are sensitive to loss of stability, therefore, calculations of critical loads of the shells are important elements of the analysis of the shells strength. Homogeneous and sandwich cylindrical shells have been extensively investigated. The results of research have been presented in many monographs, e.g. Volmir (1967), Doyle (2001). Magnucki and Ostwald (2001) paid particular attention to three-layered shells with two external faces made of a steel and relatively weak foam. Shen (1996) described the problem of the shell subjected to external pressure and axial loads. Marcinowski (2003) presented numerical, geometrically non-linear static analysis of sandwich plates and shells. In this paper, the concept of a degenerated finite element is used to model the mechanical behaviour of sandwich constructions. Błachut (2010) presented the numerical and experimental study buckling of axially compressed cylindrical shells with a non-uniform axial length. It is assumed in this paper that the initial imperfection of the length had a sinusoidal shape along the compressed edge. A review of selected problems of some aspects of the strength, static stability, and optimisation of horizontal pressure vessels was presented by Błachut and Magnucki (2008).

Current technologies allow creation of constructions of porous materials. Various methods of technological processes of porous materials were presented by Banhart (2001). Investigations and properties of these materials were presented by Bart-Smith *et al.* (2001), Ramamurty and Paul (2004).

Magnucki and Stasiewicz (2004), Magnucka-Blandzi (2008), Magnucki *et al.* (2006a), carried out analytical investigations of strength and stability of porous-cellular beams and plates. Analytical investigations of stability of the porous cylindrical shells were presented by Malinowski and Magnucki (2005), Magnucki at al. (2006b) for static problems, Belica and Magnucki (2007), Belica at al. (2011) for dynamic problems. These authors assumed the non-linear description of the deformation cross-section of the wall of the shell. The transverse shear deformation effect was taken into account. The shear effect does not occur on the external surfaces of the shell. The proposed model of deformation of a plane cross-section is included in the Higher Order Theory group, and it is the generalization of well-known theories. The higher order hypotheses including shear deformation of beams and plates were presented by Wang *et al.* (2000). That monograph illustrates how the shear deformation theories provide accurate solutions compared to the classical theory.

2. Theoretical model

The subject of the paper is an isotropic porous-cellular cylindrical shell subject to a combined load: axial force and external pressure (Fig. 1). Basic dimensions and relations are determined in the cylindrical coordinate system. The shell is simply supported on all outer edges.

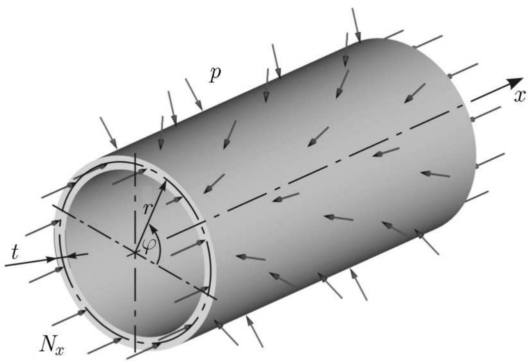

Fig. 1. Metal foam circular cylindrical shell

The shell is made of a porous metal with mechanical properties varying through the thickness of the shell. The external surfaces $(z = t/2$ and $z = -t/2)$ are made of a homogeneous material. The mechanical properties are symmetrical and continuous with regard to the middle surface of the shell (Fig. 2). Therefore, the defined porous material is a generalization of multi-layered structures.

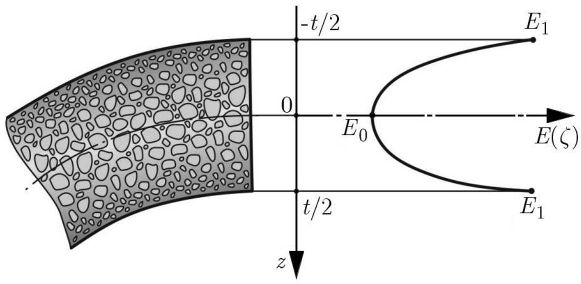

Fig. 2. Scheme of a porous-cellular shell structure

The porous metal is of continuous structure. For each layer of the ζ coordinate, the material is isotropic, while its mechanical properties vary on the thickness. The moduli of elasticity are defined in the following form

$$E(\zeta) = E_1[1 - e_0 \cos(\pi\zeta)] \qquad G(\zeta) = G_1[1 - e_0 \cos(\pi\zeta)] \tag{2.1}$$

where $e_0 = 1 - (E_0/E_1) = 1 - (G_0/G_1)$ is the dimensionless parameter of the porosity ($0 \leqslant e_0 < 1$); E_0, E_1 and G_0, G_1 are Young's and the shear moduli for $\zeta = 0$ and $\zeta = \pm 1/2$, respectively; $\zeta = z/t$ is the dimensionless coordinate; t is thickness of the shell. The relationship between the moduli of elasticity is defined in the following form $G_j = E_j/[2(1+\nu)]$, for $j = 0, 1$.

The physical model of a non-linear hypothesis of deformation of the shell plane cross-section is shown in Fig. 3. The cross-section, being initially a planar surface, becomes curved after the deformation. It is assumed that the boundaries of the curved surface of the deflected shell cross-section remain perpendicular to the outer surfaces of the shell. This geometric model is analogous to the broken line hypothesis applied to three layered structures.

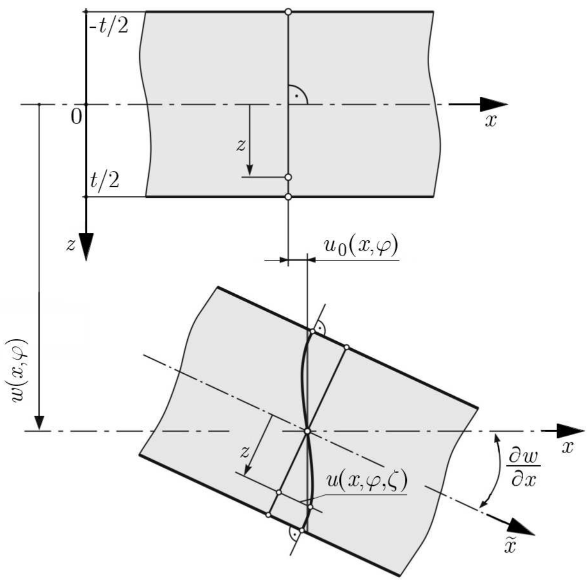

Fig. 3. Deformation of the plane cross section scheme

The field of displacement is assumed in the following form

$$
\begin{aligned}
u(x,\varphi,\zeta) &= u_0(x,\varphi) - t\Big[\zeta\frac{\partial w}{\partial x} - \frac{1}{\pi}\psi(x,\varphi)\sin(\pi\zeta)\Big]\\
v(x,\varphi,\zeta) &= v_0(x,\varphi) - t\Big[\zeta\frac{\partial w}{r\partial \varphi} - \frac{1}{\pi}\phi(x,\varphi)\sin(\pi\zeta)\Big]\\
w(x,\varphi,\zeta) &= w(x,\varphi,0) = w(x,\varphi)
\end{aligned}
\tag{2.2}
$$

where u_0, v_0 are the tangential displacements along the x and φ coordinates; ϕ and ψ are the dimensionless functions of displacements; w is the deflection of the shell (the transverse displacement along the z coordinate). The geometric relationships – components of the strain field – take the following forms

$$
\begin{aligned}
&\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\Big(\frac{\partial w}{\partial x}\Big)^2 \qquad && \varepsilon_\varphi = \frac{\partial v}{r\partial\varphi} - \frac{w}{r} + \frac{1}{2}\Big(\frac{\partial w}{r\partial\varphi}\Big)^2\\
&\gamma_{x\varphi} = \frac{\partial u}{r\partial\varphi} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{r\partial\varphi} \qquad && \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \qquad \gamma_{\varphi z} = \frac{\partial v}{\partial z} + \frac{\partial w}{r\partial\varphi}
\end{aligned}
\tag{2.3}
$$

The physical relationships, according to Hooke's law, are

$$
\begin{aligned}
&\sigma_x = \frac{E(\zeta)}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_\varphi) \qquad && \sigma_\varphi = \frac{E(\zeta)}{1-\nu^2}(\varepsilon_\varphi + \nu\varepsilon_x)\\
&\tau_{x\varphi} = G(\zeta)\gamma_{x\varphi} \qquad && \tau_{xz} = G(\zeta)\gamma_{xz} \qquad \tau_{\varphi z} = G(\zeta)\gamma_{\varphi z}
\end{aligned}
\tag{2.4}
$$

3. Equations of stability

The system of five partial differential equations obtained from the principle of stationarity of the total potential energy of the porous cylindrical shell under external pressure and intensity of the axial force

$$\delta(U_\varepsilon - W) = 0 \tag{3.1}$$

where U_ε is the potential energy of the elastic strain

$$U_\varepsilon = \frac{t}{2}\int_0^{2\pi}\int_0^L\int_{-1/2}^{1/2}(\sigma_x\varepsilon_x + \sigma_\varphi\varepsilon_\varphi + \tau_{x\varphi}\gamma_{x\varphi} + \tau_{xz}\gamma_{xz} + \tau_{\varphi z}\gamma_{\varphi z})r\;d\zeta\;dx\;d\varphi \tag{3.2}$$

W is the work of the load

$$W = \int_0^{2\pi}\int_0^L(pw)r\;dx\;d\varphi + \frac{1}{t}\int_0^{2\pi}\int_{-1/2}^{1/2}(N_xu)r\;d\zeta\;d\varphi \tag{3.3}$$

Introducing Eqs. (2.1)-(2.4) and Eqs. (3.2) and (3.3) into principle (3.1), a system of five equilibrium equations: δu_0 $(3.4)_1$, δv_0 $(3.4)_2$, $\delta\psi$ $(3.5)_1$, $\delta\varphi$ $(3.5)_2$, δw (3.6) are obtained

$$\begin{aligned}
&\frac{\partial^2 u_0}{\partial x^2} + \frac{1-\nu}{2}\frac{\partial^2 u_0}{r^2\partial\varphi^2} + \frac{1+\nu}{2}\Big(\frac{\partial w}{r\partial\varphi}\frac{\partial^2 w}{r\partial x\partial\varphi} + \frac{\partial^2 v_0}{r\partial x\partial\varphi}\Big) + \frac{\partial w}{\partial x}\Big(\frac{\partial^2 w}{\partial x^2} + \frac{1-\nu}{2}\frac{\partial^2 w}{r^2\partial\varphi^2} - \frac{\nu}{r}\Big) = 0\\
&\frac{\partial^2 v_0}{r^2\partial\varphi^2} + \frac{1-\nu}{2}\frac{\partial^2 v_0}{\partial x^2} + \frac{1+\nu}{2}\Big(\frac{\partial w}{\partial x}\frac{\partial^2 w}{r\partial x\partial\varphi} + \frac{\partial^2 u_0}{r\partial x\partial\varphi}\Big) + \frac{\partial w}{r\partial\varphi}\Big(\frac{\partial^2 w}{r^2\partial\varphi^2} + \frac{1-\nu}{2}\frac{\partial^2 w}{\partial x^2} - \frac{1}{r}\Big) = 0
\end{aligned} \tag{3.4}$$

and

$$\begin{aligned}
&C_2\frac{\partial}{\partial x}(\nabla^2 w) - C_3\Big(\frac{\partial^2\psi}{\partial x^2} + \frac{1-\nu}{2}\frac{\partial^2\psi}{r^2\partial\varphi^2} + \frac{1+\nu}{2}\frac{\partial^2\phi}{r\partial x\partial\varphi}\Big) + C_4\frac{1-\nu}{2t^2}\psi = 0\\
&C_2\frac{\partial}{r\partial\varphi}(\nabla^2 w) - C_3\Big(\frac{\partial^2\phi}{r^2\partial\varphi^2} + \frac{1-\nu}{2}\frac{\partial^2\phi}{\partial x^2} + \frac{1+\nu}{2}\frac{\partial^2\psi}{r\partial x\partial\varphi}\Big) + C_4\frac{1-\nu}{2t^2}\phi = 0
\end{aligned} \tag{3.5}$$

and

$$\begin{aligned}
&\frac{E_1 t}{1-\nu^2}\Big\{C_0\Big\{-\Big[\frac{1}{2}\Big(\frac{\partial w}{\partial x}\Big)^2 + \frac{\partial u_0}{\partial x}\Big]\Big(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{r^2\partial\varphi^2} + \frac{\nu}{r}\Big)\\
&\quad - \Big[\frac{1}{2}\Big(\frac{\partial w}{r\partial\varphi}\Big)^2 + \frac{\partial v_0}{r\partial\varphi} - \frac{w}{r}\Big]\Big(\nu\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{r^2\partial\varphi^2} + \frac{1}{r}\Big)\\
&\quad + (1-\nu)\frac{\partial^2 w}{r\partial x\partial\varphi}\Big(\frac{\partial w}{\partial x}\frac{\partial w}{r\partial\varphi} + \frac{\partial u_0}{r\partial\varphi} + \frac{\partial v_0}{\partial x}\Big)\Big\} + C_1 t^2(\nabla^4 w)\\
&\quad - C_2 t^2\Big[\frac{\partial}{\partial x}(\nabla^2\psi) + \frac{\partial}{r\partial\varphi}(\nabla^2\phi)\Big]\Big\} = p
\end{aligned} \tag{3.6}$$

where the Laplace operator is defined as $\nabla^2 = \partial^2/\partial x^2 + \partial^2/(r^2\partial\varphi^2)$, the bi-harmonic Laplace operator has the following form $\nabla^4 = \partial^4/\partial x^4 + 2\partial^4/(r^2\partial x^2\partial\varphi^2) + \partial^4/(r^4\partial\varphi^4)$ and constants are

$$C_0 = 1 - \frac{2}{\pi}e_0 \qquad C_1 = \frac{1}{12} - \frac{\pi^2-8}{2\pi^3}e_0 \qquad C_2 = \frac{1}{\pi^2}\Big(\frac{2}{\pi} - \frac{1}{4}5e_0\Big)$$
$$C_3 = \frac{1}{\pi^2}\Big(\frac{1}{2} - \frac{2}{3\pi}e_0\Big) \qquad C_4 = \frac{1}{2} - \frac{4}{3\pi}e_0$$

The loads in the directions x (N_x) and φ (N_φ) and the tangential load $S_{x\varphi}$ in the plane $x\varphi$ are defined. Now, inserting a stress function $F(x,\varphi)$ and a displacement function $\Phi(x,\varphi)$, we obtain

$$\psi = \frac{\partial \Phi}{\partial x} \qquad \phi = \frac{\partial \Phi}{r\partial \varphi} \tag{3.7}$$

The normal and tangential loads expressed with stress functions may be written as

$$N_x = \frac{\partial^2 F}{r^2 \partial \varphi^2} \qquad N_\varphi = \frac{\partial^2 F}{\partial x^2} \qquad S_{x\varphi} = -\frac{\partial^2 F}{r\partial x \partial \varphi} \tag{3.8}$$

The system of equations (3.4)-(3.6) is reduced to two differential equations thanks to introducing functions (3.7) and (3.8). When we use the equation of strain continuity, a system of three fundamental equations of stability is obtained

$$\begin{aligned} &\frac{E_1 t^3}{1-\nu^2}[C_1 \nabla^4(w) - C_2 \nabla^4(\Phi)] - L(w,F) - \frac{1}{r}\frac{\partial^2 F}{\partial x^2} - p = 0 \\ &\frac{1}{C_0 E_1 t}\nabla^4(F) = -\frac{1}{2}L(w,w) - \frac{1}{r}\frac{\partial^2 w}{\partial x^2} \qquad C_2 \nabla^4(w) - C_3 \nabla^4(\Phi) + C_4 \frac{1-\nu}{2t^2}\nabla^2(\Phi) = 0 \end{aligned} \tag{3.9}$$

where the non-linear operators are defined as

$$\begin{aligned} L(w,F) &= \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 F}{r^2 \partial \varphi^2} - 2\frac{\partial^2 w}{r\partial x \partial \varphi}\frac{\partial^2 F}{r\partial x \partial \varphi} + \frac{\partial^2 w}{r^2 \partial \varphi^2}\frac{\partial^2 F}{\partial x^2} \\ L(w,w) &= 2\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{r^2 \partial \varphi^2} - 2\Big(\frac{\partial^2 w}{r\partial x \partial \varphi}\Big)^2 \end{aligned}$$

The boundary conditions for x = 0 and x = L are formulated in the form

$$\frac{\partial^2 F}{r^2 \partial \varphi^2}\Big|_{x=0;\, x=L} = N_x^0 \qquad M_x\Big|_{x=0;\, x=L} = 0 \qquad w\Big|_{x=0;\, x=L} = 0 \tag{3.10}$$

where

$$M_x = t\int\limits_{-1/2}^{1/2} \zeta \sigma_x \, d\zeta = -\frac{E_1 t^3}{1-\nu^2}\Big[C_1\Big(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{r^2 \partial \varphi^2}\Big) - C_2\Big(\frac{\partial \psi}{\partial x} + \nu \frac{\partial \phi}{r\partial \varphi}\Big)\Big]$$

The system of three equations (3.9) is approximately solved. The two unknown functions are assumed in the following forms

$$\begin{aligned} w(x,\varphi) &= w_1 \sin X \cos Y + 2w_2 \sin^2 X \\ \Phi(x,\varphi) &= w_1 \alpha_{\Phi 1} \sin X \cos Y + 2w_2 \alpha_{\Phi 2} \sin^2 X \end{aligned} \tag{3.11}$$

where: $X = m\pi x/L$ and $Y = n\varphi$, n is the number of waves on the circuit, m is the number of half-waves in the longitudinal direction of the shell, w_1 and w_2 are the amplitude parameters of the deflection surface in form of waves occurring along axial and circumferential directions of the shell, respectively. These functions satisfy boundary conditions (3.10) for the simply supported shell.

Substitution of the equations (3.11) for Eq. $(3.9)_2$ provides the stress function

$$\begin{aligned} F = C_0 E_1 t[&w_1^2(\alpha_{f1}\cos 2X - \alpha_{f2}\cos 2Y) + w_1 w_2(\alpha_{f3}\sin 3X \cos Y - \alpha_{f4}\sin X \cos Y) \\ &- w_2 r \alpha_{f5}\cos 2X + w_1 r \alpha_{f6} \sin X \cos Y] - \frac{1}{2}(N_x^0 r^2 \varphi^2 + N_\varphi^0 x^2) \end{aligned} \tag{3.12}$$

where

$$\alpha_{f1} = \frac{k_1^2}{32} \qquad \alpha_{f2} = \frac{1}{32k_1^2} \qquad \alpha_{f3} = \frac{2k_1^2}{(9+k_1^2)^2} \qquad \alpha_{f4} = \frac{2k_1^2}{(1+k_1^2)^2}$$
$$\alpha_{f5} = \Big(\frac{k_1}{2n}\Big)^2 \qquad \alpha_{f6} = \frac{\alpha_{f4}}{2n^2} \qquad k_1 = \frac{nL}{m\pi r}$$

Equations $(3.9)_1$ and $(3.9)_3$ are solved with the use of the orthogonalization Bubnov-Galerkin (B-G) method

$$\int\limits_0^{2\pi}\int\limits_0^{L} \Re(x,\varphi)\sin X\cos Yr\,dx\,d\varphi = 0 \qquad \int\limits_0^{2\pi}\int\limits_0^{L} \Re(x,\varphi)\sin^2 Xr\,dx\,d\varphi = 0 \tag{3.13}$$

where $\Re(x,\varphi)$ stands for the left side of Eqs. $(3.9)_1$ and $(3.9)_3$.

The following parameters are obtained with equation $(3.9)_3$

$$\alpha_{\Phi1} = \frac{C_2}{C_3+k_2C_4} \qquad \alpha_{\Phi2} = \frac{C_2}{C_3+k_3C_4} \qquad k_2 = \frac{4k_3}{1+k_1^2} \qquad k_3 = \frac{1-\nu}{8}\Big(\frac{L}{m\pi t}\Big)^2 \tag{3.14}$$

Substitution of expressions (3.11), (3.12) and (3.14) for Eq. $(3.9)_1$, and using of the B-G method, gives a system of equations

$$\begin{aligned} &\frac{1}{E_1t}(N_x+N_\varphi k_1^2) - \alpha_{w1} - \alpha_{w3}\widetilde{w}_1^2 + \alpha_{w5}\widetilde{w}_2 - \alpha_{w4}\widetilde{w}_2^2 = 0 \\ &8\frac{N_x}{E_1t}\widetilde{w}_2 - 8\alpha_{w2}\widetilde{w}_2 - \alpha_{w4}\widetilde{w}_1^2\widetilde{w}_2 + \alpha_{w6}\widetilde{w}_1^2 = 0 \end{aligned} \tag{3.15}$$

From equation $(3.15)_2$, the parameter is designated $\widetilde{w}_2$ and substituted to equation $(3.15)_1$. Finally, a non-linear algebraic equation enabling the analysis of stability loss of the investigated shell is given

$$N_0 = \frac{E_1t}{k_0+k_1^2(1-k_0)}[\alpha_{w1} + (\alpha_{w3} - \alpha_{w5}\alpha_{w7})\widetilde{w}_1^2 + \alpha_{w4}\alpha_{w7}^2\widetilde{w}_1^4] \tag{3.16}$$

where $\widetilde{w}_1 = w_1/t$ and $\widetilde{w}_2 = w_2/t$ are dimensionless parameters of the deflection; N_0 [N/mm] is the external load; k_0 is the dimensionless parameter of the load, $0 \leqslant k_0 \leqslant 1$; N_φ [N/mm] is the intensity of the circumferential load, $N_\varphi = (1-k_0)N_0 = pr$; N_x [N/mm] is the intensity of the axial load, $N_x = k_0N_0$; p [MPa] is the external pressure; and the dimensionless parameters are

$$\begin{aligned} \alpha_{w1} &= C_0\alpha_{f6} + \frac{t^2}{1-\nu^2}\Big(\frac{m\pi}{L}\Big)^2(1+k_1^2)^2(C_1 - C_2\alpha_{\Phi1}) \\ \alpha_{w_2} &= \frac{4}{1-\nu^2}\Big(\frac{m\pi t}{L}\Big)^2(C_1 - C_2\alpha_{\Phi_2}) + C_0\alpha_{f_5} \\ \alpha_{w3} &= 2C_0\Big(\frac{nt}{r}\Big)^2(\alpha_{f1}+\alpha_{f2}) \qquad \alpha_{w4} = 2C_0\Big(\frac{nt}{r}\Big)^2(\alpha_{f3}+\alpha_{f4}) \\ \alpha_{w5} &= \frac{1}{2}C_0\frac{t}{r}(k_1^2 + 4\alpha_{f4}) \qquad \alpha_{w6} = C_0\frac{t}{r}(8\alpha_{f1}+\alpha_{f4}) \\ \alpha_{w7} &= \frac{\alpha_{w6}}{8\alpha_{w2} + \alpha_{w4}\widetilde{w}_1^2 - 8\frac{N_0k_0}{E_1t}} \end{aligned}$$

The static upper critical load of the shell has been obtained after the removal of geometric non-linear relationships (components of the strain) in Eqs. (3.16)

$$N_{0,cr} = \min_{m,n}\Big\{\frac{E_1}{k_0+k_1^2(1-k_0)}\Big[C_0\alpha_{f6} + \frac{t^2}{1-\nu^2}\Big(\frac{m\pi}{L}\Big)^2(1+k_1^2)^2(C_1 - C_2\alpha_{\Phi1})\Big]\Big\} \tag{3.17}$$

4. Numerical calculations

Equation (3.16) has been solved by a numerical-iterative method. It has been assumed that the maximum mistake equaled 0.01%. In the first iteration, the worth of parameter α_{w7} is assumed without the element $8N_0k_0/(E_1t)$.

The condition of allowable stress is assumed $\sigma_{eq,max} \leqslant \sigma_{all}$, where σ_{all} – allowable stress for the shell material, $\sigma_{eq,max}$ – maximum equivalent stress

$$\sigma_{eq,max} = \frac{N_{0,cr}}{C_0 t}\sqrt{1-3k_0(1-k_0)} \tag{4.1}$$

The detailed numerical analysis of the porous shell is carried out for the following data: $E_1 = 7.06 \cdot 10^4$ MPa, $\nu = 0.33$, $\sigma_{all} = 90$ MPa, $L/r = 3$, $r/t = 200$ (600).

The shells have been investigated for different coefficients of porosity for different loads: external pressure, axial load and combinations of these loads. The results of these calculations are presented in Table 1 and in the Figs. 4-7. In Table 1, the upper and lower critical loads of the shells are presented. Defining the lower critical loads, $N_{0,low}$ assumed the lowest value N_0 for the shell for different numbers m and n.

Table 1. The upper and lower critical loads of the studied shells

e_0	$N_{0,cr}/t$ [MPa]	m	n	$N_{0,low}/t$ [MPa]	m	n	$N_{0,low}/N_{0,cr}$
External pressure $N_\varphi = N_0$; $(r/t = 200)$							
0	8.02	1	6	5.80	1	5	0.72
0.5	6.33	1	6	4.57	1	5	0.72
0.9	4.98	1	6	3.59	1	5	0.72
Combined loads $N_x = 0.9N_0$; $N_\varphi = 0.1N_0$; $(r/t = 200)$							
0	62.96	1	6	37.23	1	5	0.59
0.5	49.72	1	6	26.45	1	4	0.53
0.9	37.00	1	5	19.16	1	4	0.52
Axial load $N_x = N_0$; $(r/t = 600)$							
0	71.96	4	13	13.74	5	10	0.19
0.5	53.8	3	11	10.43	5	10	0.19
0.9	38.66	4	12	7.68	5	9	0.20

In Figs. 4-6, the equilibrium paths of the cylindrical shells for the following geometric data is presented: $L/r = 3$ and $r/t = 200$ for the shell subjected to the external pressure (Fig. 4) and combined loads (Fig. 5); $L/r = 3$ and $r/t = 600$ for the shell subjected to the axial load (Fig. 6).

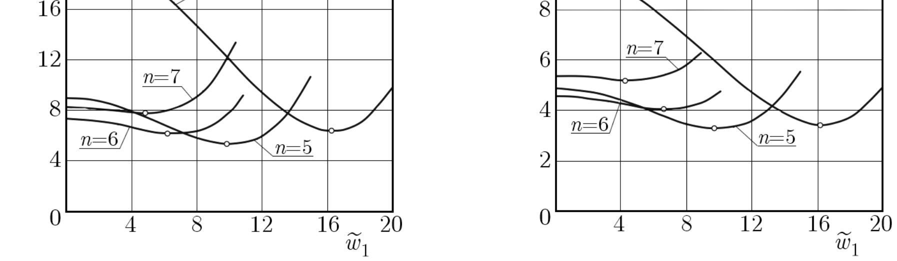

Fig. 4. Equilibrium paths of the cylindrical shells subjected to external pressure

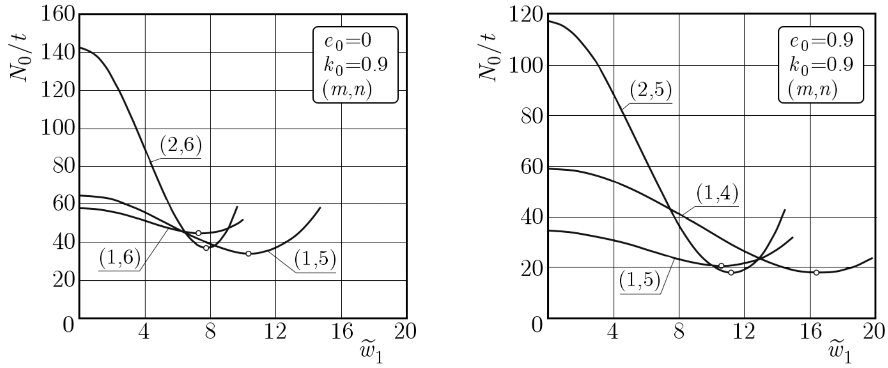

Fig. 5. Equilibrium paths of the cylindrical shells subjected to combined loads

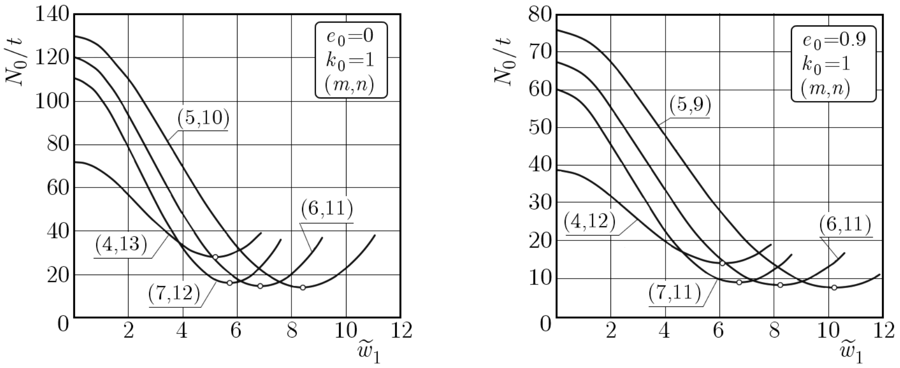

Fig. 6. Equilibrium paths of the cylindrical shells subjected to the axial load

The calculation results are presented for different values of the parameters m, n. The influence of the load parameter k_0 on the relationship between the upper and lower critical loads is shown in Fig. 7.

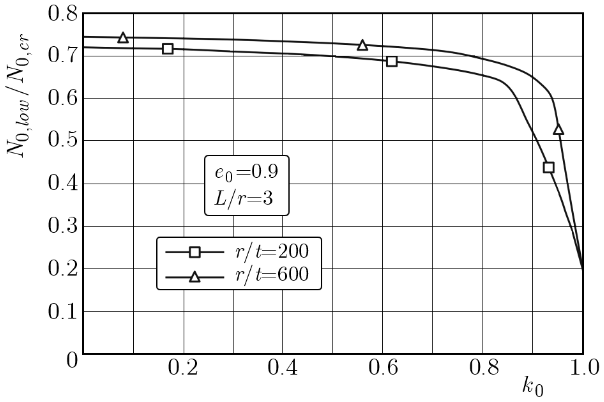

Fig. 7. Upper and lower critical loads depending on the parameter k_0

5. Conclusions

The problem of the stability of isotropic circular cylindrical shells made of a porous-cellular material, subjected to combined loads is presented in the paper. On the basis of the non--linear description of the deformation cross-section of the wall of the shell, relations between the parameters of deflection and load considering the shear effect have been obtained. The condition of free external surfaces of the shell is satisfied simultaneously. The introduced equations have the universal character in the range of:

- the change of physical properties of the material (parameter e_0); for $e_0 = 0$, the structure is homogeneous, and for $0 < e_0 < 1$, it is a porous-cellular one;
- the combination of the external load (parameter k_0); if $k_0 = 0$, then only the external pressure works, if $k_0 = 1$, then there is an axial load, and if $0 < k_0 < 1$, then there is a combined external load.

The solution of static stability problem of porous cylindrical shells were presented by Malinowski and Magnucki (2005), Magnucki at al. (2006b). In those works, only geometrically linear relations between the displacement and deformation were assumed. On the basis of these foundations, one can calculate the upper critical loads. A more difficult problem is the non-linear stability analysis of the shells, presented in this work. This analysis allows the study of dependence between the external loads and deflection of the shell as well as the designation of the lower critical loads. This is particularly important since, in most cases, the real value of the load causing buckling of the shell is in the range between the upper and lower theoretical value of the critical load. This problem confirmed the results of experimental and theoretical study conducted by many researchers due to small imperfections that every real shell (deviation from the ideal geometric shape of shell, non-homogenous of shell material, irregular load distribution, etc.) has. It is worth noting that larger imperfections of the shell cause reduction of the real value of the load at which the shell is buckled (even below the lower critical load). The exact consideration of various factors in theoretical studies of shells is not possible (only approximate solutions exist). The influence of imperfections on the buckling of porous-cellular cylindrical shells will be the subject of the further research.

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