JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
51, 1, pp. 225-234, Warsaw 2013

CONTROL OF CHARACTERISTICS OF MECHATRONIC SYSTEMS USING PIEZOELECTRIC MATERIALS

Andrzej Buchacz, Marek Płaczek, Andrzej Wróbel
Silesian University of Technology, Faculty of Mechanical Engineering, Gliwice, Poland
e-mail: andrzej.buchacz @polsl.pl; marek.placzek@polsl.pl; andrzej.wrobel@polsl.pl

This paper presents two examples of systems with piezoelectric transducers used in order to control vibration. First of them is a combination of a single piezoelectric plate glued on a mechanical subsystem surface. The piezoelectric plate is applied to suppress vibration of the system. The second model consists of a few piezoelectric plates vibrating in a thickness mode. Such kind of systems is called the piezostack and is used to enhance the transducers sensitivity or to increase the displacement. In both cases, characteristics of the considered systems were generated and presented on charts.

Key words: dynamics, vibration control, piezoelectricity, approximate methods

1. Control of vibrations with piezoelectric transducers

The piezoelectric effect discovered in 1880 by Pierre and Jacques Curie has found a lot of applications in modern technical devices. Nowadays, there are a lot of applications of both – the direct and reverse piezoelectric effects. The phenomenon of production of an electric potential when stress is applied to the piezoelectric material is called the direct piezoelectric effect. While the phenomenon of production of strain when an electric field is applied is the reverse piezoelectric effect (Moheimani and Fleming, 2006). The direct piezoelectric effect application in passive vibration damping is being considered in this section.

1.1. Idea of passive damping of vibration

In the nineteen eighties, a new idea for piezoelectric transducers application – passive damping of vibrations – was developed. It is the possibility of dissipating mechanical energy with piezoelectric transducers shunted with passive electrical circuits. A piezoelectric transducer is bonded to some mechanical subsystem (for example a beam or a shaft) and external electric circuit is adjoined to the transducer clamps. This idea was described and experimentally investigated by Forward (1981), and the first analytical models for such a kind of systems were developed by Hagood and von Flotow (1991, 1994). There are many commercial applications of this idea, such as products of ACX (Active Control eXperts inc.) – skis, snowboards or baseball bats with piezoelectric dampers (Yoshikawa *et al.*, 1998).

Two cases of passive damping of vibrations with piezoelectric transducers were described. In the first case, only a resistor is used as a shunting circuit, and in the second case, it is a circuit composed of a resistor and an inductor. Since this research, many authors have worked to improve this idea. For example Fleming *et al.* (2002) described multimode piezoelectric shunt damping systems.

This work presents analysis of a mechatronic system with piezoelectric shunt damping of vibration using a single PZT piezoelectric transducer with a resistor used as the shunting circuit. The piezoelectric transducer is used as a passive vibration damper. The resistor is adjoined to the transducer clamps. The dynamic characteristic of this system was assigned on the basis of the approximate Galerkin method. Dynamic flexibilities of the considered mechatronic system and mechanical subsystem without piezoelectric transducer were juxtaposed. The influence of geometrical and material properties of the considered system on its dynamic flexibility was determined.

The correct description of a designed system by its mathematical model during the design phase is a fundamental condition for its proper operation. A very precise mathematical model and method of the analysis of the system is very important. It is indispensable to take into consideration all geometrical and material parameters of system components (including the glue layer between these elements) because the omission of the influence of one of them results in inaccuracy in the analysis of the system (Pietrzakowski, 2001; Buchacz and Płaczek, 2010a). Therefore, the processes of modelling and testing of one-dimensional vibrating mechatronic systems are presented.

It is a part of work that was done at Gliwice research centre, where tasks of analysis and synthesis of mechatronic systems with piezoelectric transducers were considered. Tasks of analysis and synthesis were also considered and presented in previous publications, see Dzitkowski and Dymarek (2005, 2011), Zolkiewski (2010, 2011). For example, passive and active mechanical and mechatronic systems were analyzed by Białas (2008, 2009) and investigations supported by computer-aided methods by Buchacz and Wróbel (2010) and Dzitkowski (2004).

1.2. Considered system and its mathematical model

The considered system (presented in Fig. 1) consists of a cantilever beam which has a rectangular constant cross-section, length l and Young's modulus E. The piezoelectric transducer of length l_p and thickness h_p is bonded to the top surface of the beam by a glue layer of thickness h_k and Kirchhoff's modulus G. The glue layer has homogeneous properties in overall length. The external shunting circuit is adjoined to the transducer clamps. The system is loaded with a harmonic force $F(t)$, described by the equation

$$F(t) = F_0 \cos(\omega t) \tag{1.1}$$

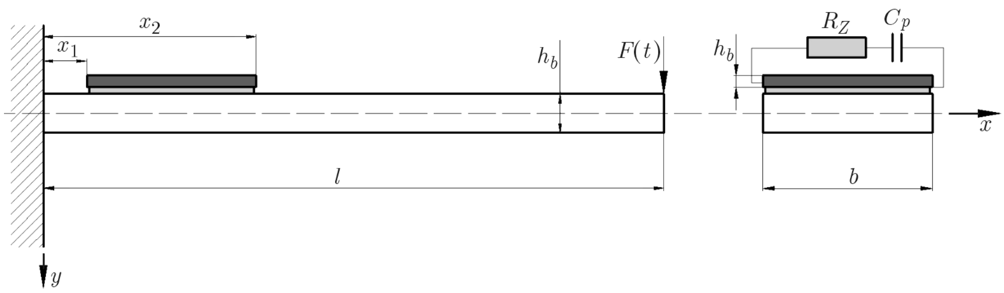

Fig. 1. The considered system with the external shunting circuit

In accordance with the idea of passive damping of vibrations with piezoelectric materials, the mechanical system affects the piezoelectric transducer which generates electric charge and produces additional stiffness of electromechanical nature, dependent on the capacitance of the transducer and the adjoined external circuit. This shunting circuit introduces electric dissipation resulting in electronic damping of vibration (Kurnik, 2004).

The piezoelectric transducer with the external shunting circuit was treated as a linear RC circuit as it is shown in Fig. 2, where R_Z is the external electric circuit resistance and C_p is the capacitance of the piezoelectric transducer (Buchacz and Płaczek, 2010b, 2011a,b).

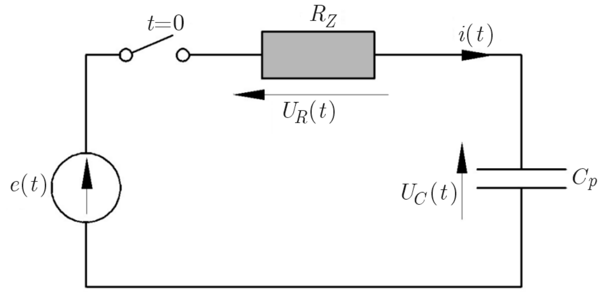

Fig. 2. Electrical representation of the piezoelectric transducer with the external shunting circuit

The equation of flexural vibration of the beam was assigned in accordance with Bernoulli-Euler's model of the beam, and the following assumptions were made:

- the material of which the system is made is subjected to Hooke's law,
- the system has a continuous, linear mass distribution,
- the system vibration is harmonic,
- planes of sections that are perpendicular to the axis of the beam remain flat during deformation of the beam,
- displacements are small compared with the dimensions of the system.

Displacements of the beam cross sections were described by the coordinate $y(x,t)$.

The dynamic equation of motion of the beam was assigned on the basis of elementary beam and transducer section dynamic equilibrium in agreement with d'Alembert's principle with the assumption on the uniaxial, homogeneous strain of the transducer and pure shear of the glue layer. Heaviside's function $H(x)$ was introduced to curb the working space of the transducer within the range from x_1 to x_2 (Buchacz and Płaczek, 2011a,b). The obtained equation of beam motion can be described as

$$\frac{\partial^2 y(x,t)}{\partial t^2} = -a^4 \frac{\partial^4 y(x,t)}{\partial x^4} + c_2 \frac{\partial}{\partial x}[\varepsilon_b(x,t)H - \varepsilon_k(x,t)H + \lambda_1(t)H] + \frac{\delta(x-l)}{\rho_b A_b} F(t) \tag{1.2}$$

where

$$a = \sqrt[4]{\frac{E_b J_b}{\rho_b A_b}} \qquad c_2 = \frac{G l_p}{2\rho_b h_k} \qquad H = H(x-x_1) - H(x-x_2) \qquad \lambda_1(t) = d_{31}\frac{U_C(t)}{h_p} \tag{1.3}$$

The symbols ε_b and ε_k are strains of the beam and glue layer surfaces, d_{31} is the piezoelectric constant and $U_C(t)$ is the electric voltage on the capacitance C_p of the piezoelectric transducer. The distribution of the external force was determined using Dirac's delta function $\delta(x-l)$.

Taking into account constitutive equations which include the relationship between mechanical and electrical properties of transducers (Preumont, 2006; Moheimani and Fleming, 2006), the equation of the piezoelectric transducer with the external electric circuit can be described as

$$R_Z C_p \frac{\partial U_C(t)}{\partial t} + U_C(t) = \frac{l_p b d_{31}}{C_p s_{11}^E} S_1(x,t) + l_p b \varepsilon_{33}^T \frac{U_C(t)}{C_p h_p}(1-k_{31}^2) \tag{1.4}$$

where

$$k_{31}^2 = \frac{d_{31}^2}{s_{11}^E \varepsilon_{33}^T} \tag{1.5}$$

is the electromechanical coupling constant of the transducer and $S_1(x,t)$ is the strain of the transducer resulting from deflection of the beam. The symbols ${}^{E}_{s11}$ and ε^{T}_{33} are respectively Young's modulus of the transducers under zero/constant electric field and dielectric constant under zero/constant stress. The electric voltage generated by the transducer as a result of its strain was assumed as

$$U_C(t) = \frac{|U_p|}{\omega C_p|Z|}\sin(\omega t + \varphi) \tag{1.6}$$

where $|Z|$ and φ are absolute value and argument of the serial circuit impedance. U_p is the electric voltage generated as a result of the strain of the transducer. Equations (1.2) and (1.4) create the mathematical model of the system.

Dynamic flexibilities of the considered mechatronic system and its mechanical subsystem were assigned on the basis of approximate Galerkin's method. The solution to the beam differential motion equation was defined as a product of time and displacement eigenfunctions (Buchacz and Płaczek, 2010c)

$$y(x,t) = A\sum_{n=1}^{\infty}\sin\Big[(2n-1)\frac{\pi}{2l}x\Big]\cos(\omega t) \tag{1.7}$$

This equation meets only two boundary conditions – displacements of the clamped and free ends of the beam, but the approximate method was verified – dynamic flexibility of the mechanical subsystem (the beam without the piezoelectric transducer) was assigned using exact Fourier's and approximate methods and the obtained results were juxtaposed. Inexactness of the approximate method was reduced by the correction coefficients introduced for the first three values of periodicity of the system (Buchacz and Płaczek, 2011a,b). It was proved that after correction this method can be used to analyse such a kind of systems, and the obtained results are very precise. Derivatives of the assumed solution were substituted into the mathematical model of the system. After transformations, the dynamic flexibility was calculated, and taking into account parameters of the system it was presented on charts.

1.3. Parameters of the system and obtained results

Parameters of the system are presented in Table 1. The dynamic flexibility of the mechatronic system, assigned on the free end of the beam $(x = l)$, separately for the first three natural frequencies $(n = 1, 2, 3)$ is presented in Fig. 3. It is juxtaposed with the dynamic flexibility of the mechanical subsystem.

Table 1. Parameters of the mechatronic system

$l = 0.24\,\mathrm{m}$	$E_b = 210000\,\mathrm{MPa}$
$b = 0.04\,\mathrm{m}$	$\rho_b = 7850\,\mathrm{kgm^{-3}}$
$h_b = 0.002\,\mathrm{m}$	$d_{31} = -240\cdot 10^{-12}\,\mathrm{mV^{-1}}$
$x_1 = 0.01\,\mathrm{m}$	$\varepsilon^{T}_{33} = 2.65\cdot 10^{-9}\,\mathrm{Fm^{-1}}$
$x_2 = 0.06\,\mathrm{m}$	$s^{E}_{11} = 63000\,\mathrm{MPa}$
$h_p = 0.0005\,\mathrm{m}$	$R_Z = 6000\,\Omega$
$h_k = 0.0001\,\mathrm{m}$	$G = 1000\,\mathrm{MPa}$

Using the developed mathematical model of the considered system and the proposed method of analysis of the system, it is possible to analyse the influence of the system parameters on its characteristic. Effects of some selected parameters on the dynamic flexibility are presented in Figs. 4 and 5.

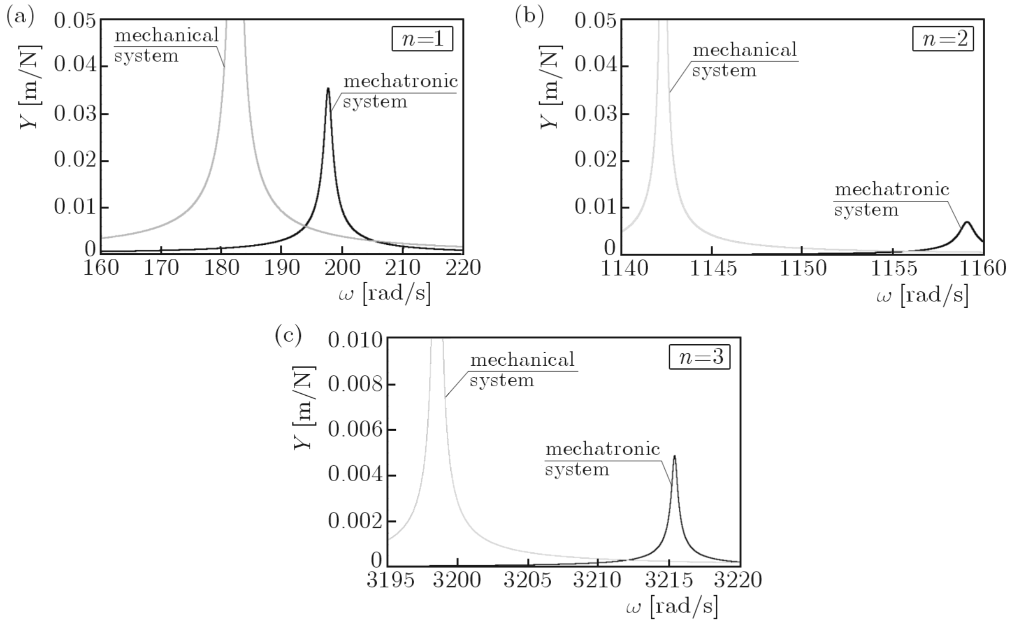

Fig. 3. Dynamic flexibility, (a) for the first natural frequency, (b) for the second natural frequency, (c) for the third natural frequency

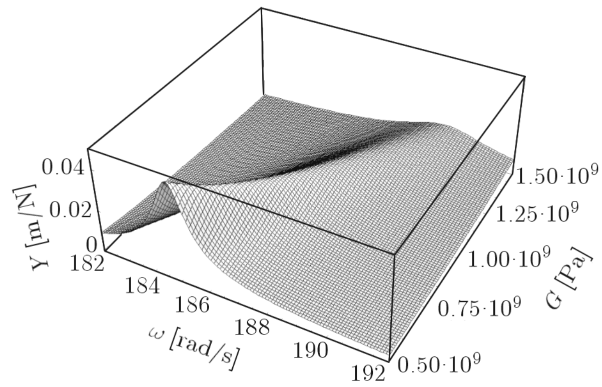

Fig. 4. Influence of modulus of shear elasticity of the glue layer, for $n = 1$

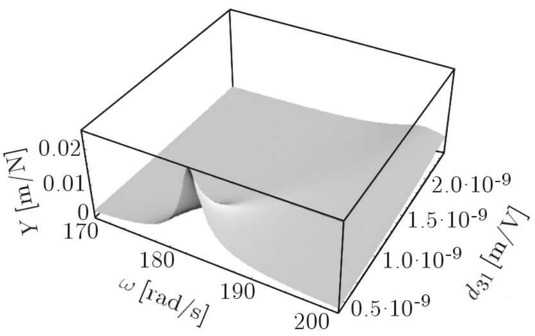

Fig. 5. Influence of the piezoelectric constant d_{31} of the transducer material, for $n = 1$

The presented mathematical model of the considered system is one of a series of models developed and presented in other publications. The developed models have different degrees of precision of the real system representation because different simplifications were introduced. The aim of this study was to develop a series of mathematical models of the considered system, to verify them and to indicate an adequate model to accurately describe the phenomena occurring in the system and effectively simplify mathematical calculations with reducing the required time for them (Buchacz and Płaczek, 2010a).

2. Vibration control of the piezoelectric stack

In the construction and operation of mamy machatronic subassemblies, the piezoelectric phenomenon has been utilised for a long time. For testing of machine elements, piezoelectric sensors are glued on their surfaces, whereas for health monitoring, transducers made of piezoelectric films are used. Piezoelectrics are often used in machine building in form of assemblies, subassemblies or actuators. In some cases, the parameters of simple piezoelectric plates are insufficient; sometimes we want to have a greater sensitivity for transducers, or expect more displacement for a piezoelectric actuator. In such cases, single piezoelectric plates are replaced by piezoelectric stacks. The piezoelectric stack is a composition of few thin piezoelectric layers, which are connected in parallel. This kind of monolithic ceramic construction is characterized by:

- good conversion of energy,
- low voltage control,
- high force,
- rapid response of the system.

The view of piezoelectric stacks with electrical connectors was shown in Fig. 6. The stacks may have different shapes and dimensions – it depends on the kind of characteristics we want to get. Piezoelectric stacks are used if bigger displacement or bigger force on the piezoelectric surface is required.

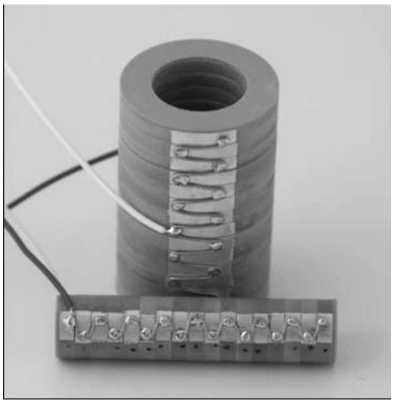

Fig. 6. A stack of piezoelectric plates

Analyzing the reverse piezoelectric phenomenon, in other words, the generation of charge on the piezoelectric linings, it is worth noting that in the construction of piezoelectric transducers the longitudinal phenomenon is most commonly used. Plates used for building of such a converter are usually in the form of plates of rolls and connected in parallel. Through such connection, an increase in the obtained charge is obtained, and thereby an increase in the sensitivity of the transducer. Proper implementation of the piezoelectric system which acts as a sensor or actuator depends on selection of geometric dimensions of the plate, or a group of plates, their numbers and their basic physical parameters.

Figures 7, 8 and 9 show the influence of various parameters such as thickness of the plate, density of the material of which the plate is made on the characteristics of the system (Buchacz and Wróbel, 2007; Wróbel, 2012).

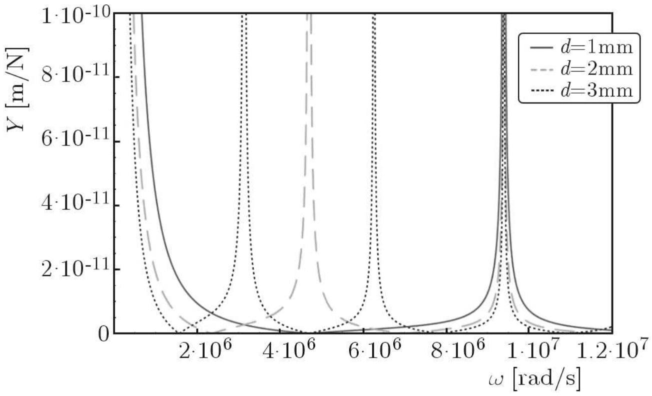

Fig. 7. Characteristic of a piezoelectric plate in the frequency domain depending on thickness of the plate

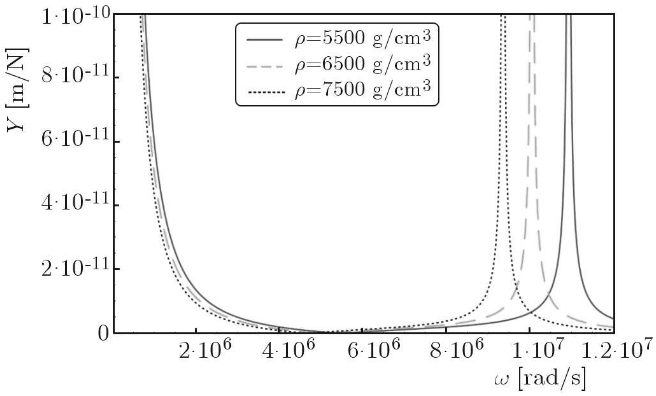

Fig. 8. Characteristic of a piezoelectric plate in the frequency domain depending on piezoelectric density

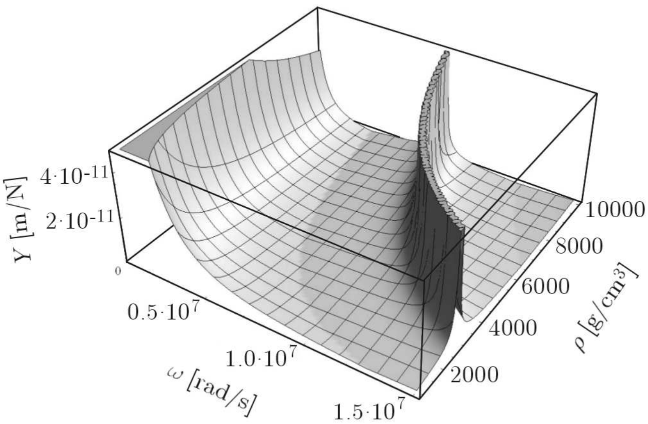

Fig. 9. 3D characteristic of the piezoelectric plate in the frequency domain in function of material density of the plate

Up to now, the proper selection has been realised by checking whether the results measure up the expectations through applying another, electrically connected, piezoelectric layer in order to increase the transducer sensitivity until achieving a satisfactory result. This section presents the possibility of controlling the characteristics of the piezoelectric stack consisting of PZT plates. The control in this case is done by changing the supply frequency, geometrical and material parameters.

Table 2. Parameters used in numerical simulations

Number	Symbol	Value	Unit
1	A	3	[cm^2]
2	d	3	[mm]
3	c_{33}^E	5.8	[GPa]
4	ρ	7500	[g/cm^3]

All characteristics in Figs. 7 to 9 shown displacements in the thickness mode. Infinite amplitudes at the resonances are due to disregarding damping in the material. In the current researches by the authors, this problem is under consideration and will be published in the future work.

Analysing a system consisting of only two piezoelectric plates and assuming that the two subsystems are characterized by identical physicochemical and geometric parameters, and assuming the same thickness of individual plates $d_1^{(i)} = 0.5d$ and $d_2^{(i+1)} = 0.5d$, the same characteristics as in the case of a single piezoelectric plate with thickness $d_1 + d_2$ were obtained.

3. Conclusions

The discussed subjet is important due to the increasing number of applications of both the simple and reverse piezoelectric phenomena in various modern technical devices. The process of modelling of technical devices with piezoelectric materials is complex and requires large amounts of time because of complexity of the phenomena occurring in such systems. The correct description of a given system by its mathematical model during the design phase is the fundamental condition for proper operation of the designed system. A mathematical model that provides the most accurate analysis of the system together with maximum simplification of used mathematical tools and minimum required amount of time should be incorporated. The identification of the optimal mathematical model that meets the assumed criteria is one of the purposes of research works realized at Gliwice research centre. It is an introduction to the task of synthesis of one-dimensional vibrating complex mechatronic systems. Tasks of analysis and synthesis of mechanical and mechatronic systems were considered at Gliwice centre in previous works (Buchacz and Wróbel, 2008, 2010). Passive and active mechanical systems and mechatronic systems with piezoelectric transducers were analyzed by Buchacz and Płaczek (2011). The examinations were also supported by computer-aided methods (Buchacz and Wróbel, 2010).

Acknowledgements

This work was partially supported by Polish Ministry of Science and Higher Education – National Science Centre as a part of the research project No. N501 064440 (2011-2013).

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Kontrola charakterystyk układów mechatronicznych przy użyciu materiałów piezoelektrycznych

Streszczenie

W artykule przedstawiono dwa przykłady układów mechatronicznych zawierających przetworniki piezoelektryczne stosowane w celu kontroli i stabilizacji drgań. W pierwszym układzie pojedyncza płytka piezoelektryczna naklejana jest na powierzchni podukładu mechanicznego. Do przetwornika piezoelektrycznego dołączony jest zewnętrzny, pasywny obwód elektryczny, przez co pełni on rolę szerokopasmowego, pasywnego tłumika drgań. Drugi z rozpatrywanych układów złożony jest ze stosu płytek piezoelektrycznych drgających grubościowo. Tego typu układy stosowane są w celu zwiększenia czułości przetworników stosowanych jako sensory lub zwiększenia odkształceń układu w przypadku stosowania go w roli wzbudnika. W przypadku obu omawianych układów drgających zaproponowano metodę ich analizy oraz wyznaczono charakterystyki, których przebiegi przedstawiono w formie graficznej.

Manuscript received June 6, 2011; accepted for print June 23, 2012