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THERMAL INSTABILITY OF A HETEROGENEOUS OLDROYDIAN VISCOELASTIC FLUID HEATED FROM BELOW IN POROUS MEDIUM

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The thermal instability of an Oldroydian heterogeneous viscoelastic fluid in a porous medium is considered. Following the linearized stability theory and normal mode analysis, the dispersion relation is obtained. For stationary convection, the medium permeability and density distribution are found to have a destabilizing effect. The dispersion relation is also analyzed numerically. Sufficient conditions for non-existence of overstability are also obtained.

Key words: thermal instability, heterogeneous Oldroydian viscoelastic fluid, porous medium, linear stability theory

1. Introduction

The problem of thermal instability in a horizontal layer of a fluid was discussed in detail by Chandrasekhar (1981). Bhatia and Steiner (1972) studied the thermal instability of a Maxwell fluid in the presence of rotation and found that the rotation has a destabilizing influence for a certain numerical range in contrast to the stabilizing effect on the Newtonian fluid. Eltayeb (1975) considered the convective instability in a rapidly rotating Oldroydian fluid. Toms and Strawbridge (1953) demonstrated experimentally that a dilute solution of methyl methacrylate in n-butyl acetate behaves in accordance with the theoretical model of the Oldroyd fluid. Hamabata and Namikawa (1983)studied the propagation of thermoconvective waves in the Oldroyd fluid. Mohapatra and Misra (1984) considered the thermal instability of a heterogeneous rotating fluid layer with free boundaries. The thermal instability of a conducting, viscous, heterogeneous and incompressible horizontal fluid layer confined between free boundaries in the presence of a uniform magnetic field and uniform rotation were considered by Sengar and Singh (1989).

The medium was considered to be non-porous in all the above studies. Lapwood (1948) studied the stability of heat convective flow in hydrodynamics in a porous medium using Rayleigh's procedure. Wooding (1960) considered the Rayleigh instability of a thermal boundary layer in flow through porous a medium. The gross effect, when the fluid slowly percolates through pores of a rock is represented by the well known Darcy's law. Generally, it is accepted that comets consist of a dusty "snowball" of a mixture of frozen gases which, in the process of their journey, changes from solid to gas and vice-versa. The physical properties of comets, meteorites and interplanetary dust strongly suggests the importance of porosity in the astrophysical context (McDonnel, 1978).

Sharma and Sharma (1977) considered the thermal instability of a rotating Maxwell fluid through a porous medium and found that, for stationary convection, the rotation has a stabilizing effect, whereas the permeability of the medium has both stabilizing as well as destabilizing effect, depending on the magnitude of rotation. In another study, Sharma (1975) studied the stability of a layer of an electrically conducting Oldroyd fluid (Oldroyd, 1958) in the presence of a magnetic field and found that the magnetic field has a stabilizing influence. Khare and Sahai (1995) considered the effect of rotation on the convection in a porous medium in a horizontal fluid layer which was viscous, incompressible and of variable density. Kumar *et al.* (2004) considered the instability of the plane interface between two Oldroydian viscoelastic superposed fluids in the presence of uniform rotation and variable magnetic field in a porous medium. Kumar and Singh (2008)studied the superposed Maxwellian viscoelastic fluids through porous media in hydromagnetics. In another study, Kumar and Singh (2010) considered the transport of vorticity in an Oldroydian viscoelastic fluid in the presence of suspended magnetic particles through porous media.

Keeping in mind the importance in various fields particularly in the soil sciences, groundwater hydrology, geophysical, astrophysical and biometrics, the thermal instability of a viscoelastic (Oldroydian) incompressible and heterogeneous fluid layer saturated with a porous medium, where density is $\rho_0 f(z)$, ρ_0 being a positive constant having the dimension of density, and $f(z)$ is a monotonic function of the vertical coordinate z, with $f(0) = 1$ has been considered in the present paper.

2. Formulation of the problem and perturbation equations

Consider an infinite horizontal layer of an incompressible and heterogeneous Oldroydian viscoelastic fluid confined between the planes $z = 0$ and $z = d$ in a porous medium of porosity ε and permeability k_1, acted on by gravity force $\mathbf{g}(0,0,-g)$. Let the axis z be directed vertically upwards. The interstitial fluid of variable density is viscous and incompressible. The initial inhomogenenity in the fluid is assumed to be of the form $\rho_0 f(z)$, where ρ_0 is the density at the lower boundary and $f(z)$ be a function of the vertical coordinate z such that $f(0) = 1$. The fluid layer is infinite in the horizontal direction and is heated from below. An adverse temperature gradient $\beta = (T_0 - T_1)/d$ is maintained across the two boundaries, where T_0 and T_1 are constant temperatures of the lower and upper boundaries. The effective density is the superposition of the inhomogeneity described by (a) $\rho = \rho_0 f(z)$, and (b) $\rho = \rho_0[1 + \alpha(T_0 - T)]$ which is caused by the temperature gradient. This leads to the effective density

$$\rho = \rho_0[f(z) + \alpha(T_0 - T)] \tag{2.1}$$

where α is the coefficient of thermal expansion.

The fluid is decribed by the constitutive relations

$$\begin{aligned} &\mathrm{T}_{ij} = -p\delta_{ij} + \tau_{ij} \qquad \Big(1 + \lambda\frac{d}{dt}\Big)\tau_{ij} = 2\mu\Big(1 + \lambda_0\frac{d}{dt}\Big)e_{ij} \\ &e_{ij} = \frac{1}{2}\Big(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\Big) \end{aligned} \tag{2.2}$$

where T_{ij}, τ_{ij}, e_{ij}, μ, λ, $\lambda_0(< \lambda)$ denote the normal stress tensor, shear stress tensor, rate-of-strain tensor, viscosity, stress relaxation time, and strain retardation time, respectively. p is the isotropic pressure, δ_{ij} is the Kroneckor delta, d/dt is the mobile operator, while u_i and x_i are velocity and position vectors, respectively. Relations of type (2.2) were first proposed by Jeffreys for Earth and later studied by Oldroyd (1958). Oldroyd (1958) also showed that many rheological equations of state, of general validity, reduce to (2.2) when linearized. If $\lambda_0 = 0$, the fluid is Maxwellian, while for $\lambda_0 \neq 0$ we shall refer to the fluid as Oldroydian. $\lambda = \lambda_0 = 0$ gives a Newtonian viscous fluid.

As a consequence of Brinkman's equation, the resistance term $-(\mu/k_1)\mathbf{u}$ will also occur with the usual viscous term in the equations of motion. Here $\mathbf{u}$ denotes the filtration velocity of the fluid.

The equations of motion and continuity for the Oldroydian viscoelastic fluid, following the Boussinesq approximation, are

$$\begin{aligned}
&\frac{1}{\varepsilon}\Big(1+\lambda\frac{\partial}{\partial t}\Big)\Big[\frac{\partial}{\partial t}+\frac{1}{\varepsilon}\Big(\mathbf{u}\cdot\nabla\Big)\Big]\mathbf{u} \\
&\qquad =\Big(1+\lambda\frac{\partial}{\partial t}\Big)\Big[-\frac{1}{\rho_0}\nabla p+\mathbf{g}\Big(1+\frac{\delta\rho}{\rho_0}\Big)\Big]+\Big(1+\lambda_0\frac{\partial}{\partial t}\Big)\Big[\frac{\nu}{\varepsilon}\nabla^2-\frac{\nu}{k_1}\Big]\mathbf{u}
\end{aligned} \tag{2.3}$$

and

$$\nabla\cdot\mathbf{u}=0 \tag{2.4}$$

The equation of heat conduction (Joseph, 1976) is

$$[\rho_0 c\varepsilon+\rho_s c_s(1-\varepsilon)]\frac{\partial T}{\partial t}+\rho_0 c(\mathbf{u}\cdot\nabla)T=k\nabla^2 T \tag{2.5}$$

where ρ_0, c, ρ_s, c_s denote the density and heat capacity of the fluid and the solid matrix, respectively, k is the thermal conductivity. Equation (2.5) can be rewritten as

$$E\frac{\partial T}{\partial t}+(\mathbf{u}\cdot\nabla)T=\xi\nabla^2 T \tag{2.6}$$

where

$$E=\varepsilon+(1-\varepsilon)\frac{\rho_s c_s}{\rho_0 c}$$

Also we have

$$\varepsilon\frac{\partial\rho}{\partial t}+(\mathbf{u}\cdot\nabla)\rho=0 \tag{2.7}$$

The kinematic viscosity $\nu(=\mu/\rho_0)$ and the thermal diffusivity $\xi(=k/(\rho_0 c))$ are assumed to be constants, where ρ_0 has the same positive value due to the coupling and Boussinesq approximation for the same fluid.

Now the initial state whose stability is to be examined is characterized by

$$\begin{aligned}
&\mathbf{u}=[0,0,0] && T=T_0-\beta z \\
&\rho=\rho_0[f(z)+\alpha\beta z] && p=p_0-\int\limits_0^1 g\rho\,dz
\end{aligned}$$

where p_0 is the pressure at $\rho=\rho_0$ and $\beta(=|dT/dz|)$ is the magnitude of the uniform temperature gradient.

Let $\delta\rho$, δp, θ, and $\mathbf{v}[u,v,w]$ denote respectively the perturbations in density ρ, pressure p, temperature T and velocity $\mathbf{u}$ (initially zero). The change in density $\delta\rho$, caused by the perturbation θ in temperature, is given by

$$\rho+\delta\rho=\rho_0[f(z)-\alpha(T+\theta-T_0)]=\rho-\alpha\rho_0\theta$$

i.e.

$$\delta\rho=-\alpha\rho_0\theta \tag{2.8}$$

Then the linearized perturbation equations for the Oldroydian viscoelastic fluid flow through the porous medium are

$$\frac{1}{\varepsilon}\Big(1+\lambda\frac{\partial}{\partial t}\Big)\frac{\partial\mathbf{v}}{\partial t}=\Big(1+\lambda\frac{\partial}{\partial t}\Big)\Big[-\frac{1}{\rho_0}\nabla\delta p-\mathbf{g}\alpha\theta\Big]+\frac{\nu}{\varepsilon}\Big(1+\lambda_0\frac{\partial}{\partial t}\Big)\Big[\nabla^2-\frac{\varepsilon}{k_1}\Big]\mathbf{v} \tag{2.9}$$

and

$$\nabla\cdot\mathbf{v}=0 \qquad \varepsilon\frac{\partial}{\partial t}\delta\rho+\rho_0 w\frac{df}{dz}=0 \qquad \Big(E\frac{\partial}{\partial t}-\xi\nabla^2\Big)\theta=\beta w \tag{2.10}$$

where w is the perturbed velocity in the z-direction.

The fluid is confined between the planes $z=0$ and $z=d$ maintained at constant temperatures. Since no perturbation in temperature is allowed and since the normal component of the velocity must vanish on these surfaces, we have

$$w=0 \qquad \theta=0 \qquad \text{at} \qquad z=0 \qquad \text{and} \qquad z=d \tag{2.11}$$

Here we consider both the boundaries to be free. The case of two free boundaries is slightly artificial, except in stellar atmospheres (Spiegel, 1965) and in certain geophysical situations where it is most appropriate. However, the case of two free boundaries allows us to obtain an analytical solution without affecting the essential features of the problem. The vanishing of tangential stresses at the free surfaces implies

$$\frac{\partial^2 w}{\partial z^2}=0 \qquad \text{at} \qquad z=0 \qquad \text{and} \qquad z=d \tag{2.12}$$

Eliminating δp between the three component equations of (2.9) and using $(2.10)_1$, we obtain

$$\Big(1+\lambda\frac{\partial}{\partial t}\Big)\Big[\frac{1}{\varepsilon}\nabla^2\frac{\partial w}{\partial t}-g\alpha\nabla_1^2\theta\Big]=\frac{\nu}{\varepsilon}\Big(1+\lambda_0\frac{\partial}{\partial t}\Big)\Big(\nabla^2-\frac{\varepsilon}{k_1}\Big)\nabla^2 w \tag{2.13}$$

where

$$\nabla_1^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2} \qquad \nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$

3. Dispersion relation and discussion

Decompose the disturbances into normal modes and assume that the perturbed quantities are of form

$$[w,\theta]=[W(z),\Theta(z)]\exp(\mathrm{i}k_x x+\mathrm{i}k_y y+nt) \tag{3.1}$$

where k_x, k_y are the wave numbers along the x- and y-directions, respectively, $k=\sqrt{(k_x^2+k_y^2}$ is the resultant wave number and n is a complex constant.

The non-dimensional form of equations (2.13) and $(2.10)_3$, with the help of expression (3.1) and $(2.10)_2$, becomes

$$\begin{aligned}(1+F\sigma)\Big[\sigma(D^2-a^2)W+\frac{g\alpha d^2\varepsilon}{\nu}a^2\Theta+\frac{ga^2d^4}{\kappa\nu}\frac{df}{dz'}W\Big]&\\ =(1+F^*\sigma)\Big(D^2-a^2-\frac{\varepsilon}{P_l}\Big)(D^2-a^2)W&\\ (D^2-a^2-Ep_1\sigma)\Theta=-\frac{\beta d^2}{\xi}W&\end{aligned} \tag{3.2}$$

where we have introduced new coordinates $(x',y',z')=(x/d,y/d,z/d)$ in new units of length d and $D=d/dz'$. For convenience, the dashes are dropped hereafter. Also we have put $a=kd$, $\sigma=nd^2/\nu$, $F=\lambda\nu/d^2$, and $F^*=(\lambda_0\nu/d^2)p_1=\nu/\xi$ is the Prandtl number and $P_l=k_1/d^2$ is the dimensionless permeability of the medium.

Eliminating Θ between equations (3.2), we get

$$(1+F\sigma)[\sigma(D^2-a^2)(D^2-a^2-Ep_1\sigma)-Ra^2+R_2a^2(D^2-a^2-Ep_1\sigma)]W \\ =(1+F^*\sigma)\Big(D^2-a^2-\frac{\varepsilon}{P_l}\Big)(D^2-a^2)(D^2-a^2-Ep_1\sigma)W \tag{3.3}$$

where

$$R=\frac{g\alpha\beta d^4\varepsilon}{\nu\xi}$$

is the modified Rayleigh number for the porous medium and

$$R_2=\frac{gd^4}{\kappa\nu}\frac{df}{dz}$$

Boundary conditions (2.11) and (2.12) transform to

$$W=0 \qquad D^2W=0 \qquad \Theta=0 \qquad \text{at} \qquad z=0 \qquad \text{and} \qquad z=1 \tag{3.4}$$

Using (3.4), it can be shown that all the even order derivatives of W must vanish for $z=0$ and $z=1$, and hence the proper solution to equation (3.3) characterizing the lowest mode is

$$W=A\sin(\pi z) \tag{3.5}$$

where A is a constant. Substituting (3.5) into equation (3.3), we obtain the dispersion relation

$$R_1=\frac{1}{x^*}[\mathrm{i}\sigma_1(1+x^*)(1+x^*+\mathrm{i}\sigma_1Ep_1)-R_3\pi^2x^*(1+x^*+\mathrm{i}\sigma_1p_1)] \\ +\frac{1+\mathrm{i}F^*\sigma_1\pi^2}{x^*(1+\mathrm{i}\sigma_1\pi^2F)}\Big(1+x^*+\frac{\varepsilon}{P}\Big)(1+x^*)(1+x^*+\mathrm{i}E\sigma_1p_1) \tag{3.6}$$

where we have put

$$x^*=\frac{a^2}{\pi^2} \qquad R_1=\frac{R}{\pi^4} \qquad R_3=\frac{R_2}{\pi^4}$$
$$\mathrm{i}\sigma_1=\frac{\sigma}{\pi^2} \qquad P=\pi^2P_l \qquad \mathrm{i}=\sqrt{-1}$$

4. The stationary convection

For stationary convection $\sigma=0$, and equation (3.6) reduces to

$$R_1=-R_3\pi^2(1+x^*)+\frac{(1+x^*)^2\Big(1+x^*+\frac{\varepsilon}{P}\Big)}{x^*} \tag{4.1}$$

Thus for stationary convection, the stress relaxation time F and the strain retardation time parameter F^* vanish with σ, and the Oldroydian fluid behaves like an ordinary Newtonian fluid.

To study the effects of medium permeability and density distribution, we examine the nature of dR_1/dP and dR_1/dR_3 analytically.

Equation (4.1) yields

$$\frac{dR_1}{dP}=-\frac{(1+x^*)^2\varepsilon}{x^*P^2} \tag{4.2}$$

which is always negative, meaning thereby that the permeability of the medium has a destabilizing effect on the viscoelastic heterogeneous Oldroydian fluid, for stationary convection.

Also from equation (4.1), we have

$$\frac{dR_1}{dR_3} = -\pi^2(1+x^*) \tag{4.3}$$

which is always negative, meaning thereby that the density distribution R_3 has a destabilizing effect on the viscoelastic heterogeneous Oldroydian fluid, for stationary convection.

Dispersion relation (4.1) is also analysed numerically. In Fig. 1, R_1 is plotted against x^* for $\varepsilon = 0.5$, $R_3 = -5$ and $P = 10$, 100. The destabilizing role of the medium permeability is clear from the decrease of the Rayleigh number with the increase in the permeability parameter P. The minor differences between the effects of P on R_1 are due to taking large values of P.

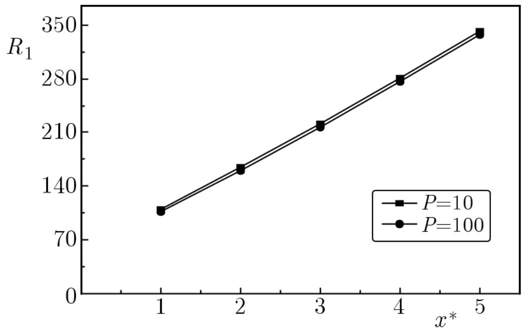

Fig. 1. Variation of the Rayleigh number R_1 with x^* for $\varepsilon = 0.5$, $R_3 = -5$ and $P = 10$, 100

In Fig. 2, R_1 is plotted against x^* for $\varepsilon = 0.5$, $P = 10$ and $R_3 = -5, \ -1$. The value of R_1 decreases with the increase in the density distribution R_3, showing thereby the destabilizing role of the density distribution.

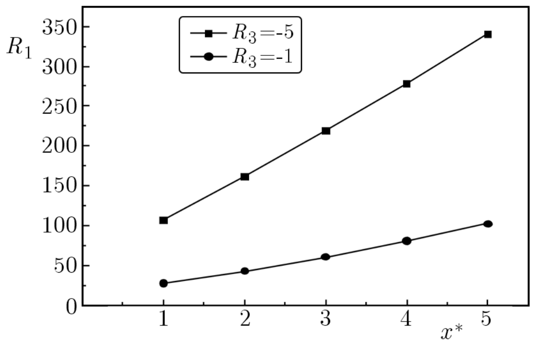

Fig. 2. Variation of the Rayleigh number R_1 with x^* for $\varepsilon = 0.5$, $p = 10$ and $R_3 = -5, \ -1$

5. The case of overstability

Here we examine the possibility of whether instability may occur as overstability. Since for overstability we wish to determine the critical Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find the condition under which equation (3.6) will admit solutions with real values of σ_1. Putting $b = 1 + x^*$ and equating the real and imaginary parts of equation (3.6), we get

$$\begin{aligned} R_1(b-1) &= -\sigma_1^2 bEp_1 - R_3\pi^2(b-1)b - \sigma_1^2\pi^2 Fb^2 + \sigma_1^2\pi^4 FR_3(b-1)Ep_1 \\ &\quad + b^2\Big(b + \frac{\varepsilon}{P}\Big) - \Big(b + \frac{\varepsilon}{P}\Big)F^*\sigma_1^2\pi^2 Ep_1 \end{aligned} \tag{5.1}$$

and

$$
\begin{aligned}
R_1(b-1)\pi^2F = b^2 - R_3\pi^2(b-1)Ep_1 - \sigma_1^2\pi^2FbEp_1 - \pi^4FR_3(b-1)b \\
+ b\Big(b+\frac{\varepsilon}{P}\Big)Ep_1 + b^2\Big(b+\frac{\varepsilon}{P}\Big)F^*\pi^2
\end{aligned}
\tag{5.2}
$$

Eliminating R_1 between equations (5.1) and (5.2), we obtain

$$
\sigma_1^2 = -\frac{b^2 - R_3x^*Ep_1 + b\Big(b+\frac{\varepsilon}{P}\Big)(Ep_1 - b\pi^2F) + b^2\Big(b+\frac{\varepsilon}{P}\Big)F^*\pi^2}{\pi^4F^2b^2 - \pi^6F^2R_3x^*Ep_1 + b\Big(b+\frac{\varepsilon}{P}\Big)F^*\pi^4Ep_1F}
\tag{5.3}
$$

Since σ_1 is real in the case of overstability, σ_1^2 should always be positive. Equation (5.3) shows that this is clearly impossible, i.e. if σ_1^2 is always negative if

$$
R_3 < 0 \qquad \text{i.e.} \qquad \frac{df}{dz} < 0 \qquad \text{and} \qquad Ep_1 > b\pi^2F
$$

which implies that

$$
\frac{df}{dz} < 0 \qquad \text{and} \qquad k^2 < \frac{E}{\xi\lambda} - \frac{\pi^2}{d^2}
\tag{5.4}
$$

Thus if $df/dz < 0$ and $k^2 < (E/\xi\lambda) - (\pi^2/d^2)$, the overstability is not possible. Inequalities (5.4) are, therefore, the sufficient conditions for the non-existence of the overstability.

6. Conclusions

An attempt has been made to investigate thermal instability of a heterogeneous Oldroydian viscoelastic fluid layer through a porous medium under the linear stability theory. The investigation of thermal instability is motivated by its direct relevance to soil sciences, groundwater hydrology, geophysical, astrophysical and biometrics. The main conclusions from the analysis of this paper are as follows:

- For the case of stationary convection, the following observations are made:
  - the stress relaxation time F and the strain retardation time parameter F^* vanish with σ, and the Oldroydian fluid behaves like an ordinary Newtonian fluid
  - the medium permeability and density distribution have destabilizing effect on the system
- It is also observed from Figs. 1 and 2 that the medium permeability and density distribution have a destabilizing effect on the system
- Inequalities $df/dz < 0$ and $k^2 < E/(\xi\lambda) - \pi^2/d^2$ are the sufficient conditions for the non-existence of overstability.

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Termiczna niestabilność niejednorodnej lepko-sprężystej cieczy Oldroyda w ośrodku porowatym ogrzewanym od spodu

Streszczenie

W artykule przedstawiono zagadnienie termicznej niestabilności niejednorodnej cieczy Oldroyda wypełniającej ośrodek porowaty. W wyniku zastosowania zlinearyzowanej teorii stateczności i analizy postaci normalnych określono funkcję dyspersji. Dla stacjonarnej konwekcji stwierdzono, że przepuszczalność ośrodka oraz rozkład gęstości destabilizują ciecz. Funkcję dyspersji wyznaczono także numerycznie. Znaleziono również warunki wystarczające do wykluczenia nadstabilności układu.

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