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APPLICATION OF MODAL ANALYSIS SUPPORTED BY 3D VISION-BASED MEASUREMENTS

Piotr Kohut
Piotr Kurowski

AGH-University of Science and Technology, Deparetment of Robotics and Mechatronics, Cracow, Poland; e-mail: pko@agh.edu.pl; kurowski@agh.edu.pl

In the paper, applications of the 3D vision techniques to the modal analysis method are presented. The goal of the project was to develop a methodology for vibration amplitude measurements and a software tool for modal analysis based on visual data. For this purpose, dedicated procedures and algorithms based on the vision technique methods were developed. 3D measurements of vibrations and structure geometry were obtained by application and developing passive 3D vision techniques. The amplitude of vibrations was calculated for selected points on the structure. Necessary vision data were received from the high-speed digital camera "X-Stream Vision" in form of "avi" files that were used as the input data for the developed software tool. The amplitude of vibration displacements obtained from vision-based measurements were treated as the input data for operational modal analysis algorithms. In this domain, the Balanced Realization algorithm has been used.

Key words: passive 3D vision techniques, structure from motion, vibration measurements, modal analysis

1. Introduction

The recent tendency in the realm of experimental modal analysis is reduction of time associated with preparing and carrying out modal tests. Non-contact techniques of signal registration have conformed to the new tendency in the construction design, and they meet all contemporary modal-analysis requirements. The basics of modal-analysis requirements can be classified as: test-accuracy increasing, increasing of frequency-bandwidth and measuring-points number, reducing testing-preparation time and result-analysis time, facilitating analyses and tests. Vibration measurement tools are employed in realisation of modal analysis. Non-contact optical techniques of displacement and vibration measurement are often encountered (Chen *et al.*, 2003; Freymann *et al.*, 1996; Mitchell, 2005; Moreno *et al.*, 2005; Peeters *et al.*, 2004; Schmidt *et al.*, 2003; Synnergren and Sjödahl, 1999; Van der Auweraer *et al.*, 2002a,b; Vanlanduit *et al.*, 1998; Zisserman and Hartley, 2004). Among many various methods based on 3D optical measurements, two categories of systems in the area of vision techniques prevail: active and passive. In the case of active methods for the purpose of depth measurement, supplementary devices (e.g. lasers, LCD projector) for generating suitably formed light (e.g. in form of a regular grille) are used (Moreno *et al.*, 2005; Van der Auweraer *et al.*, 2002a,b, *polytec*). Great advantages of active methods are their high accuracy (up to *pm*, *polytec*), the downside being their cost. They are however expensive, but getting cheaper. Moreover, these methods are completely insensitive to diversity of texture in the scene and they are not always feasible, especially for modelling distant or fast-moving objects.

Passive methods (Kohut and Kurowski, 2005, 2006; Peeters *et al.*, 2004; Schmidt *et al.*, 2003) refer to the measurement of visible radiation already present in the scene. In the case of this methods, depth measurements are carried out on the basis of image sequences captured by one or more cameras. They acquire images of a scene observed from different viewpoints and possibly different illuminations. Based on these images, the scene shape and reflectance at every surface point is computed. Many systems comprise combination of passive imaging devices (one or more cameras) and active devices (lasers/ LCD projectors) mutually calibrated (Mitchell, 2005; Synnergren and Sjödahl, 1999; Van der Auweraer *et al.*, 2002a,b). A three-dimensional scene structure is determined by means of geometrical triangulation. Among active techniques used to modal estimation of parameters, the most popular are methods based on lasers (Chen *et al.*, 2003; Freymann *et al.*, 1996; Van der Auweraer *et al.*, 2002a,b; Vanlanduit *et al.*, 1998). There are two basic approaches: older technology engages Doppler vibrometers or scanning vibrometers. Another basic type of laser measurement systems are interferometric systems, including ESPI (Electronic Speckle Pattern Interferometry). They present a new technology which uses holography and coherent and monochromatic property of laser rays.

Vision systems for two and three-dimensional measurements of geometry and object motion are employed in modal analysis (Kohut and Kurowski, 2005, 2006; Peeters *et al.*, 2004; Schmidt *et al.*, 2003). They basically relay on passive methods. A great challenge for designers of vision systems is to create systems estimating three dimensional scenes based on obtained images and external illuminations only. In contrast to the active techniques, the passive methods are often more flexible, but computationally more expensive and dependent on the structure of the scene itself. Stereovision methods belong to the most natural ones, since they make use of two images of the same scene to inference about three-dimensional properties of the scene. Another approach to stereovision techniques involves, in general, replacing a pair of cameras by a single moving camera. In this case, the single camera records two images of a scene at two different locations and at two different time moments. The reconstruction procedure is identical to stereovision: the three-dimensional structure is determined by means of triangulation, based upon two obtained images. The advantage of this approach is its low cost (single camera) and ergonomic properties.

The paper explains the implementation of the method of "structure from motion" belonging to the group of passive techniques. Vibration amplitude characteristics for selected structural points will be presented as well as 3D geometry of the structure, found on the manufactured test stand by means of devices for realisation of modal analysis and estimation of quantities characterising dynamic properties of the structure.

2. Experimental set-up

In the experimental research, a test stand enabling automatic two-axis control of a camera (Fig. 1a) was designed and manufactured. A frame structure was built, in which the guiding-rail system enables straight-line motion of the camera. Additionally, a system realising rotational motion of the camera was built-in. In order to control the test stand, software making it possible to combine the hardware-software part of the stand with the software part of the vision system was created. The tool was developed for the purpose of modal analysis and estimation of the quantities characterising dynamic properties of the structure based on vision signals. The produced software comprises the set of functions and procedures written in the Matlab environment dedicated for the purpose of determining vibrations of visible areas of mechanical systems.

System parameters:

frequency range	0.1-150 Hz
measurement accuracy	0.08-1.00 mm
range of camera straight-line motion	1000 mm
camera rotation range	360°

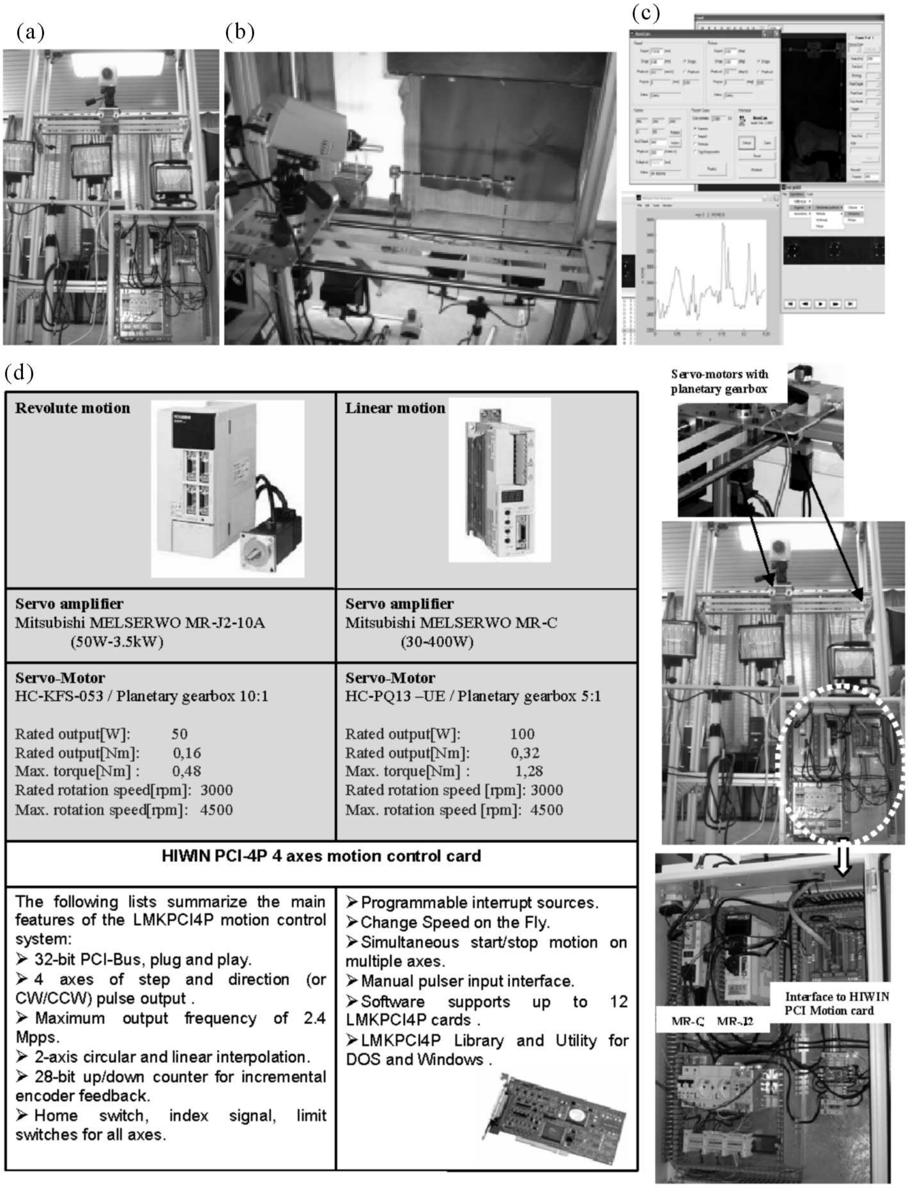

Fig. 1. (a), (b) Developed and constructed vision system for 3D measurements of geometry and motion of analysed objects. (c) Developed and manufactured tool for carrying out modal analysis and estimation of quantities characterising dynamic properties of the structure based on vision signals. (d) Hardware specification (linear and revolute motion based on two Mitsubishi servo motors)

height of camera attachment to the frame base	350-1800 mm
resolution of measurement of angle location of the camera	(1/1820)°
resolution of measurement of camera location during straight-line motion	0.05 mm

VIOMA software (Virtual In Operation Modal Analysis, Kurowski (2001)), which is available at the Department of Robotics and Mechatronics was extended. VIOMA is a toolbox consisting of advanced numerical procedures implemented in the Matlab programming environment. The main task of VIOMA is to estimate parameters of the experimental modal model. Such estimation can be performed based on the classic experiment with measurement of the extortion force or based on exploitation analysis performed during regular work of the device examined. VIOMA has been developed at the Department of Robotics and Mechatronics. It is commercially available and gained numerous positive references from users in educational and industrial sectors. Below, the general purpose algorithm of the created software is presented. The algorithm was designed in accordance with its stages that had been carried out during execution of the modal experiment. The first stage is connected with preliminary analysis of the examined structure. In the second stage, a geometrical model of the examined object should be built. Third stage involves execution of measurements on the examined object. The purpose of the next stage is to prepare measurements for carrying out the further analysis. This stage is extremely important in the case of preserving data in form of time histories during the measurement. Data in such a form should be processed to a form acceptable by suitable modal analysis algorithms. The next two stages are: realisation of modal analysis and visualization of the results.

In order to fulfil the above mentioned assumptions the software toolkit was introduced in the Matlab programming environment. In oreder to facilitate dealing with the whole set of tools, a Graphical User Interface – GUI was created (Fig. 1c), integrating and consolidating all the tools in one coherent form. The GUI interface also allows integrating a tool with VIOMA toolbox.

Basic properties of the tool are as follows:

1) Camera calibration based on the available tools (Camera Calibration Toolbox for Matlab by Bouguet and Perona (1998); Calibration Toolbox for Matlab by Heikkilä (2000)).
2) Determination of vibration characteristics and amplitude, which is realised in several stages according to the following algorithm:
   a) Areas are selected, in which vibration tracking will be executed. There should be at least a few characteristic points that can be

easily distinguished in each image frame. These points will be automatically selected by the tracking algorithm and, as a result of this selection, vibrations will be determined in consecutive stages of the program (Fig. 2a).

b) Algorithm of vibration identification must be selected. Three different algorithms are being currently implemented into the tool based on 'structure from motion' techniques (with orthographic model, scaled and para-perspective one (Fig. 2b).

c) Carrying out vibration identification (Fig. 2c).

d) Visualization of vibrations and its registering (Fig. 2d).

3) Modal analysis operation.

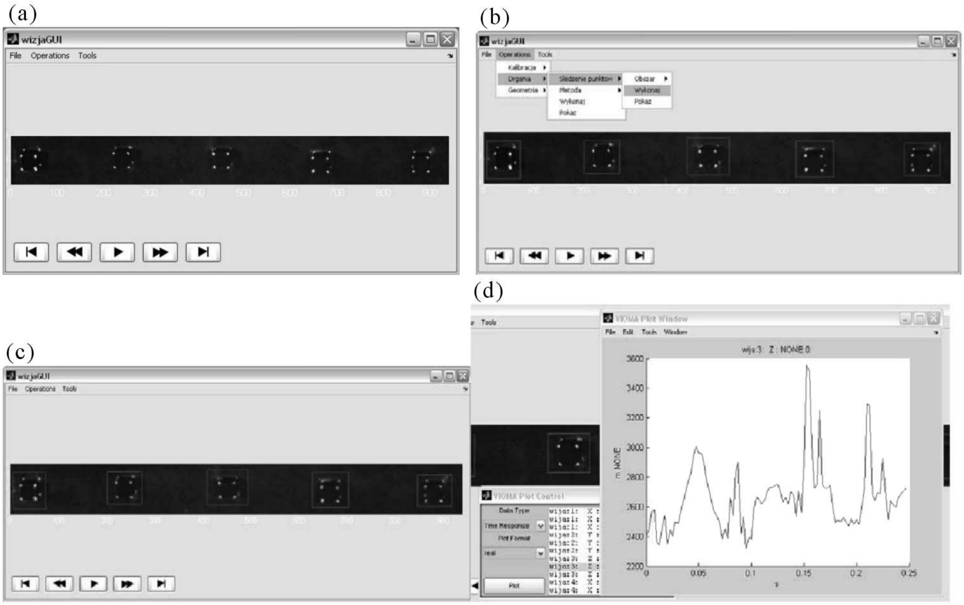

Fig. 2. Example of developed software toolkit: (a) embedded objects ready for vibration-characteristic tracking, (b) selection of areas on objects subject to tracking procedure, (c) tracking result: white spots – markers subject to tracking, red spots – markers resulted from tracking, (d) time characteristic of vibration found after conducting tracking procedure

In order to curry out modal analysis, a module of VIOMA software is applied. Data conversed by the module described above are ready to use by modal analysis modules.

3. Methodology – structure from motion

In order to obtain 3D geometry of an object and vibration measurements in selected measurement points, algorithms based on the 'structure from motion' method were drawn up. The feature tracking method was based on the tracker described by Ma *et al.* (2004), Trucco and Verri (1998).

The factorization method belongs to the group of sparse methods: structure from motion. The key features of the factorization approach are as follows: the knowledge concerning the number of objects is not required, no initial segmentation is necessary and the measurement matrix is globally factorized into two matrices (motion matrix and structure matrix) that are highly robust to noise. It is simple to implement and gives very good results for objects viewed from large distances.

The assumptions originally taken by Tomasi and Kanade (1991), Poelman and Kanade (1997) are summarized below:

- The camera model is orthographic; the position of n image points (u_{fp}, v_{fp}) has been tracked in F frames $(F \geqslant 3)$; n image points corresponding to scene points P.
- The problem can be stated: given (u_{fp}, v_{fp}), the positions of n image points that have been tracked in F frames $1 \leqslant f \leqslant F$, $1 < p \leqslant n$, compute motion of the camera from one frame to another.
- The aim is to track (u_{fp}, v_{fp}) in f frames for p: points. After subtracting the mean 2D position, the measurement equations can be written in the following form

$$u_{fp} = \boldsymbol{i}_f^\top \boldsymbol{s}_p \qquad v_{fp} = \boldsymbol{j}_f^\top \boldsymbol{s}_p \tag{3.1}$$

where

$\boldsymbol{i}_f$ – rotation
$\boldsymbol{s}_p$ – position.

The measurement matrix can be calculated as follows

$$\widetilde{\mathbf{W}} = \mathbf{RS} \tag{3.2}$$

where
$\mathbf{R}$ – rotation matrix, $\mathbf{R} = (\boldsymbol{i}_1, \ldots, \boldsymbol{i}_F, \boldsymbol{j}_1, \ldots, \boldsymbol{j}_F)^\top$
$\mathbf{R}$ – shape matrix (in a coordinate system attached to the object centroid), $\mathbf{S} = (\boldsymbol{s}_1, \ldots, \boldsymbol{s}_P)$

The size of the measurement matrix is as follows: $\widetilde{\mathbf{W}} = \mathbf{R}_{2F\times3}\mathbf{S}_{3\times P}$.

The factorization method algorithm is based on SVD decomposition

$$\widetilde{\mathbf{W}} = \mathbf{U}\mathbf{\Lambda}\mathbf{V} \tag{3.3}$$

in which $\mathbf{\Lambda}$ must be of rank 3. When noise is present, the adjusted matrices are defined

$$\mathbf{\Lambda}' = \mathbf{\Lambda}(1:3,1:3) \qquad \mathbf{U}' = \mathbf{U}(:,1:3) \qquad \mathbf{V}' = \mathbf{V}(:,1:3) \tag{3.4}$$

and constructed

$$\widehat{\mathbf{R}} = \mathbf{U}'\sqrt{\mathbf{\Lambda}'} \qquad \widehat{\mathbf{S}} = \sqrt{\mathbf{\Lambda}'}\mathbf{V}'^{\top} \tag{3.5}$$

The factorization of $\widetilde{\mathbf{W}}$ is straightforward via SVD but is not unique

$$\begin{aligned}
&\mathbf{W} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}\\
&\mathbf{W} = \widehat{\mathbf{R}}\widehat{\mathbf{S}} = \widehat{\mathbf{R}}(\mathbf{Q}\mathbf{Q}^{-1})\widehat{\mathbf{S}} = (\widehat{\mathbf{R}}\mathbf{Q})(\mathbf{Q}^{-1}\widehat{\mathbf{S}})\\
&(\widehat{\mathbf{R}}\mathbf{Q})(\mathbf{Q}^{-1}\widehat{\mathbf{S}}) = \widehat{\mathbf{R}}(\mathbf{Q}\mathbf{Q}^{-1})\widehat{\mathbf{S}}) = \widehat{\mathbf{M}}\widehat{\mathbf{S}} = \widehat{\mathbf{W}}
\end{aligned} \tag{3.6}$$

Fortunately, two constrains can be added:

- the norms of 3D vectors forming the rows of $\mathbf{R}$ must be unit,
- in $\mathbf{R}$, the $\boldsymbol{i}_i^{\top}$ must be orthogonal to the corresponding $\boldsymbol{j}_i^{\top}$.

The rows of the matrix $\mathbf{R}$ do not satisfy the constraints mentioned above but when looking for a (correction) matrix $\mathbf{Q}$ such that

$$\begin{aligned}
|\boldsymbol{m}_f|^2 &= \boldsymbol{i}_i^{\top}\mathbf{Q}\mathbf{Q}\boldsymbol{i}_i^{\top} = 1\\
|\boldsymbol{n}_f|^2 &= \boldsymbol{j}_i^{\top}\mathbf{Q}\mathbf{Q}\boldsymbol{j}_i^{\top} = 1\\
\boldsymbol{m}_f\boldsymbol{n}_f &= \boldsymbol{i}_i^{\top}\mathbf{Q}\mathbf{Q}\boldsymbol{j}_i^{\top} = 0
\end{aligned} \tag{3.7}$$

new matrices $\mathbf{R} = \widehat{\mathbf{R}}\mathbf{Q}$ and $\mathbf{S} = \mathbf{Q}^{-1}\mathbf{R}$ still factorize $\widetilde{\mathbf{W}}$, and the rows of $\mathbf{R}$ satisfy the constraints.

In the case of a orthographic model of a camera, the method does not determine camera motion along the optic axis. Therefore, shape reconstruction of the resulting object is usually deformed. To avert the problem, two methods are applied (Christy and Horaud, 1996; Poelman and Kanade, 1997; Tomasi and Kanade, 1991) that enable approximation of perspective camera character. The first method adapts *scaled orthographic* model of a camera, also referred to as weak perspective; whereas the other one *para-perspective* projection. In the case of weak perspective, it is assumed that diversity in the object depth in the direction of optic axis is negligible compared to the distance from which the object is recorded. The method introduces an effect of scaling the image coordinates by the ratio of focal length-to-depth. The second method much better constitutes approximation of the camera model, since apart from the scaling effect the effect of location is introduced. This also means that it models nearer or farther objects as an effect of observation at different angles. The two methods impose other metric limitations on the matrix $\mathbf{Q}$.

Let x_f, y_f, and z_f designate relative camera locations. Three models of the camera can be distinguished with the limitations presented in Table 1.

Table 1. Three models of metric limitations

Orthographic	$\lvert \boldsymbol{m}_f\rvert^2 = 1$ $\lvert \boldsymbol{n}_f\rvert^2 = 1$ $\boldsymbol{m}_f\boldsymbol{n}_f = 0$
Weak perspective	$\lvert \boldsymbol{m}_f\rvert^2 = \lvert \boldsymbol{n}_f\rvert^2 = 1/z_f^2$ $\boldsymbol{m}_f\boldsymbol{n}_f = 0$ $\lvert \boldsymbol{m}_1\rvert^2 = 1$
Para-perspective	$\lvert \boldsymbol{m}_f\rvert^2/(1+x_f^2) = \lvert \boldsymbol{n}_f\rvert^2/(1+y_f^2) = 1/z_f^2$ $\boldsymbol{m}_f\boldsymbol{n}_f = 0$ $\lvert \boldsymbol{m}_1\rvert^2 = 1$

where

z_f – depth to the object center of mass
x_f, y_f – components of translation between the origins of the camera and fixed coordinate system
f – index indicating f-th frame.

The next step of the before-mentioned methods is, based on metric limitations presented in Table 1, determination of the matrix $\mathbf{Q}$. In this research, the correction matrix $\mathbf{Q}$ was determined by means of the Newton-Raphson method. The research provided algorithms and procedures enabling determination of all three models, the purpose of which can be specified as follows:

1. Given, in the sequence of m image frames $(m \geqslant 3)$, n mutually corresponding image points x_{ij}, $j = 1, \ldots, n$, $i = 1, \ldots, m$
2. Objective: determine affinite camera matrices M^i, t^i and 3D points X_j such that the reprojection error is minimized for all M^i, t^i, X_j

$$\min_{M^i,t^i,X_j} \sum_{ij} \|x_{ij} - \widetilde{x}_{ij}\|^2 = \min_{M^i,t^i,X_j} \sum_{ij} \|x_{ij} - (M^i X_j + t^i)\|^2$$

The following algorithm was developed and implemented for all the three cases: orthographic, scaled orthographic and para-perspective, according to metric limitations spelled out in Table 1:

1. Calculation of SVD of $\widetilde{\mathbf{W}} = \mathbf{UDV}$
2. Specification of geometry and orientation: $\widehat{\mathbf{R}} = \mathbf{U}'\sqrt{\mathbf{D}'}$, $\widehat{\mathbf{S}} = \sqrt{\mathbf{D}'}\mathbf{V}'^{\top}$
3. Determination of the correction matrix $\mathbf{Q}$ by imposing metric limitations. Application of the Newton-Rasphon method
4. Determination of the motion matrix $\mathbf{M}$ and structure $\mathbf{S}$
5. Setting the first camera as the reference system relative to global coordinates.

3.1. Feature tracking

The feature tracking algorithm with pyramid decomposition (Harris and Stephens, 1988; Ma *et al.*, 2004; Tomasi and Kanade, 1991; Trucco and Verri, 1998) was based on the translation model in which the displacement $\boldsymbol{h}$ of the feature point $\boldsymbol{x}$ between two consecutive frames can be calculated by minimizing the sum of square differences between two images $I_i(\boldsymbol{x})$ and $I_{i+1}(\boldsymbol{x}+\boldsymbol{h})$ in a small window $W(\boldsymbol{x})$ around the feature point $\boldsymbol{x}$. The minimization problem for the displacement $\boldsymbol{h}$ can be described as follows

$$\min_{\boldsymbol{h}} E(\boldsymbol{h}) = \sum_{\widetilde{\boldsymbol{x}}\in W(\boldsymbol{x})} [I_2(\widetilde{\boldsymbol{x}}+\boldsymbol{h}) - I_1(\widetilde{\boldsymbol{x}})]^2 \tag{3.8}$$

The closed-form solution is given by

$$\boldsymbol{h} = -\mathbf{G}^{-1}\boldsymbol{b} \tag{3.9}$$

where

$$\mathbf{G} = \begin{bmatrix} \sum\limits_{W(\boldsymbol{x})} I_x^2 & \sum\limits_{W(\boldsymbol{x})} I_x I_y \\ \sum\limits_{W(\boldsymbol{x})} I_x I_y & \sum\limits_{W(\boldsymbol{x})} I_y^2 \end{bmatrix} \qquad \boldsymbol{b} = \begin{bmatrix} \sum\limits_{W(\boldsymbol{x})} I_x I_t \\ \sum\limits_{W(\boldsymbol{x})} I_y I_t \end{bmatrix} \tag{3.10}$$

where: I_x, I_y are image gradients, $I_t = I_2 - I_1$ – temporal image derivative.

4. Experiment – application of 'Structure from motion' to modal analysis techniques

The estimation of modal model parameters was carried out based on determined vibration characteristics. For the estimation, a time method was selected, whose input was fed with functions of internal and mutual correlations. For the purpose of improving data quality and smoothing the correlation function, the average of several measuring sessions was applied. The correlation functions were subsequently used for conducting modal analysis. Since no measurement of exerting force was carried out, the only solution to the problem was execution of operational modal analysis. The Balanced Realization (BR) algorithm was implemented. The analysis was carried out in the whole measurement band, i.e. 0-200 Hz. For the purpose of determining a stabilization diagram, models from the 2nd to 50th row were estimated. Figure 6 presents an exemplary stabilization diagram in the discussed case. It is noticeable that in spite of poor quality of time characteristics (Fig. 5a) being analysed, the stabilization of poles is remarkably good, which means that for a given frequency, the poles for individual rows form stabilized vertical lines. In every analysis, a pole of frequency of 100 Hz and relatively low damping coefficient (not exceeding 0.2%) occurs. In a classic measurement with the application of accelerometer sensors, this frequency could be the evidence of the adverse influence of interference coming from electrical mains. In the case of vision measurement, the sole explanation of the occurence of this frequency is the negative lighting effect on the conducted measurement.

The developed method of "structure from motion" with para-perspective model was tested on the data obtained from a series of simulations. After verification, experimental tests were conducted on the special laboratory steel frame stand (Fig. 1). The upper part of the frame was considered. In order to enhance the accuracy of vibration measurements (Kohut and Kurowski, 2005, 2006), the upper beam of the frame was divided into three segments (Fig. 3). In each measurement session, the measurements were carried out on a different frame part. For each segment, 4 measurement sessions were carried out. The excitation was exerted by means of random noise.

Based on the developed method, vibration amplitude characteristics were obtained for all measurement points of the segmented upper beam of the frame. Examplary vibration characteristic for a separated measurement point is shown in Fig. 5a.

Additionally, by means of the developed method of 'structure from motion' with para-perspective model, geometry of the whole frame was obtained (Fig. 5b).

As a result of the conducted experiment, vibration characteristics were obtained for all measurement points (all parts of the examined frame). The algorithm of 'structure from motion' correctly specified the amplitude characteristics of vibrating object displacements.

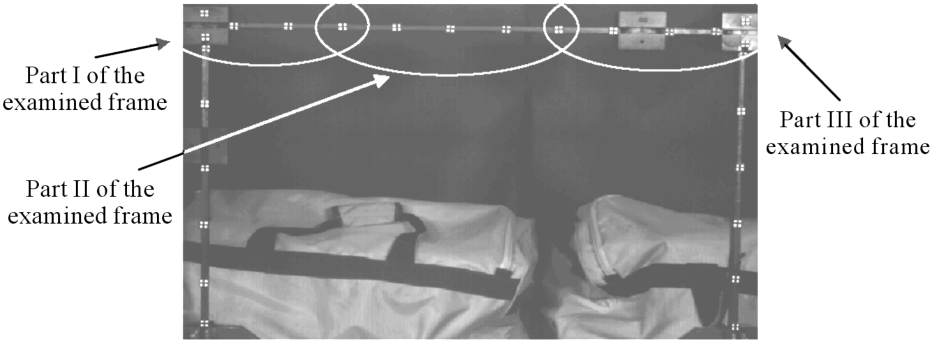

Fig. 3. Examined upper beam of the frame divided into three parts. Each part of the frame was measured in a separate measurement session

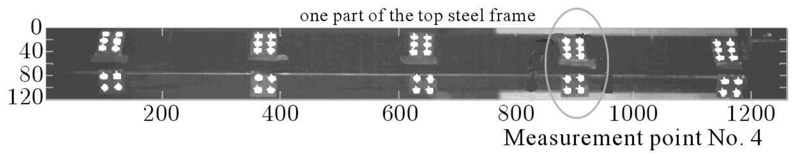

Fig. 4. Segment II of the upper beam along with a separate measurement point

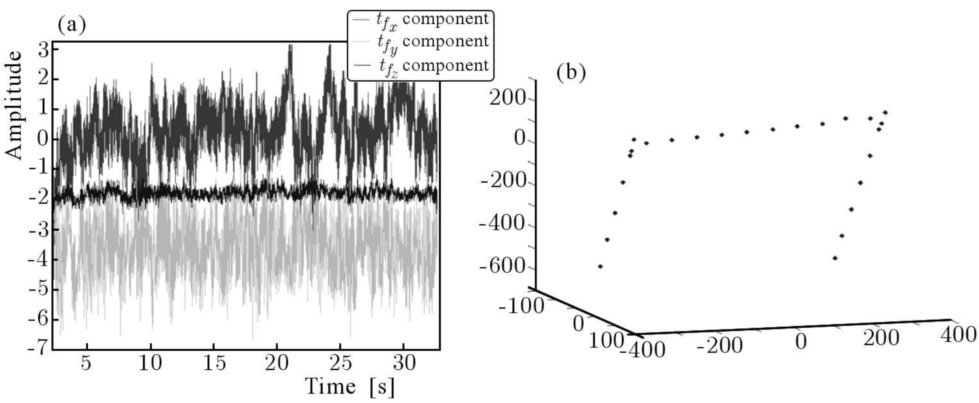

Fig. 5. (a) Vibration amplitude of the examined object for a selected measurement point. Displacement components X, Y, Z presented in the camera coordinate system. (b) Reconstruction of model geometry

4.1. Modal analysis of the upper section of the frame

The results of experimental measurements were applied as input data to the algorithm of estimation of modal parameters based on visual data measuremnts. Modal analysis was carried out by means of the Balanced Realization algorithm in the frequency range of 2-200 Hz. Order and poles of the modal model were selected on the basis of the stabilization diagram. The diagram was created for the estimation of modal models from the 2nd to 50th order with a step 1. The obtained stabilization diagram is presented in Fig. 6. Analysis of the diagram shows clear stabilization with strong vertical lines. Basing on the diagram, modal model parameters were estimated. Results are collected in Table 2. In Fig. 7, a few selected modeshapes obtained during estimation process are additionally presented. According to the measurement data, the presented modeshapes are related only with the top beam of the measured frame.

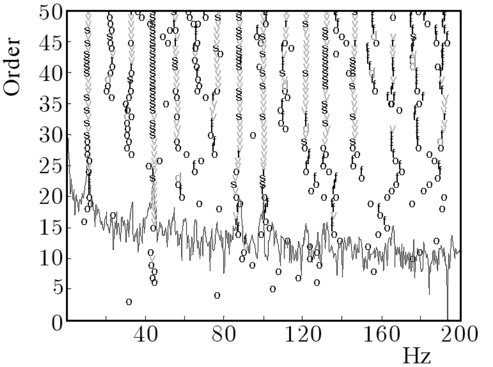

Fig. 6. Stabilization diagram obtained during estimation

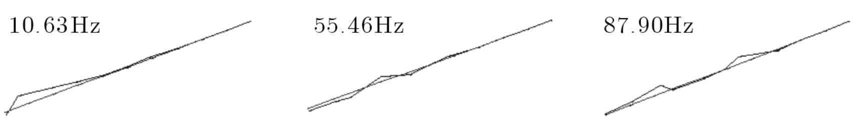

Fig. 7. Selection of the obtained modeshapes

Table 2. Modal coefficients obtained after parameters estimation

No.	Frequency [Hz]	Damping coefficient [%]	No.	Frequency [Hz]	Damping coefficient [%]
1	10.63	3.02	8	99.92	0.23
2	33.01	2.76	9	109.72	1.38
3	43.91	0.99	10	121.46	0.32
4	55.46	2.22	11	131.87	1.23
5	74.19	1.53	12	146.05	1.53
6	76.48	1.32	13	165.37	0.29
7	87.90	1.19	14	191.60	1.93

5. Conclusions

In the project, methodology and software for realisation of modal analysis and estimation of quantities characterising dynamic properties of the structure based on visual data were developed. The carried out research proved that it is possible to apply vision systems to modal analysis. The developed tool facilitates vibration detection based on vision images and modal analysis. The structure was created and a prototype of vision system dedicated for modal analysis purposes was manufactured. A laboratory test stand integrated with the vision system was designed and made. For realisation of the experiment, software, user's graphical interface and algorithms controlling motion of the camera along two axes were elaborated.

In relation to new demands of modal analysis, new methodology algorithms and procedures enabling automation of the preliminary stage of modal analysis were designed, i.e. algorithms for automatic geometry representation and measurement points localization.

As a result of the investigation and estimation of the modal model, it can be stated that frequencies and damping factors found from the vision methods correspond with the results obtained by means of classical methods (Kohut and Kurowski, 2005, 2006). The representation of 3D vibration forms indicates the necessity of taking up further research on the improvement of spatial resolution of vibrations obtained from developed methods and algorithms.

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Trójwymiarowe wizyjne metody pomiarowe do wspomagania analizy modalnej

Streszczenie

W artykule zaprezentowano wykorzystanie trójwymiarowych technik wizyjnych w metodach analizy modalnej. Przedstawiono opracowaną metodykę pomiarów amplitudy drgań oraz narzędzie programowe do realizacji analizy modalnej bazującej na danych wizyjnych. Opracowano dedykowane procedury i algorytmy wykorzystujące metody wizyjne. Amplituda drgań została obliczona w wybranych punktach pomiarowych struktury. Trójwymiarowe pomiary drgań oraz geometrię struktury uzyskano poprzez zastosowanie i opracowanie trójwymiarowych pasywnych metod wizyjnych. Jako dane wejściowe do opracowanego narzędzia programowego użyto sygnały wizyjne otrzymane z „szybkiej" kamery cyfrowej X-StreamVision w postaci plików 'avi'. Uzyskana z systemu wizyjnego amplituda drgań stanowiła dane wejściowe do algorytmu operacyjnej analizy modalnej. W tej dziedzinie wykorzystany został algorytm zbilansowanej realizacji (ang. *Balanced Realization*).

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