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ON THE STRESS INTENSITY FACTORS FOR TRANSIENT THERMAL LOADING IN AN ORTHOTROPIC THIN PLATE WITH A CRACK

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This paper is concerned with an orthotropic thin plate containing a crack perpendicular to its surfaces. It is assumed that the transient thermal stress is set up by the application of a heat flux as a function of time and position along the crack edge and the heat flow by convection from the plate surfaces. The exact analytical solutions for the stress intensity factor and crack-opening displacement are derived. Numerical examples show, among others, a dependence of the stress intensity factor on the thermal and elastic constants of the orthotropic material.

Key words: orthotropic plate, crack problem, transient thermal loading, stress intensity factor

1. Introduction

The study of thermoelastic problems has always been an important branch in solid mechanics (see Nowacki, 1986; Nowiński, 1978). In particular, the thermoelastic fracture problems subjected to various types of thermal boundary conditions have been discussed extensively in the literature. Most of research works discuss the steady-state crack problems and axisymmetric cases for which the Hankel transform technique and the theory of dual integral equations were usually employed (Sneddon, 1966). Recently, a report on a penny-shaped or external crack subjected to temperature and heat flux, arbitrarily acting in a transversely isotropic medium, was presented by the author (Rogowski, 2003). The corresponding fundamental solution can play an important role in the boundary element method of thermoelastic fracture analysis. Some metallic materials, such as zinc, magnesium, cadmium are transversely isotropic (Hearmon, 1961). Many fibrous composites may also be modeled as transversely isotropic materials (Christensen, 1979). There have been many reports on crack analysis in transversely isotropic and orthotropic thermoelastic materials. Among the studies, Tsai (1983a,b) calculated the stress intensity factors of a penny shaped crack in a transversely isotropic material due to a thermal loading, while Rogowski (2001a,b) presented analysis of a crack system in transversely isotropic materials. Many of research works discuss the two-dimensional thermal crack problem in the literature. Sumi (1981, 1982), Aköz and Tauchert (1972), Atkinson and Clement (1977), Ghosh (1977), Clements and Tauchert (1979), Clements (1983), Tsai (1983a,b) and Rogowski (1982) solved various problems in anisotropic thermoelastic solids. Gladwell *et al.* (1983) considered the radiation boundary conditions. But, perhaps because of mathematical complexity, the three-dimensional crack problem of an anisotropic medium under transient thermal loading have not yet received much attention. Among the studies, Koizumi and Niwa (1977) performed the analysis of an edge crack in a semi-infinite plate under transient thermal loading. Noda and Matsunaga (1986) investigated the transient crack problem in an infinite medium, while Ishida (1987) calculated the stress intensity factor for a transient thermal loading in a transversely isotropic material. Ting and Jacobs (1979) solved the problem for transient thermal stress in a cracked solid. Many problems of thermoelasticity were solved in a book by Podstrigach and Kolyano (1972).

This paper considers the transient thermal problem of a crack in an orthotropic thin plate. The method of solution involves the use of Fourier and Laplace's transforms and displacement potentials to reduce the mixed boundary value problem to a pair of dual integral equations. The solution is given in an exact analytical form. The stress intensity factor of mode I and the crack-opening displacement for a heat flux arbitrarily acting on the crack surface, are determined. The numerical results are shown graphically to demonstrate the influence of thermal and mechanical anisotropic parameters.

2. Analysis

2.1. Temperature field

Consider an orthotropic thin plate of thickness $2h$ containing a crack. Figure 1a shows the geometry of the problem where the position of the point is defined by Cartesian co-ordiantes (x, y, z). In this co-ordinate system, the crack occupies the region $y = 0$, $|x| \leqslant a$, $|z| \leqslant h$. We shall suppose that the crack is opened out by the heat flux depending on time and position applied to its surfaces. Referring to the semi-infinite region $y \geqslant 0$, the boundary conditions in the problem can be assumed as shown in Fig. 1b, since the thermal and mechanical conditions on $y = 0^+$ are identical with those on $y = 0^-$. Additionally, for a thin plate the unknown temperature distribution $T(x, y, t)$ is assumed to be constant over the thickness, giving the heat exchange by convection on both surfaces of the plate, which equals $-2\gamma T$, where T is the temperature change and γ is the heat transfer coefficient on the plane surfaces.

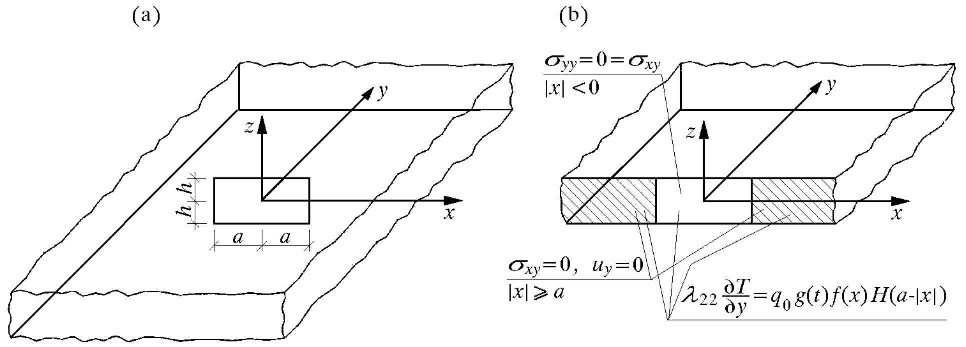

Fig. 1. Geometry and co-ordinate system (a) and boundary conditions (b)

The equation heat conduction governing an unsteady-state temperature field in an orthotropic thin plate with heat dissipation at both plane surfaces is (Nowacki, 1986)

$$\lambda_{11}\frac{\partial^2 T}{\partial x^2} + \lambda_{22}\frac{\partial^2 T}{\partial y^2} - \frac{\gamma}{h}T = c\rho\frac{\partial T}{\partial t} \tag{2.1}$$

where c is the specific heat, ρ is the mass density and λ_{11} and λ_{22} are the thermal conductivities in the x- and y-directions, respectively.

The initial and boundary conditions for the temperature field are

$$\begin{aligned} &T = 0 && \text{at} \quad t = 0 \\ &\lambda_{22}\frac{\partial T}{\partial y} = q_0 g(t) f(x) H(a - |x|) && \text{on} \quad y = 0 \end{aligned} \tag{2.2}$$

where q_0 is the heat flux per unit area and unit time, and $H(\cdot)$ denotes Heaviside's step function. The problem is symmetric with respect to the plane $y = 0$, the temperature $T(x, y, t)$ is an even function of y and differentiable with respect to y at $y = 0$; in consequence, the heat flux is equal to zero for $y = 0$, $|x| > a$. Applying Laplace's transform to time and Fourier cosine transform to the variable x, and using the convolution theorem for inverse Laplace's transform, the solution to (2.1) which satisfies (2.2) and (2.3) may be expressed by

$$T = \int\limits_0^t g(t-\tau)\Big[-\frac{4}{\pi^2}\frac{q_0\chi\lambda^2}{\lambda_{22}}\int\limits_0^\infty \overline{f}(s)\cos(sx)ds\int\limits_0^\infty \cos(py)\mathrm{e}^{-\chi(m^2+s^2+\lambda^2p^2)\tau}dp\Big]d\tau = \tag{2.3}$$

$$= \int\limits_0^\infty \Big[\int\limits_0^\infty \theta(s,p,t)\cos(py)\;dp\Big]\cos(sx)\;ds$$

where

$$\begin{aligned} \theta(s,p,t) &= -\frac{4}{\pi^2}\frac{q_0\chi\lambda^2}{\lambda_{22}}\overline{f}(s)\int\limits_0^t g(t-\tau)\mathrm{e}^{-\chi(m^2+s^2+\lambda^2p^2)\tau}\;d\tau \\ \overline{f}(s) &= \int\limits_0^a f(x)\cos(sx)\;dx \\ m^2 &= \frac{\gamma}{\lambda_{11}h} \qquad\qquad \chi = \frac{\lambda_{11}}{c\rho} \qquad\qquad \lambda^2 = \frac{\lambda_{22}}{\lambda_{11}} \end{aligned} \tag{2.4}$$

From $(2.4)_2$ it follows that only the symmetric problem with respect to the y axis is considered, since it is assumed that $f(x)$ is an even function. For the general case of the function $f(x)$, its odd part will be associated with Fourier's sine transform and the solution can be obtained in a similar manner; formally by replacement of $\cos(sx)$ with $\sin(sx)$ functions in Eq. $(2.4)_2$.

2.2. Thermal stress and displacement

We consider the stress and displacement field. The stress-strain equations for an orthotropic medium under a plane stress state are

$$\begin{aligned} \sigma_{xx} &= c_{11}e_{xx} + c_{12}e_{yy} - \beta_1 T \\ \sigma_{yy} &= c_{12}e_{xx} + c_{22}e_{yy} - \beta_2 T \\ \sigma_{xy} &= 2Ge_{xy} \end{aligned} \tag{2.5}$$

where e_{ij} are the strain components, σ_{ij} are the stress components, c_{ij} are the moduli of elasticity of the material, G is the shear modulus, $\beta_1 = c_{11}\alpha_1+c_{12}\alpha_2$,

$\beta_2 = c_{12}\alpha_1 + c_{22}\alpha_2$ and α_1, α_2 are the thermal expansion coefficients along the x- and y-directions, respectively. The strain components are

$$e_{xx} = \frac{\partial u_x}{\partial x} \qquad e_{yy} = \frac{\partial u_y}{\partial y} \qquad e_{xy} = \frac{1}{2}\Big(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\Big) \tag{2.6}$$

where u_x and u_y are the displacement components along the axis. The equations of equilibrium for the plane stress in the absence of the body forces are

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0 \qquad \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0 \tag{2.7}$$

From (2.5), (2.6) and (2.7), it follows

$$\begin{aligned} c_{11}\frac{\partial^2 u_x}{\partial x^2} + G\frac{\partial^2 u_x}{\partial y^2} + (c_{12}+G)\frac{\partial^2 u_y}{\partial x \partial y} &= \beta_1 \frac{\partial T}{\partial x} \\ G\frac{\partial^2 u_y}{\partial x^2} + c_{22}\frac{\partial^2 u_y}{\partial y^2} + (c_{12}+G)\frac{\partial^2 u_x}{\partial x \partial y} &= \beta_2 \frac{\partial T}{\partial y} \end{aligned} \tag{2.8}$$

The general solution to equilibrium equations (2.8) may be obtained as the superposition of two fields. The first corresponds to the solution to homogeneous equation (2.8), for which (Rogowski, 1975)

$$\begin{aligned} u_x &= \frac{\partial}{\partial x}(k\varphi_1 + \varphi_2) & u_y &= \frac{\partial}{\partial y}(\varphi_1 + k\varphi_2) \\ \sigma_{xx} &= -G(k+1)\frac{\partial^2}{\partial y^2}(\varphi_1 + \varphi_2) & \sigma_{yy} &= -G(k+1)\frac{\partial^2}{\partial x^2}(\varphi_1 + \varphi_2) \\ \sigma_{xy} &= G(k+1)\frac{\partial^2}{\partial x \partial y}(\varphi_1 + \varphi_2) & &\frac{\partial^2 \varphi_i}{\partial x^2} + \frac{1}{s_i^2}\frac{\partial^2 \varphi_i}{\partial y^2} = 0 \qquad (i=1,2) \end{aligned} \tag{2.9}$$

where $(i = 1,2)$

$$Gc_{22}s_i^4 - (c_{11}c_{22} - c_{12}^2 - 2c_{12}G)s_i^2 + Gc_{11} = 0 \qquad k = \frac{c_{22}s_1^2 - G}{c_{12} + G} \tag{2.10}$$

The second may be obtained in terms of the thermoelastic displacement potential function $\psi(x,y,t)$, defined as follows

$$u_x = \frac{\partial \psi}{\partial x} \qquad u_y = l\frac{\partial \psi}{\partial y} \tag{2.11}$$

Equations (2.8) are satisfied if

$$
\begin{aligned}
&c_{11}\frac{\partial^2\psi}{\partial x^2}+G\frac{\partial^2\psi}{\partial y^2}+l(c_{12}+G)\frac{\partial^2\psi}{\partial y^2}=\beta_1 T\\
&Gl\frac{\partial^2\psi}{\partial x^2}+c_{22}l\frac{\partial^2\psi}{\partial y^2}+(c_{12}+G)\frac{\partial^2\psi}{\partial x^2}=\beta_2 T
\end{aligned}
\tag{2.12}
$$

A suitable expression for ψ defined by (2.12) for temperature distribution in (2.3) is in the form

$$
\psi=\int\limits_0^\infty\Big[\int\limits_0^\infty C(s,p,t)\cos(py)\;dp\Big]\cos(sx)\;ds
\tag{2.13}
$$

This satisfies both equations (2.12) providing

$$
\begin{aligned}
&C(s,p,t)[c_{11}s^2+Gp^2+lp^2(c_{12}+G)]=-\beta_1\theta(s,p,t)\\
&C(s,p,t)[l(c_{22}p^2+Gs^2)+s^2(c_{12}+G)]=-\beta_2\theta(s,p,t)
\end{aligned}
\tag{2.14}
$$

i.e.

$$
\begin{aligned}
l(s,p)&=\frac{\beta_1 s^2(c_{12}+G)-\beta_2(c_{11}s^2+p^2G)}{\beta_2 p^2(c_{12}+G)-\beta_1(c_{22}p^2+s^2G)}\\
C(s,p,t)&=\theta(s,p,t)\frac{\beta_2 p^2(c_{12}+G)-\beta_1(c_{22}p^2+s^2G)}{(c_{11}s^2+Gp^2)(c_{22}p^2+Gs^2)-(c_{12}+G)^2p^2s^2}
\end{aligned}
\tag{2.15}
$$

Appropiate solutions to Eqs $(2.9)_6$ are

$$
\begin{aligned}
\varphi_1(x,y)&=-\frac{s_2}{G(k+1)(s_1-s_2)}\int\limits_0^\infty s^{-1}A(s)\mathrm{e}^{-s_1 sy}\cos(sx)\;ds\\
\varphi_2(x,y)&=\frac{s_1}{G(k+1)(s_1-s_2)}\int\limits_0^\infty s^{-1}B(s)\mathrm{e}^{-s_2 sy}\cos(sx)\;ds
\end{aligned}
\tag{2.16}
$$

Using the above obtained potentials, we find

$$\begin{aligned}
u_x(x,y,t) &= \frac{1}{G(k+1)(s_1-s_2)} \int\limits_0^\infty [ks_2 A(s)\mathrm{e}^{-s_1 sy} - s_1 B(s)\mathrm{e}^{-s_2 sy}] \sin(sx) ds - \\
&- \int\limits_0^\infty \Big[\int\limits_0^\infty sC(s,p,t)\cos(py)\; dp\Big] \sin(sx)\; ds
\end{aligned} \tag{2.17}$$

$$\begin{aligned}
u_y(x,y,t) &= \frac{s_1 s_2}{G(k+1)(s_1-s_2)} \int\limits_0^\infty [A(s)\mathrm{e}^{-s_1 sy} - kB(s)\mathrm{e}^{-s_2 sy}] \cos(sx)\; ds - \\
&- \int\limits_0^\infty \Big[\int\limits_0^\infty pC(s,p,t) l(s,p)\sin(py)\; dp\Big] \cos(sx)\; ds
\end{aligned}$$

$$\begin{aligned}
\sigma_{xx}(x,y,t) &= \frac{s_1 s_2}{s_1-s_2} \int\limits_0^\infty s[s_1 A(s)\mathrm{e}^{-s_1 sy} - s_2 B(s)\mathrm{e}^{-s_2 sy}] \cos(sx)\; ds + \\
&+ G\int\limits_0^\infty \Big\{\int\limits_0^\infty p^2 C(s,p,t)[l(p,s)+1]\cos(py)\; dp\Big\} \cos(sx)\; ds \\
\sigma_{yy}(x,y,t) &= -\frac{1}{s_1-s_2} \int\limits_0^\infty s[s_2 A(s)\mathrm{e}^{-s_1 sy} - s_1 B(s)\mathrm{e}^{-s_2 sy}] \cos(sx)\; ds + \\
&+ G\int\limits_0^\infty \Big[\int\limits_0^\infty s^2 C(s,p,t)[l(p,s)+1]\cos(py)\; dp\Big] \cos(sx)\; ds \\
\sigma_{xy}(x,y,t) &= -\frac{s_1 s_2}{s_1-s_2} \int\limits_0^\infty s[A(s)\mathrm{e}^{-s_1 sy} - B(s)\mathrm{e}^{-s_2 sy}] \sin(sx)\; ds + \\
&+ G\int\limits_0^\infty \Big[\int\limits_0^\infty psC(s,p,t)[l(p,s)+1]\sin(py)\; dp\Big] \sin(sx)\; ds
\end{aligned} \tag{2.18}$$

The mechanical boundary conditions on the plane $y = 0$ are

$$\sigma_{xy} = 0 \tag{2.19}$$

$$\sigma_{yy} = 0 \quad \text{on} \quad |x| < a \qquad\qquad u_y = 0 \quad \text{on} \quad |x| \geqslant a \tag{2.20}$$

Applying $(2.18)_3$ to boundary condition (2.19), we obtain $A = B$.

Substituting $(2.17)_2$ and $(2.18)_2$ into boundary conditions (2.20) and using $A = B$, we obtain the following dual integral equations for $A(s)$

$$\begin{aligned} &\int_0^\infty sA(s)\cos(sx)\,ds = \int_0^\infty s^2 F(s,t)\cos(sx)\,ds && \text{on} \quad |x| < a \\ &-\frac{1}{GC}\int_0^\infty A(s)\cos(sx)\,ds = 0 && \text{on} \quad |x| \geqslant a \end{aligned} \tag{2.21}$$

where

$$\begin{aligned} F(s,t) &= -G\int_0^\infty C(s,p,t)[l(p,s)+1]\,dp \\ C &= (k+1)(k-1)^{-1}(s_2^{-1} - s_1^{-1}) \end{aligned} \tag{2.22}$$

Equation $(2.21)_1$ may be replaced by the following equation

$$\int_0^\infty A(s)\sin(sx)\,ds = \int_0^\infty sF(s,t)\sin(sx)\,ds \tag{2.23}$$

We introduce an integral representation of the function $A(s)$

$$A(s) = \int_0^a x'h(x')J_0(sx')\,dx' \tag{2.24}$$

where $J_0(sx')$ is the Bessel function of the first kind and zero order, and $h(x')$ is a new unknown function. This representation satisfies equation $(2.21)_2$ and converts equation (2.23) to the Abel integral equation for $h(x')$

$$\int_0^x \frac{x'h(x')}{\sqrt{x^2 - x'^2}}\,dx' = \int_0^\infty sF(s,t)\sin(sx)\,ds \qquad |x| < a \tag{2.25}$$

The solution to this equation is

$$h(x') = \int_0^\infty s^2 F(s,t)J_0(sx')\,ds \tag{2.26}$$

where the following integral were employed

$$\frac{2}{\pi}\frac{1}{x'}\frac{d}{dx'}\int_0^{x'} \frac{x\sin(sx)}{\sqrt{x'^2 - x^2}}\,dx = sJ_0(sx') \tag{2.27}$$

Thus, the solution to the dual integral equations of form (2.21) is

$$A(s)=\int\limits_0^a x'J_0(sx')\Big[\int\limits_0^\infty q^2J_0(qx')F(q,t)\;dq\Big]\;dx' \tag{2.28}$$

The above formula is exactly the same as that obtained by Sneddon (1966, p.98) for dual integral equations of type (2.21). Therefore, we obtain the complete solution to the problem by substituting (2.28) and $(2.22)_1$ into (2.17) and (2.18).

The singular stress $\sigma_{yy}(x,0)$ is obtained as follows

$$\sigma_{yy}(x,0)=\frac{xh(a)}{\sqrt{x^2-a^2}}=\frac{x}{\sqrt{x^2-a^2}}\int\limits_0^\infty s^2F(s,t)J_0(sa)\;ds \qquad \text{as} \quad x\to a^+ \tag{2.29}$$

Therefore, the stress intensity factor K_I of mode I is defined as

$$K_I=\lim_{x\to a^+}\sqrt{2\pi(x-a)}(\sigma_{yy})_{y=0}=\sqrt{\pi a}\int\limits_0^\infty s^2F(s,t)J_0(sa)\;ds \tag{2.30}$$

Note that

$$\begin{aligned}GC(s,p,t)[l(p,s)+1]&=-E_1\frac{\alpha_1p^2+\alpha_2p^2}{\delta^2s^4+2\mu s^2p^2+p^4}\theta(s,p,t)=\\&=-E_1\Big(\frac{\alpha_1+c_0}{s_1^2s^2+p^2}-\frac{c_0}{s_2^2s^2+p^2}\Big)\theta(s,p,t)\end{aligned} \tag{2.31}$$

where s_1^2 and s_2^2 are the roots of algebraic equation $(2.10)_1$, which may be written in an equivalent form

$$s_i^4-2\mu s_i^2+\delta^2=0 \qquad \mu=\frac{E_1}{2G}-\nu_{21} \qquad \delta^2=\frac{E_1}{E_2} \tag{2.32}$$

and

$$c_0=\frac{\alpha_1s_2^2-\alpha_2}{s_1^2-s_2^2}=\frac{\alpha_1(\mu-\sqrt{\mu^2-\delta^2})-\alpha_2}{2\sqrt{\mu^2-\delta^2}} \tag{2.33}$$

Here E_1 and E_2 are Young's moduli in the x- and y-directions, respectively, and ν_{21} is Poisson's ratio.

The stress intensity factor K_I is calculated from (2.30), and we get the formula

$$K_I = -\frac{4\sqrt{a}}{\pi\sqrt{\pi}}\frac{q_0\chi\lambda^2 E_1}{\lambda_{22}}\int\limits_0^\infty s^2\overline{f}(s)J_0(as)\int\limits_0^\infty\Big(\frac{\alpha_1+c_0}{s_1^2s^2+p^2}-\frac{c_0}{s_2^2s^2+p^2}\Big)\cdot \\ \cdot\int\limits_0^t g(t-\tau)\mathrm{e}^{-\chi(m^2+s^2+\lambda^2p^2)\tau}\,d\tau\,ds\,dp \tag{2.34}$$

For an isotropic material we have $s_1 = s_2 = 1$, $\alpha_1 = \alpha_2 = \alpha$, $\lambda = 1$, and stress intensity factor (2.34) assume the form

$$(K_I)_{iso} = -\frac{4\sqrt{a}}{\pi\sqrt{\pi}}\frac{q_0\chi E\alpha}{\lambda_{iso}}\int\limits_0^\infty s^2\overline{f}(s)J_0(as)\int\limits_0^\infty\frac{1}{s^2+p^2}\cdot \\ \cdot\int\limits_0^t g(t-\tau)\mathrm{e}^{-\chi(m^2+s^2+p^2)\tau}\,d\tau\,ds\,dp \tag{2.35}$$

The displacement $u_y(x,0)$ is obtained in the form

$$u_y(x,0) = \frac{4\sqrt{2}}{\pi^2}\frac{q_0\chi\lambda^2\delta\sqrt{\mu+\delta}}{\lambda_{22}}\int\limits_x^a\frac{x'\,dx'}{\sqrt{x'^2-x^2}}\int\limits_0^\infty s^2\overline{f}(s)J_0(sx')\cdot \\ \cdot\int\limits_0^\infty\Big(\frac{\alpha_1+c_0}{s_1^2s^2+p^2}-\frac{c_0}{s_2^2s^2+p^2}\Big)\int\limits_0^t g(t-\tau)\mathrm{e}^{-\chi(m^2+s^2+\lambda^2p^2)\tau}\,d\tau\,ds\,dp \tag{2.36}$$

The displacement $u_x(x,0)$ is given by the formula

$$u_x(x,0) = -\frac{4}{\pi^2}\frac{q_0\chi\lambda^2(\delta-\nu_{21})}{\lambda_{22}(s_2^2+\nu_{21})}\int\limits_0^\infty s\overline{f}(s)\sin(sx)\cdot \\ \cdot\int\limits_0^\infty\Big(\frac{c_1+c_2}{s_1^2s^2+p^2}-\frac{c_2}{s_2^2s^2+p^2}\Big)\int\limits_0^t g(t-\tau)\mathrm{e}^{-\chi(m^2+s^2+\lambda^2p^2)\tau}\,d\tau\,ds\,dp \tag{2.37}$$

where

$$
\begin{aligned}
a_1 &= 2 - \frac{G}{E_1}\nu_{12} & b_1 &= \frac{G}{E_1} & c_1 &= a_1\alpha_1 - b_1\alpha_2 \\
a_2 &= 1 - \frac{G}{E_2}\nu_{12} & b_2 &= \frac{G}{E_2} & c_2 &= \frac{c_1 s_2^2 - b_2\alpha_1 - a_2\alpha_2}{s_1^2 - s_2^2} \\
\frac{\nu_{12}}{E_2} &= \frac{\nu_{21}}{E_1}
\end{aligned}
\qquad (2.38)
$$

3. Numerical results

In calculating the temperature and the stress intensity factors, the following dimensionless quantities are introduced

$$
\begin{aligned}
\xi &= \frac{x}{a} & \eta &= \frac{y}{a} & t' &= \chi\frac{t}{a^2} \\
M^2 &= a^2m^2 = \frac{\gamma a^2}{\lambda_{11}h} & \lambda^2 &= \frac{\lambda_{22}}{\lambda_{11}} & \alpha &= \frac{\alpha_2}{\alpha_1} \\
\delta^2 &= E = \frac{E_1}{E_2} & \mu &= \frac{E_1}{2G} - \nu_{21} & \overline{T} &= \frac{T\lambda_{22}}{q_0 a} \\
\overline{K}_I &= \frac{K_I\lambda_{22}}{\alpha_1 E_1 a q_0\sqrt{a}} & \overline{\sigma}_{yy} &= \frac{\sigma_{yy}\lambda_{22}}{\alpha_1 E_1 a q_0}
\end{aligned}
$$

Numerical calculations were carried out for two types of the heat supply q

Case 1: $q = q_0 g(t) f(x) = q_0$

Case 2: $q = q_0 g(t) f(x) = q_0 \mathrm{e}^{-t'}$

where t' is the Fourier number.

Figure 2 shows the temperature at $\eta = 0$ for various values of λ^2. The temperature at $\eta = 0$ increases with the ratio of thermal conductivity. Figures 3a,b show the effects of λ^2 and α on the normal stress $\overline{\sigma}_{yy}$ at $\eta = 0$ for case 1 of thermal loading.

Figures 4a-d show the effects of anisotropies of the material constants on the stress intensity factor for case 1 and case 2. It is assumed that only one of the material constants λ^2, α, E, μ indicates various anisotropies, while the other constants are kept equal to those of isotropic conditions.

Figures 2-4 show that the anisotropy effects of the material constants λ^2, α, E and μ on the stress intensity factor are large. In the figures we can notice

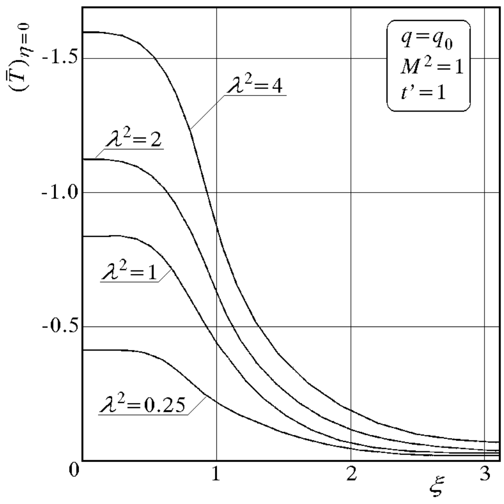

Fig. 2. Variation of temperature at $\eta = 0$ with ξ for various λ^2

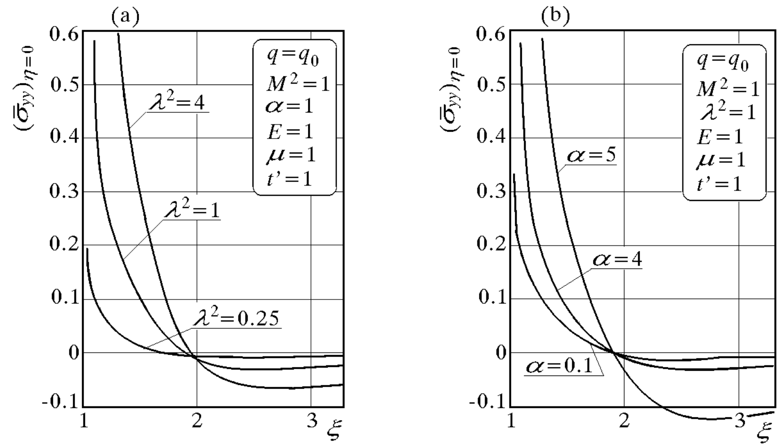

Fig. 3. Variation of normal stress $\overline{\sigma}_{yy}$ at $\eta = 0$ with ξ for various values of λ^2 (a) and α (b)

that the stress intensity factor increases with the thermal conductivity, Young's modulus and thermal expansion coefficient in the direction perpendicular to the crack plane.

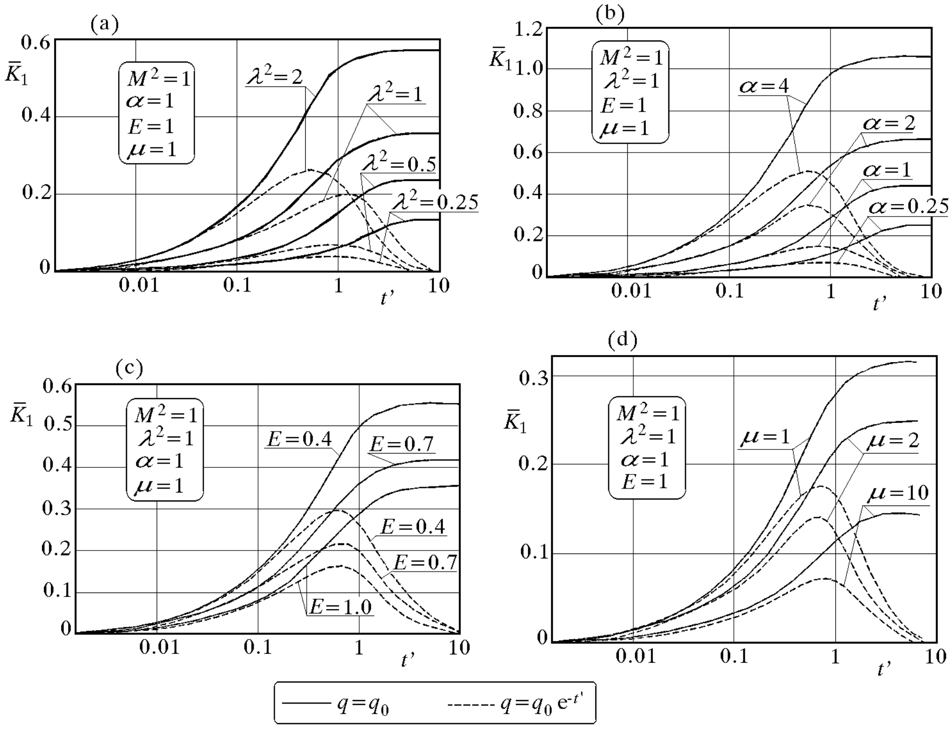

Fig. 4. Variation of stress intensity factor with t' for various values of λ^2 (a), α (b), E (c) and μ (d)

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O współczynnikach intensywności naprężenia dla nieustalonego termicznego obciążenia w ortotropowej cienkiej płycie ze szczeliną

Streszczenie

Rozpatrzono zagadnienie ortotropowej cienkiej płyty zawierającej szczelinę prostopadłą do jej brzegów. Założono, że nieustalone naprężenia termiczne powstają w wyniku przepływu przez powierzchnie szczeliny strumienia ciepła będącego funkcją czasu, miejsca i konwekcyjnego przepływu ciepła przez powierzchnie płyty. Znaleziono ścisłe, analityczne rozwiązanie określające współczynnik intensywności naprężeń i rozwarcie szczeliny. Przykłady numeryczne pokazują zależności temperatury, naprężeń i współczynnika intensywności naprężenia od parametrów geometrycznych i stałych określających własności termiczne i sprężyste materiału.

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