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GEARBOX DYNAMIC MODELLING

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The paper reports on current developments in gearbox dynamic modelling. It refers to Müller's one-mass two-parameter (stiffness and damping) gearbox model with rectilinear vibration. The paper shows that there is a need to develop a new model, which would incorporate torsional vibration. The paper refers to the previous papers on gearbox dynamic modelling published by the author in journals and conference proceedings. In the present paper, the influence of the clutch damping coefficient and one random parameter value from the three-parameter error mode, and that of the interaction between error parameters on the vibration generated by a gearbox system, is analyzed.

Key words: gearbox model, mathematical modelling, computer simulation

1. Introduction

The modelling of gearbox dynamic behaviour belongs to the fundamental problems of mechanical system modelling. The problem has been the subject of many papers in Poland and abroad: Wang (1974), Mark (1978), Velex and Maatar (1996), Smith (1998). The model presented by Müller in 1979 has received a lot of attention in Poland. It was discussed, for example, by Ryś (1977), Wilk (1981), and Dąbrowski (1992). The model is a two-parameter (stiffness and damping) model in which the inertia of two wheels has been reduced to one mass equivalent to a one-stage gearbox.

The author of the present paper has found that more sophisticated models are needed to describe the gearbox dynamics properly; Bartelmus (1994, 2000). Mathematical modelling and computer simulation can be applied to gearbox dynamics to support diagnostic signal evaluation for diagnostic inference. This is the main aim of the present research. The computer simulation is based on a mathematical model developed by Bartelmus (1994, 2000). General information on gearing needed for the computer simulation of gearbox behaviour was given by Bartelmus (2000). Some results of computer simulations supporting diagnostic inference were presented by Bartelmus (1996-1999). The papers show that mathematical modelling and computer simulation enable the detailed investigation of the dynamic properties of a gearing system. All the basic factors such as: design, technology, operation and change of the gearing system conditions which have a bearing on the vibration generated by a gearing can be investigated. The causes of vibration in gearboxes are mainly the tooth errors and vibration is an indication of them. The computer simulation results are referred to the laboratory investigation results presented by Rettig (1977) and to the field measurements reported by Penter (1991), Bartelmus (1992), and Tuma et al. (1994).

As mentioned above, the vibration of a gearbox indicates that there are tooth errors in it. The errors appear at the production stage. The nature of the gear-wheel interaction is such that non-linear phenomena occur caused by friction, intertooth backlash, impact-like intertooth forces and periodic changes in tooth stiffness. As a result, the intertooth forces may exceed the force values, which follow from the gearbox system's rated moment. Mathematical description enabling to include these phenomena in the equation of motion is given by Bartelmus (2000).

The intertooth forces increase dramatically under unstable conditions. A one-stage gear system operates under resonance conditions and is unstable when the gearbox system's mesh frequency is equal to its natural frequency. In such conditions, the intertooth forces are two times greater than the rated forces. The phenomenon of resonance has not been investigated fully for gearbox systems.

Computer simulations reveal that conditions similar to the ones occurring at resonance may result as errors (pitting, scuffing of teeth flanks and failure of bearings) increase during the service of a gearbox system. In the present paper, current developments in gearbox modelling are presented with reference to the previous papers by the present author. It is shown that a flexible clutch and an error mode random parameter have an influence on the gearbox stability (teeth separation). An error mode is described by several parameters, i.e. maximum error value, shape of error plot and random error fluctuation depth.

2. Modelling of gearbox system

The Müller (1979) one-stage gearbox model is shown in Fig. 1.

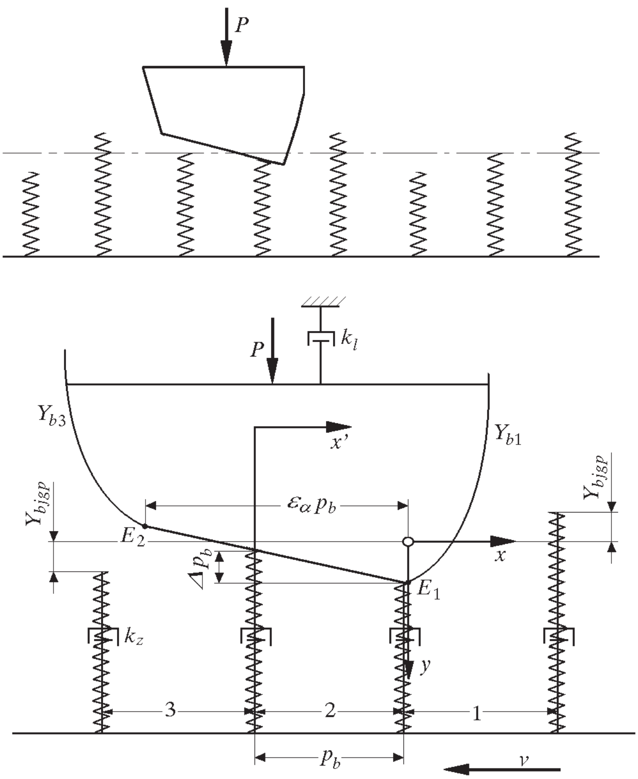

Fig. 1. One-mass, two-parameter model of gearbox by Müller (1979), where k_z – gearing's stiffness, l_z – bearing's stiffness, ε_α – tooth contact index, Y_{b1} – input tooth contact ordinate, Y_{b3} – output tooth contact ordinate, p_b – gear base pitch, Δ_p – gear base pitch error, Y_{bjgp} – gear pitch error, v – pitch line velocity, P – intertooth force

It is a two-parameter (stiffness and damping) model. The inertia of the two gear-wheels is reduced to one mass. The motion of the mass is equivalent to the relative motion of the two gear-wheels. The motion is caused by the relative motion of springs (having different lengths) hitting the mass. The motion of the springs with velocity v [m/s] is equivalent to the pitch-line velocity of the wheels. As one can see in the model shown in Fig. 1, the motion of the mass has no influence on the instant change of v as in actual gearboxes. This weakness of the Müller (1979) model and lack of possibility of building multistage gearbox

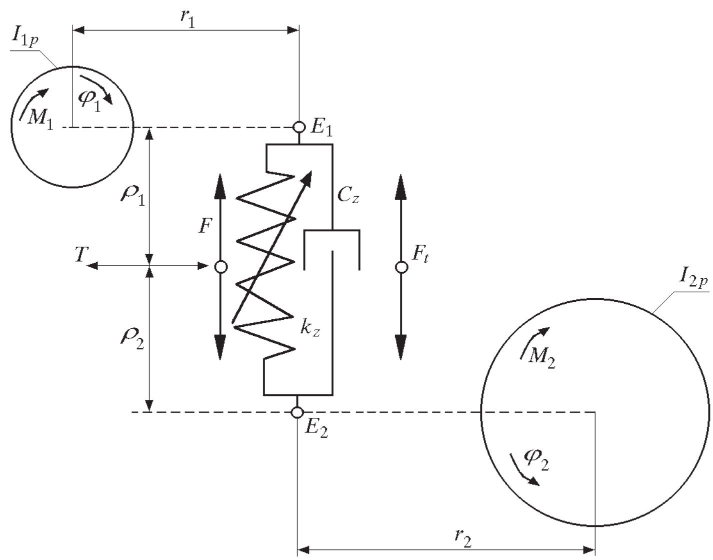

Fig. 2. Two-wheel, two-parameter model of gearbox

models made the author look for a new model. It is more convenient to use a model with the rotary motion of the wheels and torsional vibration, and thus to overcome the weakness of the Müller model. The simplest model of this kind is shown in Fig. 2. The rotation equations of motion may be given by statements according to the second Newton's law of motion

$$\begin{aligned} I_{1p}\ddot{\varphi}_1 &= M_1 - r_1(F + F_t) + M_{zt1} \\ I_{2p}\ddot{\varphi}_2 &= r_2(F + F_t) - M_2 - M_{zt2} \end{aligned} \tag{2.1}$$

To get closer to reality, a more sophisticated model is considered. The model is shown in Fig. 3. The mathematical model of torsional vibration of the system is given by the following equations

$$\begin{aligned} I_s\ddot{\varphi}_1 &= M_s(\dot{\varphi}_1) - M_1 - M_{1t} \\ I_{1p}\ddot{\varphi}_2 &= M_1 + M_{1t} - r_1(F + F_t) + M_{zt1} \\ I_{2p}\ddot{\varphi}_3 &= r_2(F + F_t) - M_2 - M_{zt2} \\ I_m\ddot{\varphi}_4 &= M_2 - M_r \end{aligned} \tag{2.2}$$

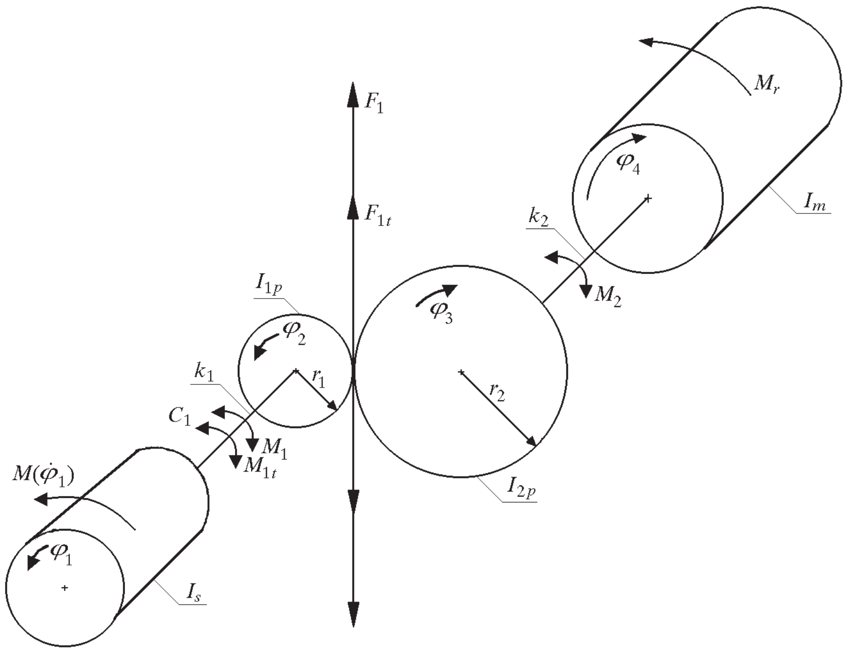

Fig. 3. System with one-stage gearbox

The values of forces and moments are given by

$$M_1 = k_1(\varphi_1 - \varphi_2) \qquad M_2 = k_2(\varphi_3 - \varphi_4) \qquad M_{1t} = C_1(\dot{\varphi}_1 - \dot{\varphi}_2) \tag{2.3}$$

Using the functions min and max one may write a formula for the inter-tooth force

$$\begin{aligned} F &= k_z(p_{om}, g)\Big(\max\Big(A - l_u + E(p_{om}, a, e, r), \\ &\qquad \min\big(A + l_u + E(p_{om}, a, e, r), 0\big)\Big)\Big) \\ F_t &= C_z(A') \qquad\qquad A = r_1\varphi_2 - r_2\varphi_3 \end{aligned} \tag{2.4}$$

where

$\varphi, \dot{\varphi}, \ddot{\varphi}$	–	rotation angle, angular velocity, angular acceleration
$M_s(\dot{\varphi})$	–	electric motor driven moment characteristic
M_1, M_2	–	moments of shaft stiffness

I_s, I_m	–	moments of inertia for electric motor and driven machine
M_{1t}	–	damping moment of clutch/coupler
C_1	–	damping coefficient of the coupler
F, F_t	–	stiffness and damping intertooth forces
k_1, k_2	–	stiffness of shafts
M_{zt1}, M_{zt2}	–	intertooth moment of friction, $M_{zti} = T\rho_i$, $i = 1, 2$
T	–	intertooth force of friction, Fig. 2
$k_z(p_{om}, g)$	–	gearing stiffness function
r_1, r_2	–	gear base radii
l_u	–	intertooth backlash
$E(p_{om}, a, e, r)$	–	error mode function
p_{om}, a, e, r	–	parameters of error function, $p_{om} = \mathrm{frac}(\varphi_2 z_1/(2\pi))$, where "frac" denote fractional part
z_1	–	number of teeth in the pinion.

A full description of the model is given in Bartelmus (1994) and after modification – in Bartelmus (2000).

For a given pair of teeth, the value of an error is random and it can be denoted by

$$e(random) = [1 - r(1 - l_i)]e \tag{2.5}$$

where

e	–	maximum error value
r	–	coefficient of error scope, scope $(0-1)$
l_i	–	random value, scope $(0-1)$.

A symbolic description of an error characteristic (error mode) is $E(a, e, r)$, parameters of the error mode were described earlier above, a is given in Fig. 4b and Fig. 4c, changes in the range $(0-1)$, and it indicates the position of the maximum error value on the line of action. As an example, an error mode for $E(0.5, 10, 0.3)$ is given in Fig. 4b and for $E(0.1, 10, 0.3)$ in Fig. 4c.

As it follows from the above discussion, gearing's dynamic properties depend on several factors. The factors may be divided, according to Bartelmus (1992, 1998a), into:

- design factors
- technology factors
- operation factors
- change of condition factors.

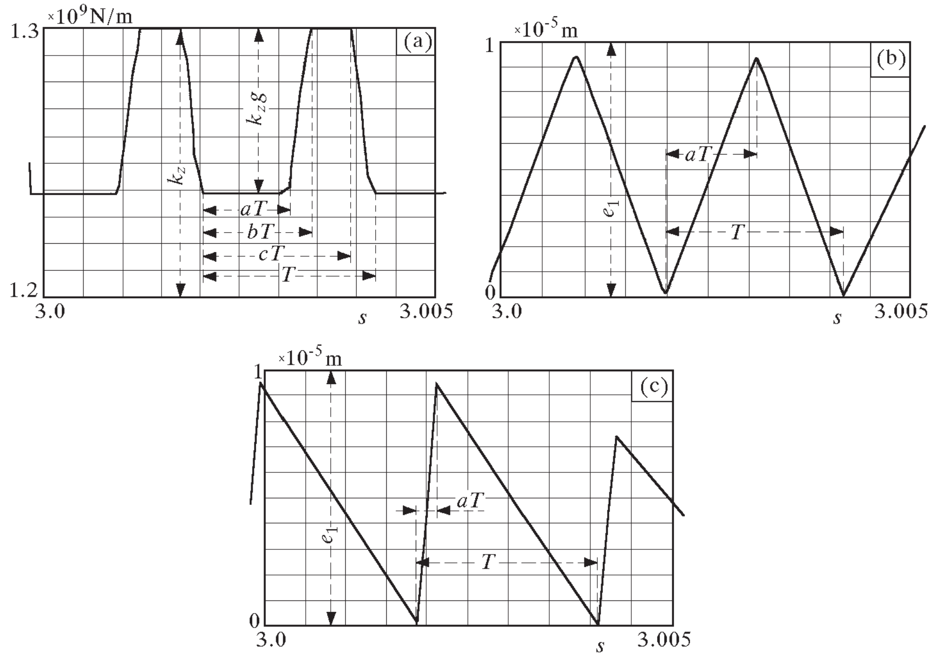

Fig. 4. (a) Stiffness function for gearing, (b) error function for error mode $E(0.5, 10, 0.3)$, (c) error function for error mode $E(0.1, 10, 0.3)$

Numerical solutions of the differential equations are obtained by means of CSSP (Continuous System Simulation Program) developed by Siwicki (1992), using the England procedure of integration. The procedure is a general procedure of the Runge-Kutta type and it assures the stability of integration, even in the case of discontinuity, as well as it enables error estimation and the automatic change of the integration step.

3. Results of computer simulations

The results of computer simulations published by the author (Bartelmus, 1996-2000) have been compared with the results of rig investigations by Rettig (1977) and field investigations by Penter (1991), Bartelmus (1992), Tuma (1994). As mentioned above, the aim of the present paper was to study the influence of random intertooth errors on the vibration generated by the gearbox system (Fig. 3) for different clutch damping coefficients: $C_1 = 0$ and $C_1 > 0$.

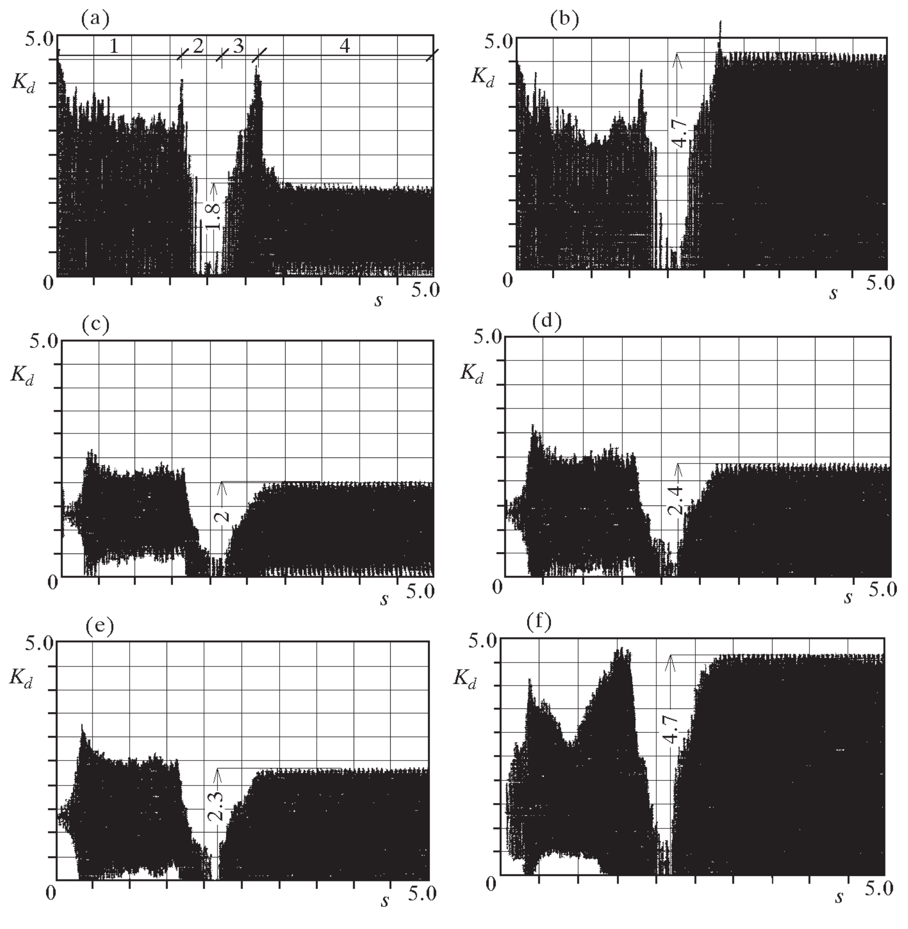

Fig. 5. (a) Plot of K_d for error mode $E(0.5, 12, 0.3)$, stiffness coefficient $C_1 = 0$; (b) plot of K_d for $E(0.5, 15, 0.3)$, $C_1 = 0$; (c) plot of K_d for $E(0.5, 15, 0.3)$, $C_1 = 1000$; (d) plot of K_d for $E(0.5, 20, 0.3)$, $C_1 = 1000$; (e) plot of K_d for $E(0.5, 20, 0.15)$, $C_1 = 1000$; (f) plot of K_d for $E(0.1, 20, 0.3)$, $C_1 = 1000$

The random intertooth error is described by error mode $E(a,e,r)$, where $e(random)$ is given by (2.5). Author's previous studies on gearing dynamics were for the error mode described by two parameters, $E(a,e)$ and $C_1 = 0$. According to Bartelmus (1996), an increase in intertooth error $e\,[\mu\mathrm{m}]$ causes a linear increase of dynamic factor K_d, but only to a certain value of e. At this value, a nonlinear effect caused by unstable running (teeth separation) is observed. Dynamic factor K_d is defined as a ratio of the current intertooth force to the rated intertooth force. The value of e at which unstable running (teeth separation effect) is observed depends also on the value of parameter a in error mode $E(a,e)$. The unstable running of a gearbox occurs when $K_d > 2$. Also parameter r, which specifies the relative scope of the intertooth fluctuation, is connected with the instability. In conditions when the error mode is given by $E(0.5,12,0.3)$ and $C_1 = 0$, the system shown in Fig. 3 runs unstably ($K_d = 1.8$, Fig. 5a). If the error mode is changed to $E(0.5,15,0.3)$ and $C_1 = 0$, the running of the gearbox system will be unstable, as shown in Fig. 5b, for a constant rated rotation and a constant outer load. Fig. 5 shows period (1) in which gear rotation increases from 0 to 107 rad/s. No external load is applied to the gearbox system, the gearbox system is loaded only by the inertia forces. In period (2) the gearbox system runs without any load. In period (3) the outer load increases from 0 to the rated load. In period (4) the gearbox system runs under a stable rated load. In the present paper, period (4) of the gearbox system run is the focus of attention. Bearing in mind that the damping clutch coefficient has an influence on the gearbox system dynamics, the plot of K_d for a run at $E(0.5,15,0.3)$ and $C_s = 1000$ Nms will be as the one shown in Fig. 5c. The plots for gearing conditions described by $E(0.5,20,0.3)$, $E(0.5,20,0.15)$ and $C_1 = 1000$ Nms are shown in Fig. 5d and Fig. 5e. The gearbox system runs then at slight instability: $K_d = 2.4$ and 2.3. If parameter a is decreased from 0.5 to 0.1 at error mode $E(0.1,20,0.15)$ and $C_1 = 1000$ Nms, the plot of factor K_d is like the one shown in Fig. 5f, where dynamic factor $K_d = 4.7$. If parameter e is further increased, the value of the dynamic factor does not change.

4. Conclusions

The paper has shown a detailed analysis of a mutual influence of the error mode parameters and influence of a clutch damping on a dynamic factor K_d, which reveals the gearing condition. The error mode is described by three parameters: a – shape parameter, e – maximum value of the error, r – scope of the error random change. The analysis proves that, in general conditions of a gearing change (change of e and r), the linear increase of K_d does not hold, as it is stated in Bartelmus (1992) (experiment on a real object) and 1996 (computer simulation experiment). In the experiment on a real object and in the computer simulation experiment, only the value of e was taken into consideration.

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Modelowanie dynamiki przekładni zębatych

Streszczenie

Praca przedstawia aktualny rozwój zagadnień związanych z modelowaniem dynamiki przekładni zębatych. Nawiązuje ona do jednomasowego, dwuparametrowego (sztywność i tłumienie) modelu przekładni Müllera o ruchu prostoliniowym. Pokazuje potrzebę opracowania nowego modelu, który uwzględni drgania skrętne. Nawiązuje do wcześniejszych publikacji autora pracy na temat modelowania przekładni publikowanych w czasopismach i materiałach konferencyjnych. W prezentowanej pracy przedstawiono analizę wpływu tłumienia w sprzęgle, wpływ losowego parametru w trójparametrowej funkcji błędu na drgania generowane przez przekładnię zębatą.

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