JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
4, **39**, 2001

NUMERICAL SIMULATION OF SOLIDIFICATION OF A CASTING TAKING INTO ACCOUNT FLUID FLOW AND HEAT TRANSFER PHENOMENA. THE AXISYMMETRICAL PROBLEM

Ryszard Parkitny
Leszek Sowa

Institute of Mechanics and Machine Design, Technical University of Częstochowa
e-mail: parkitny@imipkm.pcz.czest.pl

In the paper, a mathematical and a numerical model of solidification of a cylindrical shaped casting, which takes into account the process of filling the mould cavity with the molten metal, has been proposed. The feeding of the casting by a riser head during solidification has also been taken into consideration. Velocity fields were obtained by solving the Navier-Stokes equations and the continuity equation, while thermal fields were obtained by solving the conduction equation containing the convection term. The changes in the thermophysical parameters, with respect to temperature, were taken into consideration. The problem was solved by the finite element method.

Key words: solidification, molten metal flow, mould filling, mathematical modelling

1. Introduction

This paper concerns modelling of a solidification process taking into account phenomena of heat transfer and fluid flow during the initial stage of a steel casting process in metal moulds. During this period, the molten metal motions have essential influence on the solidification kinetics. Analysis of solidification kinetics, by determining velocity and temperature fields in a system of riser-casting-metal moulds, was made for the case of pouring from the middle of the bottom (centrally). The isolines of temperature, which limit the solid and liquid-solid regions, were used to predict the closing zone of such a region. The liquid phase feeding process was then stopped and the shrinkage cavity was formed. Numerical simulation of casting solidification both with and without taking into account the molten metal motion was carried out. In the first case the solidification process, taking into account both the fluid flow and heat transfer phenomena, was considered. This is a complex and difficult problem to solve numerically. In the second case, only the heat conductivity equation was solved. The mould cavity was assumed to be fully filled by the molten metal at the pouring temperature as an initial condition for computation. The influence of the metal motion on the solid phase growth kinetics during the solidification process was estimated.

2. Mathematical modelling of casting solidification taking into account liquid phase motion in a cylindrical axisymmetry coordinate system

The proposed model for numerical simulation of solidification gives consideration to both the motions of the metal liquid phase during the mould cavity filing process and convective motions after pouring. It is based on solving the following system of differential equations (Dantzig, 1989; Dhatt and Gao, 1990; Mishima and Szekely, 1989; Sowa, 1998):
— the Navier-Stokes equations and the continuity ones

$$\begin{gathered}\mu\Big(\frac{\partial^2 v_r}{\partial r^2}+\frac{1}{r}\frac{\partial v_r}{\partial r}+\frac{\partial^2 v_r}{\partial z^2}-\frac{v_r}{r^2}\Big)-\frac{\partial p}{\partial r}+\rho g_r\beta(\Theta-\Theta_\infty)=\rho\frac{dv_r}{dt}\\ \mu\Big(\frac{\partial^2 v_z}{\partial r^2}+\frac{1}{r}\frac{\partial v_z}{\partial r}+\frac{\partial^2 v_z}{\partial z^2}\Big)-\frac{\partial p}{\partial z}+\rho g_z\beta(\Theta-\Theta_\infty)=\rho\frac{dv_z}{dt}\end{gathered} \tag{2.1}$$

$$\frac{\partial v_r}{\partial r}+\frac{v_r}{r}+\frac{\partial v_z}{\partial z}=0 \tag{2.2}$$

— the heat conduction equation containing the convection term

$$\frac{\lambda}{r}\frac{\partial\Theta}{\partial r}+\frac{\partial}{\partial r}\Big(\lambda\frac{\partial\Theta}{\partial r}\Big)+\frac{\partial}{\partial z}\Big(\lambda\frac{\partial\Theta}{\partial z}\Big)-\rho C_{ef}\Big(v_r\frac{\partial\Theta}{\partial r}+v_z\frac{\partial\Theta}{\partial r}\Big)-\rho C_{ef}\frac{\partial\Theta}{\partial t}=0 \tag{2.3}$$

— the equation of the state, reduced to the form

$$f(p,\rho,\Theta)=\rho(\Theta) \tag{2.4}$$

where

ρ	–	density, [kg/m^3]
p	–	pressure, [N/m^2]
v_r, v_z	–	rth and zth component of the velocity, respectively, [m/s]
μ	–	dynamic viscosity, [Ns/m^2]
g_r, g_z	–	rth and zth component of the acceleration of gravity, respectively, [m/s^2]
β	–	volume coefficient of thermal expansion, [1/K]
Θ	–	temperature, [K]
Θ_∞	–	reference temperature $\Theta_\infty = \Theta_{in}$, [K]
C_{ef}	–	effective specific heat, [J/(kgK)]
λ	–	thermal conductivity, [W/(mK)]
r, z	–	coordinates of the vector of a given node position, [m]
r	–	radius, [m]
t	–	time, [s].

In the applied model of solid phase growth, the internal heat sources are not evident in the equation of heat conductivity, because they are in the effective specific heat of the mushy zone (Bokota and Parkitny, 1991; Mishima and Szekely, 1989; Parkitny et al., 1998).

The system of equations (2.1)-(2.4) is completed by appropriate initial and boundary conditions.

The initial conditions for the velocity and temperature fields are given as

$$
\begin{gathered}
v(r,z,t_0) = v_0(r,z) = v_{in}|_{\Gamma_{1-1}} \\
\Theta(r,z,t_0) = \Theta_0(r,z) = \begin{cases} \Theta_{in} & \text{on } \Gamma_{1-1} \\ \Theta_A & \text{in } \Omega_A \\ \Theta_M & \text{in } \Omega_M \end{cases}
\end{gathered}
\tag{2.5}
$$

The boundary conditions, on the indicated surfaces (Fig. 1), specified in the considered problem are as follows:

— for velocity (Mishima and Szekely, 1989; Sowa, 1998)

$$
\begin{array}{lll}
v_n|_{\Gamma_{1-1}} = v_{in} & v_t|_{\Gamma_{1-1}} = v_t|_{\Gamma_{2-2}} = v_n|_{\Gamma_{2-2}} = 0 & \\
v_n|_{r=0} = 0 & v_n|_{\Gamma_G} = 0 & v_t|_{\Gamma_G} = 0 \\
v_n|_{\Gamma_{AL}} = v_z^* & \left.\dfrac{\partial v_t}{\partial n}\right|_{r=0} = 0 &
\end{array}
\tag{2.6}
$$

where

$$
v_z^* = \frac{d_1^2}{d_0^2} v_{in} \qquad \text{or} \qquad v_z^* = \frac{d_1^2}{d_n^2} v_{in}
$$

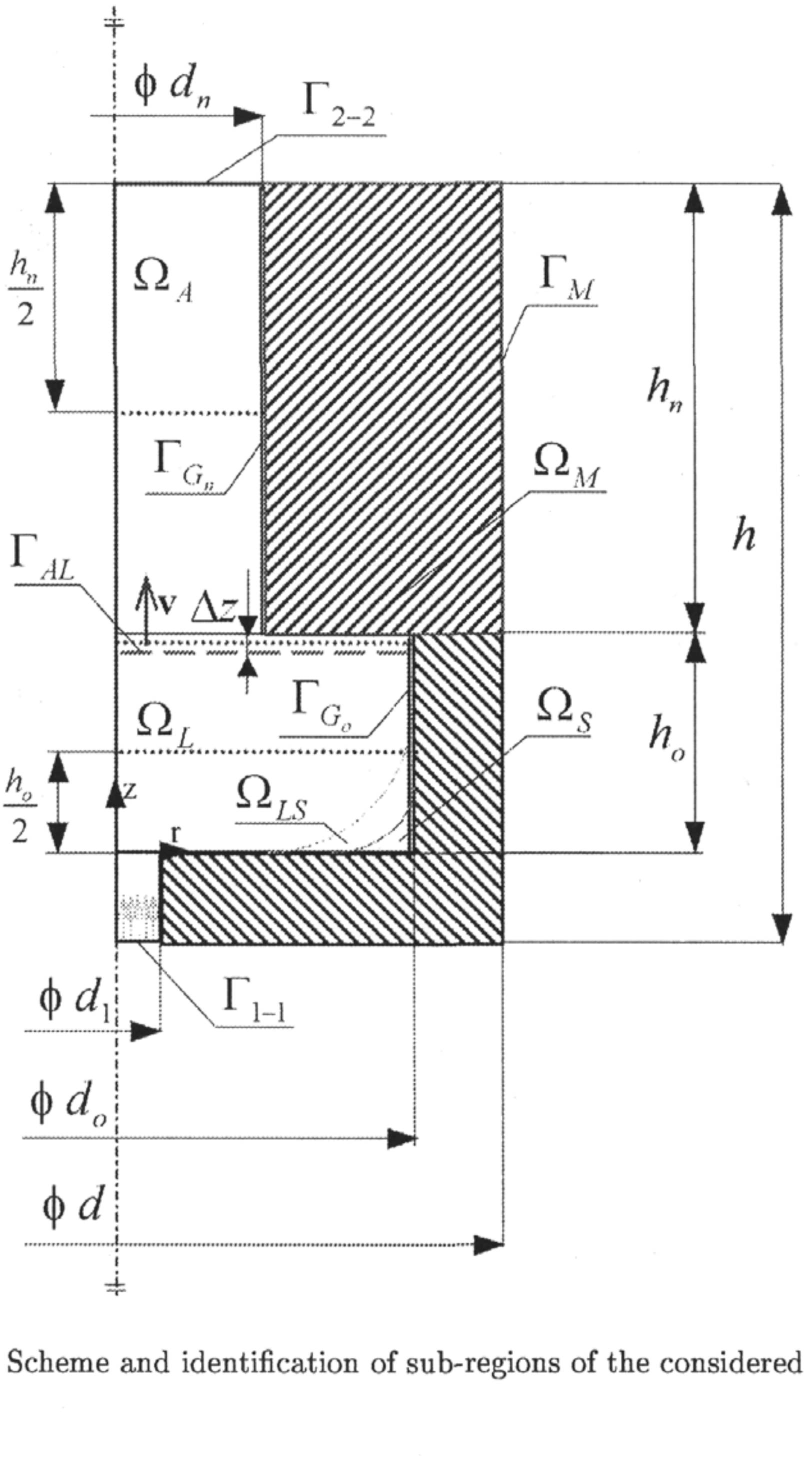

Fig. 1. Scheme and identification of sub-regions of the considered region

— for temperature (Dhatt and Gao, 1990; Parkitny et al., 1998)

$$
\begin{array}{ll}
\Theta|_{\Gamma_{1-1}} = \Theta_{in} & \dfrac{\partial \Theta}{\partial n}\Big|_{r=0} = 0 \\
\dfrac{\partial \Theta}{\partial n}\Big|_{\Gamma_{2-2}} = 0 & \lambda_M \dfrac{\partial \Theta_M}{\partial n}\Big|_{\Gamma_M} = -\alpha_M \Big(\Theta_M|_{\Gamma_M} - \Theta_a \Big) \\
\lambda_S \dfrac{\partial \Theta_S}{\partial n}|_{\Gamma_{G-}} = \lambda_G \dfrac{\partial \Theta_G}{\partial n}\Big|_{\Gamma_{G-}} & \lambda_G \dfrac{\partial \Theta_G}{\partial n}\Big|_{\Gamma_{G+}} = \lambda_M \dfrac{\partial \Theta_M}{\partial n}\Big|_{\Gamma_{G+}} \\
\lambda_L \dfrac{\partial \Theta_L}{\partial n}\Big|_{\Gamma_{AL-}} = \lambda_{AL} \dfrac{\partial \Theta_{AL}}{\partial n}\Big|_{\Gamma_{AL-}} & \lambda_{AL} \dfrac{\partial \Theta_{AL}}{\partial n}\Big|_{\Gamma_{AL+}} = \lambda_A \dfrac{\partial \Theta_A}{\partial n}\Big|_{\Gamma_{AL+}}
\end{array}
\tag{2.7}
$$

where

Θ_a – ambient temperature, [K]

α_M – heat-transfer coefficient between the mould and surrounding environment, $[\mathrm{W/(m^2K)}]$.

The isolated zone, which is considered to be the protective coating with the thermal conductivity $\lambda_{AL} = \lambda_{G_n}$, is assumed to be on the free surface. The problem has been solved by the finite element method (cf. e.g. Bokota and Parkitny, 1991; Sowa, 1998).

3. Example of numerical calculations

The calculations were carried out for the casting-mould-surrounding system with the longitudinal section shown in Fig. 1. The given dimensions of essential elements of that system were as follows: $d = 0.260\,\mathrm{m}$, $d_0 = 0.200\,\mathrm{m}$, $d_n = 0.100\,\mathrm{m}$, $d_1 = 0.020\,\mathrm{m}$, $h = 0.250\,\mathrm{m}$, $h_0 = 0.070\,\mathrm{m}$, $h_n = 0.150\,\mathrm{m}$. The considered regions were discretized by a net of quadrilateral finite elements $n_e = 2104$, unequivocally determined by $n_n = 2202$ nodes located in corners of the elements.

Thermophysical properties of a cast steel alloy poured into a cast iron mould were taken from literature (Bokota and Parkitny, 1991). The linear change of the density ρ and thermal conductivity λ was assumed in the $\Theta_L \div \Theta_S$ temperature interval. The variability of the dynamic viscosity μ with respect to temperature was determined according to Hirai's relationship (Hirai, 1993). The dependence $\mu(\Theta)$ was valid up to 0.9 of the volume fraction of the solid phase Φ in the mushy zone. The viscosity was observed to suddenly increase above 0.9 of Φ, and that was the reason to assume $\mu_S = 10^3\,\mathrm{Ns/m^2}$

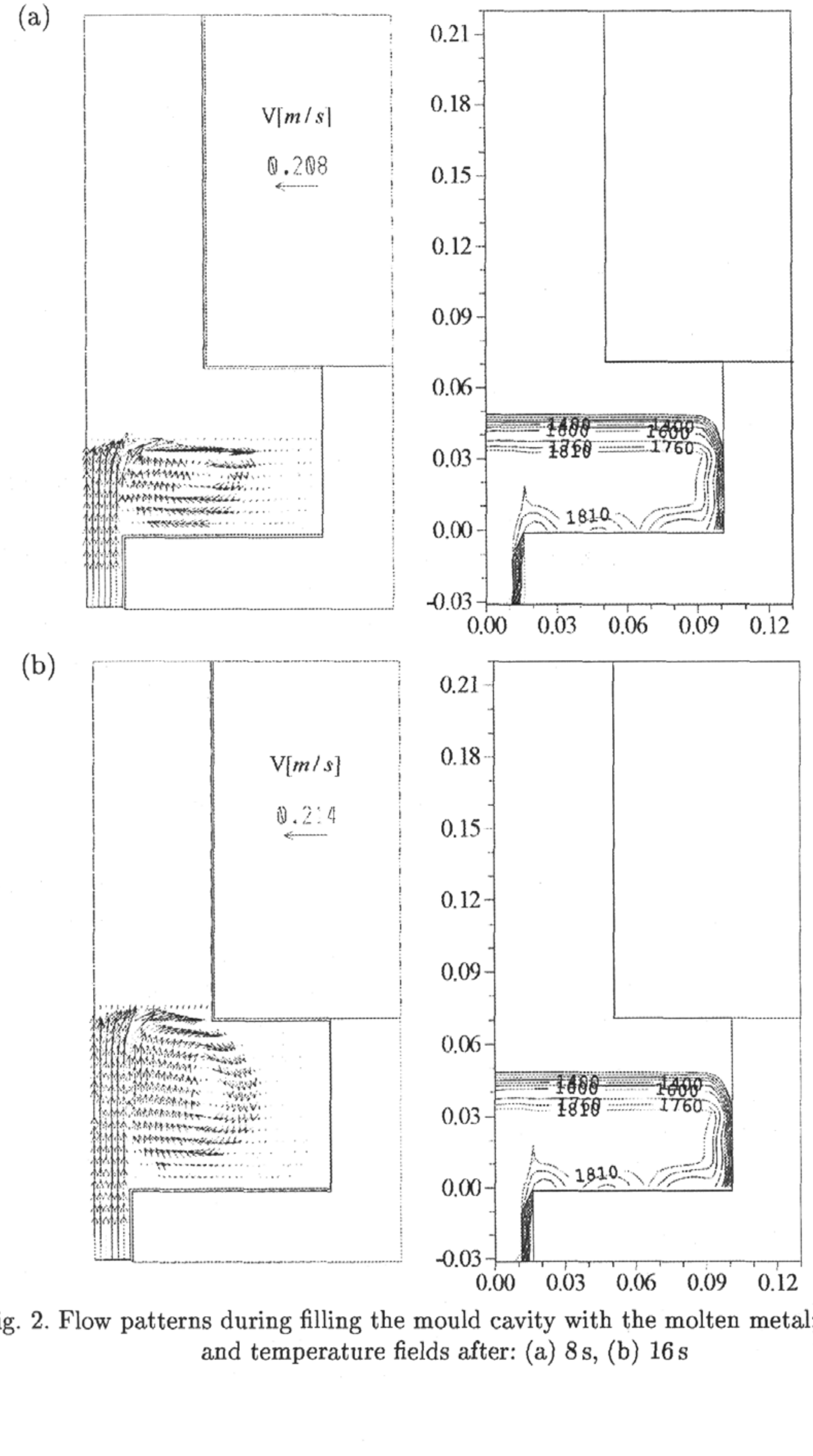

Fig. 2. Flow patterns during filling the mould cavity with the molten metal; velocity and temperature fields after: (a) 8 s, (b) 16 s

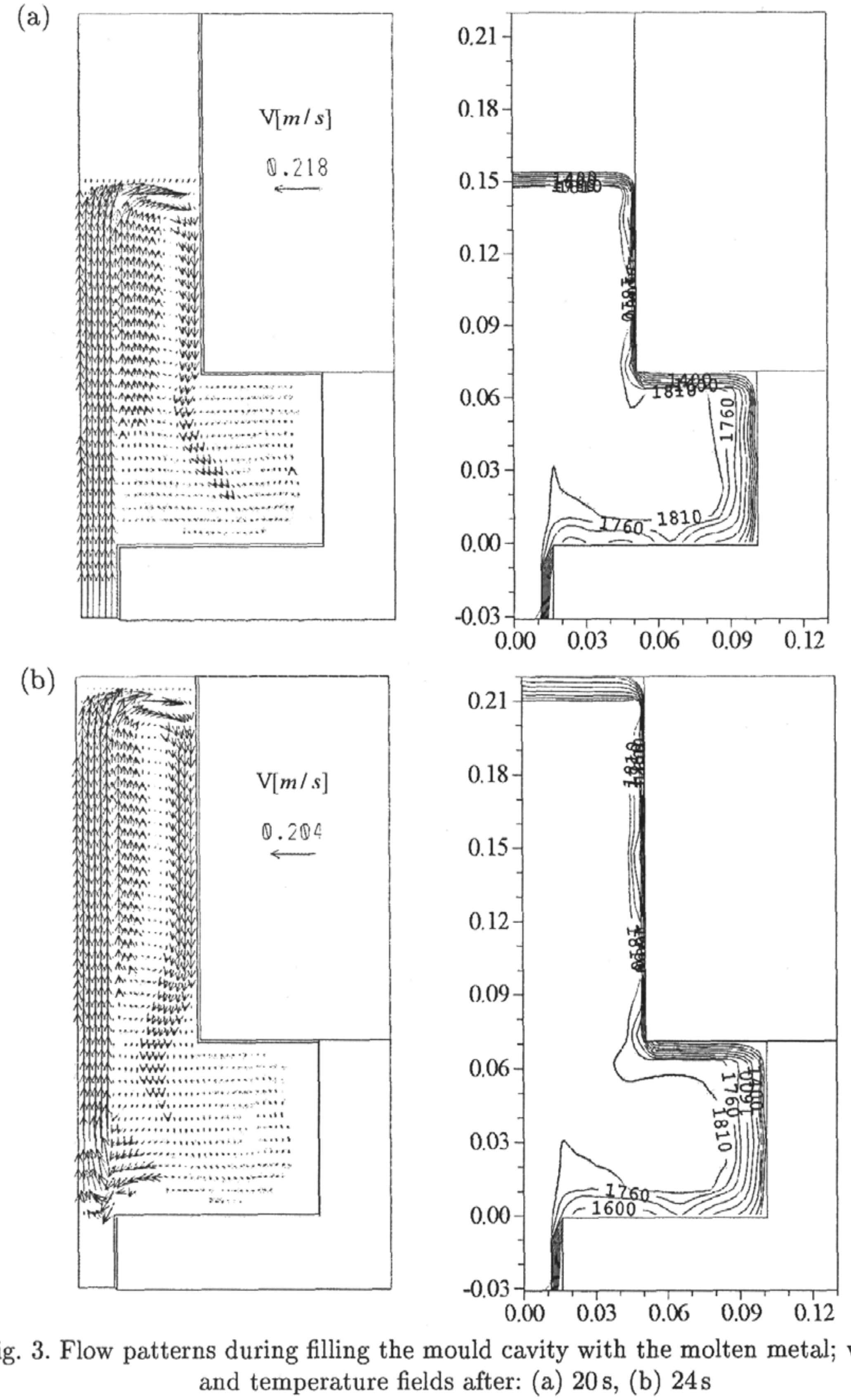

Fig. 3. Flow patterns during filling the mould cavity with the molten metal; velocity and temperature fields after: (a) 20 s, (b) 24 s

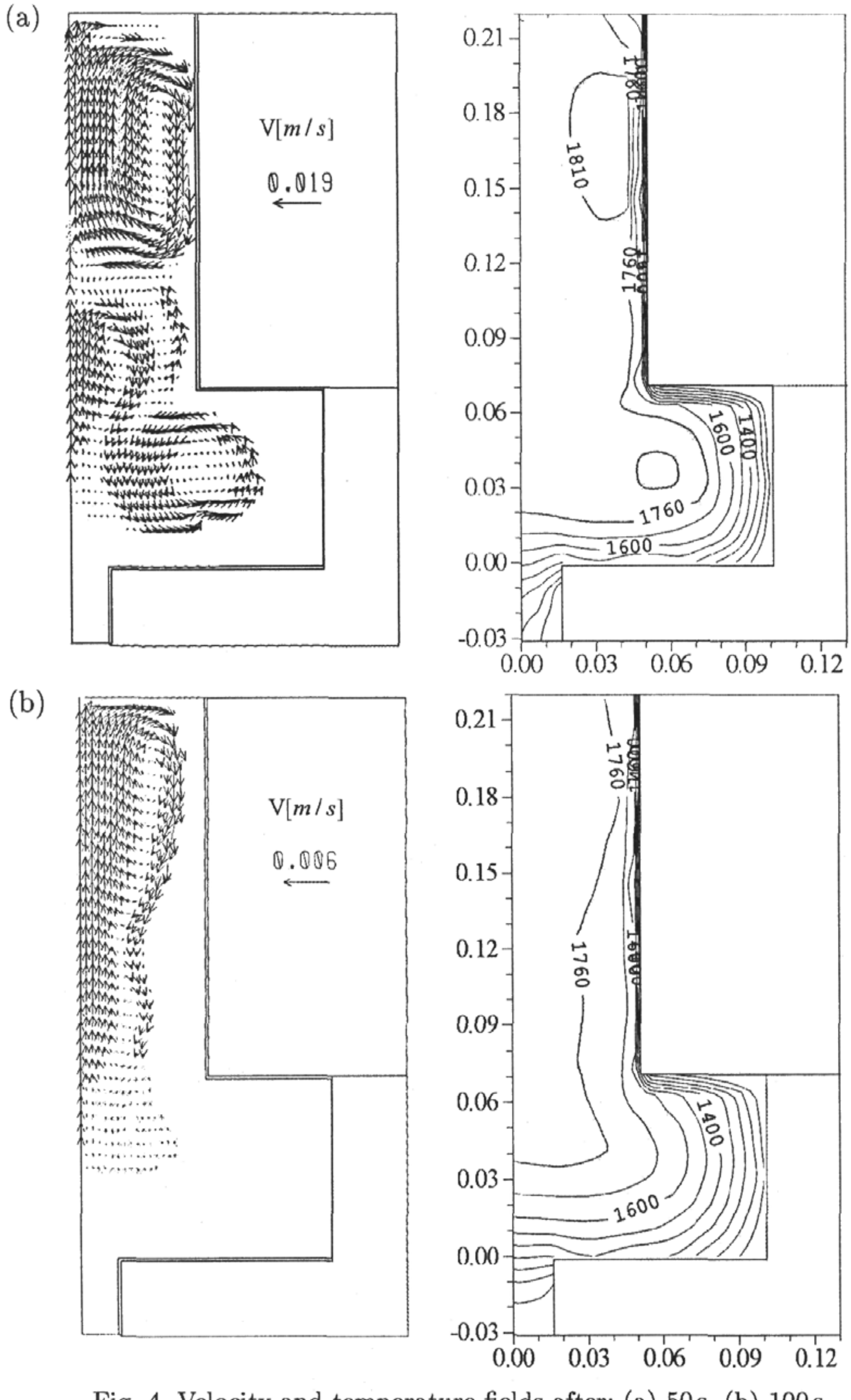

Fig. 4. Velocity and temperature fields after: (a) 50 s, (b) 100 s

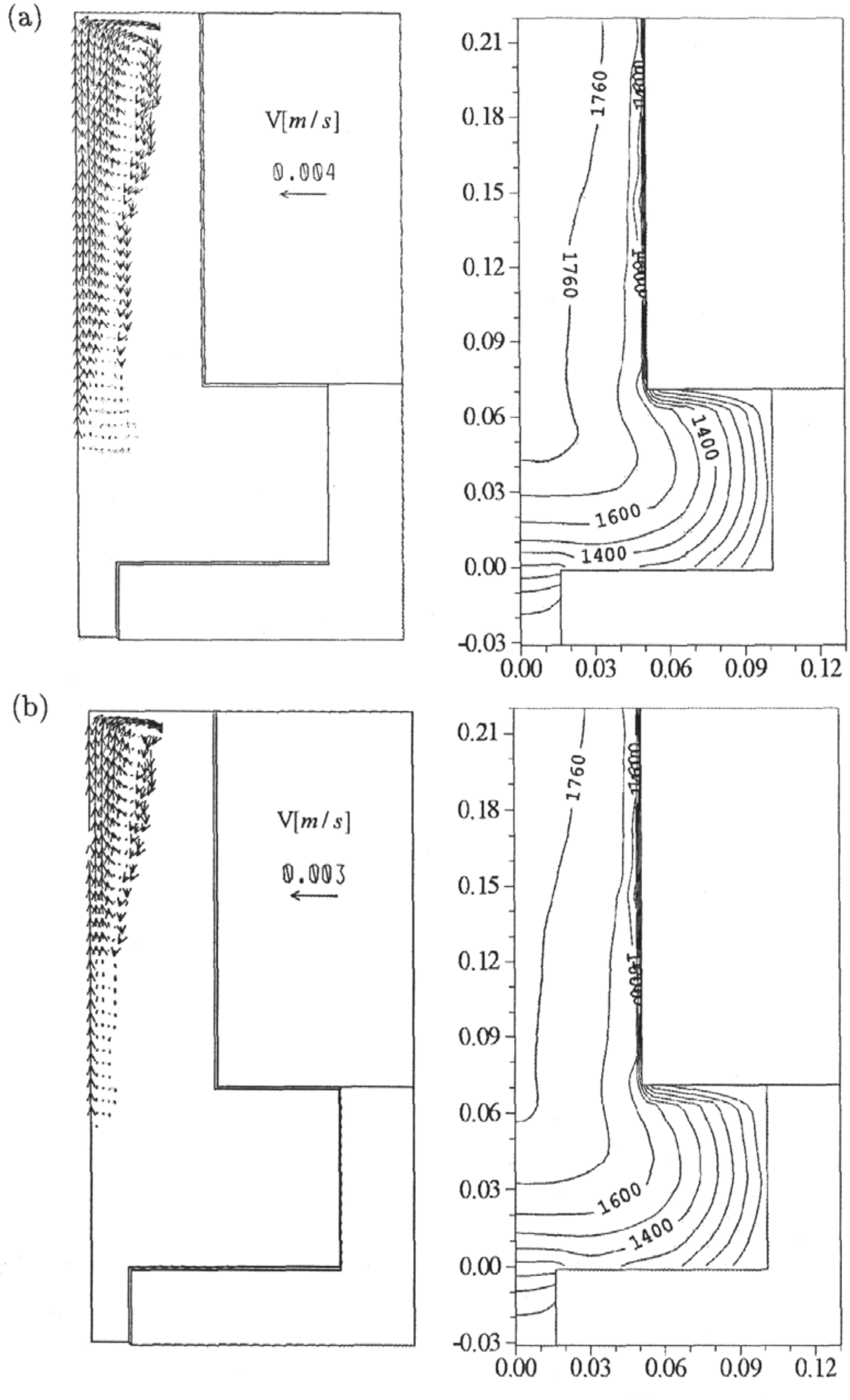

Fig. 5. Velocity and temperature fields after: (a) 120 s, (b) 140 s

to preserve stability of the calculations. In that way, the insignificant molten metal motion (or even lack of it) was taken into account near the solidus line.

A superheated metal of temperature $\Theta_{in} = 1850\,\mathrm{K}$ and velocity $v_{in} = 0.2\,\mathrm{m/s}$ was poured into a mould with the initial temperature $\Theta_M = 400\,\mathrm{K}$. The heat-transfer coefficient between the mould and surrounding $\alpha_M = 100\,\mathrm{W/(m^2K)}$, and the thermal conductivity coefficients of the protective coating $\lambda_{G_0} = 1$ and $\lambda_{G_n} = 0.12\,\mathrm{W/(mK)}$ were assumed. The other temperatures were: $\Theta_L = 1810\,\mathrm{K}$, $\Theta_S = 1760\,\mathrm{K}$, $\Theta_a = 300\,\mathrm{K}$, $\Theta_A = 420\,\mathrm{K}$. The flat free surface of the metal was established and moved with the net scale in consecutive time steps, which satisfied the continuity condition.

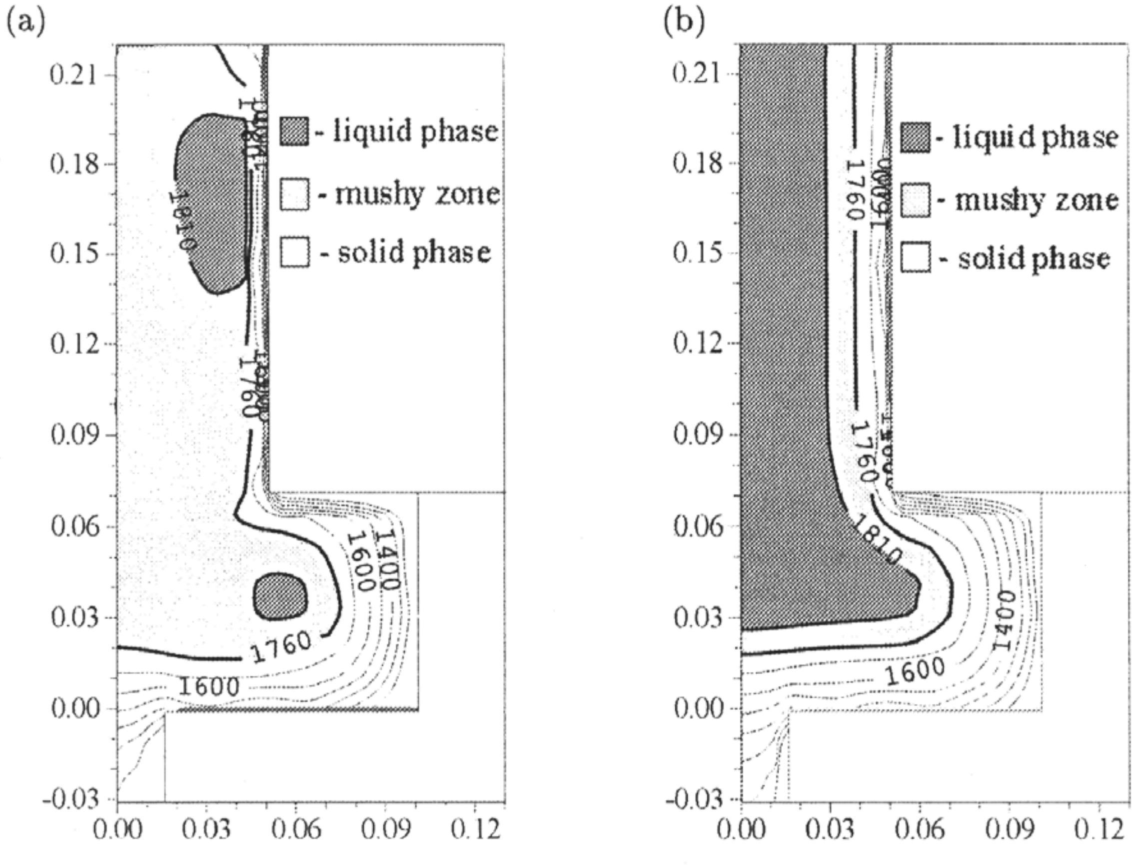

Fig. 6. Temperature fields after 50 s: (a) solidification with the metal motion taken into account, (b) solidification with the disregarded metal motion

The heat transfer and fluid flow phenomena, proceeding in the mould cavity during the filling (Fig. 2 and Fig. 3) and after it up to the completion of solidification (Fig. 4 and Fig. 5), were analysed. The influence of the metal motions on the solidification kinetics (Fig. 6) and on the shrinkage cavity location, determined by the position of the solidus line in the final moment of the casting solidification (Fig. 7), was determined.

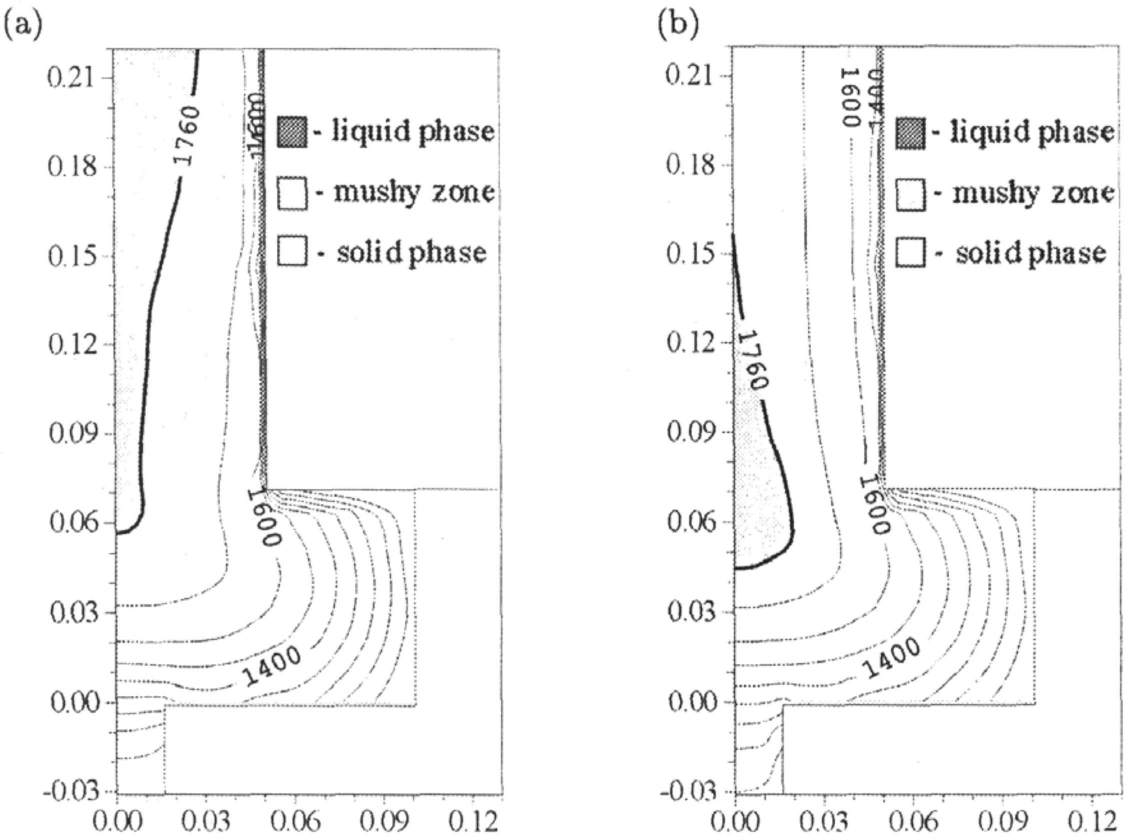

Fig. 7. Temperature fields after 140 s: (a) solidification with the metal motion taken into account, (b) solidification with the disregarded metal motion

4. Conclusions

This paper presents a coupled model of solidification for evaluation of transient fluid flow and heat transfer during casting solidification processes. Changes in thermophysical parameters, with respect to temperature, were taken into consideration in the model. The problem was solved by the finite element method. Numerical analysis included the filling process of the mould cavity with the molten metal, fluid flow, convectional motions of the molten metal and the solidification process. The calculation results were compared with those obtained from computations of statical solidification, in which motion of the molten metal was neglected and the mould cavity was assumed to be fully filled with the molten metal at the temperature of pouring as the initial condition for computation.

Calculations of the solidification, with taking into account the molten metal motions, give a picture of the non-uniform increment of the solid phase with the possible presence of circulatory motions (Fig. 4 and Fig. 6). The mushy zone of such solidification (cf. Fig. 6a and Fig. 6b) occupies an extensive region, in comparison to the statical solidification. The position of the shrinkage cavity (position of the solidus line in the final phase of solidification) is subjected to significant modification (cf. Fig. 7). Thus, taking into account the filling and convectional motions of the molten metal, enables detailed evaluation of casting conditions and more precise identification of the position of the shrinkage cavity.

Acknowledgement

The work was supported by the State Committee for Scientific Research (KBN) under grant No. 7 T07A 009 17.

References

1. BOKOTA A., PARKITNY R., 1991, Elastic-Plastic States in Solidifying Casting, *Archives Mechanics*, **43**, 2-3, 249-269
2. DANTZIG J.A., 1989, Modelling Liquid-Solid Phase Changes with Melt Convection, *Int. J. for Num. Meth. in Engrg.*, **28**, 1769-1785
3. DHATT G., GAO D.M., 1990, A Finite Element Simulation of Metal Flow in Moulds, *Int. J. for Num. Meth. in Engrg.*, **30**, 821-831
4. HIRAI M., 1993, Estimation of Viscosities of Liquid Alloys, *ISIJ International*, **33**, 2, 251-258
5. MISHIMA S., SZEKELY J., 1989, The Modelling of Fluid Flow and Heat Transfer in Mould Filling, *ISIJ International*, **29**, 4, 324-332
6. PARKITNY R., BOKOTA A., SOWA L., 1998, Numerical Modelling of the Casting Solidification During Filling Process of Mould Cavity (in Polish), *Scientific Works of Department of Applied Mechanics Silesian Technical University*, **6**, 287-292
7. SOWA L., 1998, Model of the Casting Solidification Taking into Consideration the Motion of Liquid Phase (in Polish), *Archives of Mechanical Technology and Automatization*, **18**, 287-296

Symulacja numeryczna krzepnięcia odlewu z uwzględnieniem zjawisk cieplno-przepływowych. Zadanie osiowo-symetryczne

Streszczenie

W pracy sformułowano model matematyczny i numeryczny procesu krzepnięcia odlewu o kształcie cylindrycznym z uwzględnieniem procesu wypełniania wnęki formy ciekłym metalem i zasilania odlewu przez nadlew podczas jego krzepnięcia. Uwzględniono wzajemną zależność zjawisk cieplnych i dynamicznych. Pola prędkości otrzymano z rozwiązania równań Naviera-Stokesa i równania ciągłości, natomiast pola temperatury z rozwiązania równania przewodzenia ciepła z członem konwekcyjnym. Uwzględniano zmianę parametrów termofizycznych w funkcji temperatury. Postawione zadanie rozwiązano metodą elementów skończonych.

Manuscript received March 28, 2000; accepted for print June 28, 2001