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ACTIVE DAMPING OF LAMINATED PLATES BY SKEWED PIEZOELECTRIC PATCHES

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In the paper the problem of active damping of transverse vibration of a rectangular viscoelastic laminated plate by piezoelectric elements with skewed material axes is studied. The symmetric specially orthotropic and simply supported laminate is subjected to a harmonic excitation. The control system consists of rectangular piezoelectric patches parallel to the plate edges, glued to the upper and lower plate surface and working as the sensor and actuator with a velocity feedback. The dynamic analysis is based on the classical laminated plate theory and the static-coupling model with the assumption of perfect bonding. Due to this model the interaction between the actuator and the laminate can be approximated by the equivalent moments uniformly distributed along the actuator edges. Considering the orthotropic electromechanical properties of the piezoelectric material and two-dimensional piezoelectric effect develops the analysis. The governing equations of the system are formulated for a non-zero skew angle between the natural axes of the piezoelectric material and the plate reference axes. The numerical results in terms of the frequency response show the influence of applying the skewed piezoelectric elements on the efficiency of active damping of transverse displacements.

Key words: active damping, skewed piezoelectric element, laminate, frequency response

1. Introduction

Piezoelectric materials like PZT (lead zirconate titanate) ceramics and PVDF (polyvinylidene fluoride) films have become popular in the field of mechatronics and other engineering applications. The idea of using piezoelectric elements as distributed sensors and actuators for control purposes of flexible structures has been studied and experimentally investigated by numerous researches. Some works have been focused on piezoelectric actuation and active damping of transverse vibrations of beams with bonded or embedded piezoelectric elements. The analysis is commonly simplified by assuming a static model of the interaction between the actuators and the main structure, and by neglecting the effect of piezoelectric elements on the mass and stiffness of the beam (cf Bailey and Hubbard, 1985; Clarc et al., 1991). Crawley and de Luis (1987) developed the static coupling model by the presence of an elastic bonding layer between the actuator and the beam. They showed that extremely high stiffness of the bonding layer (perfect bonding) results in equivalent concentrated moments acting at the actuator edges. The dynamic approach involving the tangential inertia forces of the actuator was proposed by Jie Pan et al. (1991). Tylikowski (1993) developed the dynamic coupling model including the bonding layer with the finite shearing stiffness. This approach was applied to active damping of beams (Pietrzakowski, 1997) and stabilisation of beam parametric vibrations (Tylikowski, 1999). The comparison of the coupling models presented by Pietrzakowski (2000) shows that the static approximation is quite good if the piezoelectric patches are sufficiently thin and bonded by stiff glue layers.

The two-dimensional piezoelectric effect can be utilised for active control of transverse vibrations of plates or shells. Basing on the classical laminated plate theory, Lee (1990) formulated constitutive equations of piezoelectric laminates, i.e. laminated composites with piezoelectric plies working as sensors and actuators. He also introduced the concept of modal damping by tailoring effective electrodes covering the piezoelectric layers. Dimitriadis et al. (1991) and Wang and Rogers (1991) confirmed that actuator patches perfectly bonded to an isotropic or laminated plate generate resultant moments along the actuator edges and indicated the possibility to control particular modes by changing the pattern of piezoelectric patches. The steady-state response of thin circular plates to excitation by actuators with annular and sectional shapes was studied by Tylikowski (2000). The dynamic problems of cylindrical shells controlled by active piezoelectric layers were considered by Baz and Chen (2000). Other papers, such as by Tzou and Tzeng (1990) and Ha et al. (1992) developed the finite element formulation for modelling the dynamic response of controlled structures basing on the classical laminated plate theory or the shear deformation theory, see Chandrashekhara and Agarval (1993) among others.

This paper deals with active damping of transverse vibrations of a simply supported viscoelastic laminated plate by means of piezoelectric patches, which serve as a sensor and actuator. The control strategy is based on a velocity feedback with a constant gain. Both the plate and piezoelectric elements are rectangular in shape and oriented parallel to each other. The piezoelectric patches are perfectly bonded to the upper and lower surfaces and their influence on the global properties of the laminate is neglected. The governing equations of the system are obtained due to the classical laminated plate theory. Assuming electromechanical orthotropy of the piezoelectric material develops the model of two-dimensional actuating and sensing effect. It is also assumed that the natural axes of the piezoelectric material are skewed with respect to the laminate reference axes. Therefore, beside the bending moment resultants acting at the actuator edges the twisting moment resultant occurs. The numerical results in terms of the frequency response function show the influence of applying the skewed piezoelectric elements on active damping efficiency.

2. Statement of the problem and the governing relations

The considered system is a thin rectangular composite plate simply supported at the edges. The plate is symmetrically laminated and composed of orthotropic layers the number or orientation of natural axes of which gives the global orthotropic behaviour. Passive energy dissipation results from the material viscoelasticity described by the Kelvin-Voigt rheological model. The active damping is obtained by the control system consisting of piezoelectric spatially distributed collocated sensors and actuators, which operate in a closed loop with a constant- gain velocity feedback. The piezoelectric rectangular patches are fully electroded and perfectly bonded to the top and bottom plate surfaces. The axes of the piezoelectric material properties 1, 2 are oriented at the angle θ with respect to the plate co-ordinate axes x, y while the axis 3 (polling direction) coincides with the axis z defined as normal to the laminate (see Fig. 1). Orthotropic electromechanical properties describe e.g. uniaxially stretched PVDF films or other piezoelectric materials similarly fabricated.

The transverse motion of the plate is excited by a uniformly distributed and harmonic disturbance of the intensity q_0 acting on the full or a limited surface.

2.1. Constitutive relations of the piezoelectric element

The coupled electromechanical behaviour of piezoelectric materials differs depending upon the axis of the applied electric field or the axis of the stress or

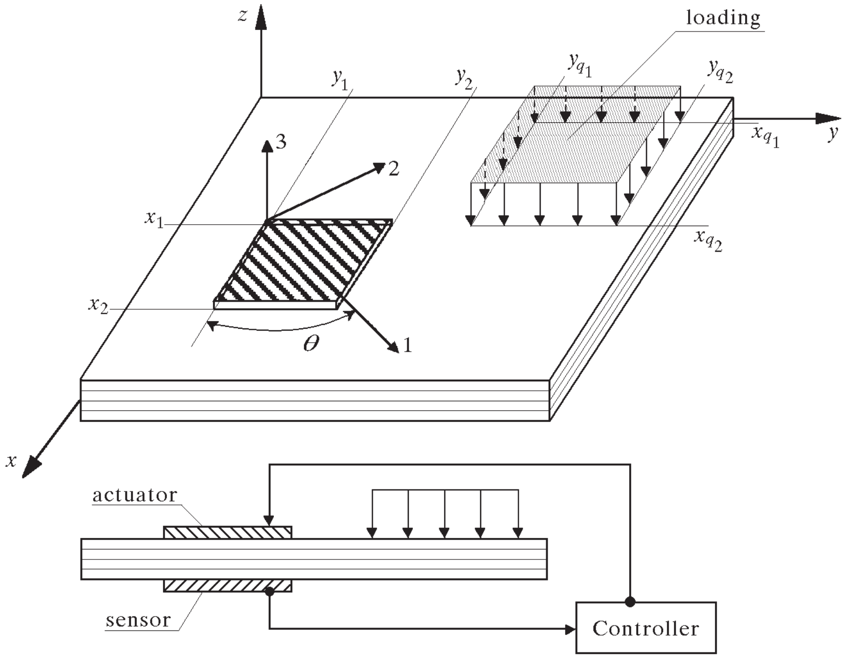

Fig. 1. Model of a laminated plate with the control device and a field of excitation

strain. The linear constitutive equations with respect to the principal material axes can be expressed in a matrix notation as (Cady, 1964)

$$\begin{aligned} \boldsymbol{\sigma} &= \mathbf{c}^{(E)}\boldsymbol{\varepsilon} - \mathbf{e}\boldsymbol{E} \\ \boldsymbol{D} &= \mathbf{e}^{\top}\boldsymbol{\varepsilon} + \in^{(\varepsilon)} \boldsymbol{E} \end{aligned} \tag{2.1}$$

where

$\boldsymbol{\sigma}, \boldsymbol{\varepsilon}$	–	representation of the stress and strain, respectively
$\boldsymbol{D}$	–	electric displacement
$\boldsymbol{E}$	–	electric field intensity
$\mathbf{c}$	–	stiffness matrix (symmetric)
$\in$	–	permittivity matrix (diagonal)
$\mathbf{e}$	–	piezoelectric stress/charge coefficient matrix.

The superscripts in brackets indicate the constant or vanishing field.

Governing relation $(2.1)_1$ explains that the stress in the piezoelectric material is proportional to both the applied strain and the applied electric field (converse piezoelectric effect). According to Eq $(2.1)_2$ the electric displacement is proportional to both the applied strain (direct piezoelectric effect) and the applied electric field (dielectric effect). The permittivity in Eq $(2.1)_2$ is measured at the constant strain condition.

In the considered piezoelectric patch the poling axis is the thickness or 3 axis, and the piezoelectric stress/charge coefficient matrix $\mathbf{e}$ is given by

$$\mathbf{e}^\top = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{24} & 0 & 0 \\ e_{31} & e_{32} & e_{33} & 0 & 0 & 3_{36} \end{bmatrix} \tag{2.2}$$

where

e_{31}, e_{32}, e_{33} – piezoelectric stress constants in the 1, 2 and 3 axis, respectively

e_{24}, e_{15}, e_{36} – piezoelectric shearing stress constants in the 2-3, 1-3 and 1-2 plane, respectively.

It should be noted that the piezoelectric shearing effect in the 1-2 plane is induced by a non-zero skew angle between arbitrarily chosen axes for cutting the element and the piezoelectric anisotropy axes.

Due to the control concept, the geometry and polarisation of the piezoelectric elements, the applicable direction of an electric field is along the 3 axis $(E_1 = E_2 = 0)$. For a sufficiently thin piezoelectric patch it is reasonable to suppose a plane stress state. Therefore, constitutive equations (2.1) can be rewritten in the two-dimensional version as follows

$$\begin{gathered} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{bmatrix} = \mathbf{c} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{bmatrix} - E_3 \begin{bmatrix} e_{31} \\ e_{32} \\ e_{36} \end{bmatrix} \\ D_3 = [e_{31}, e_{32}, e_{36}] \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{bmatrix} + \in_{33} E_3 \end{gathered} \tag{2.3}$$

The stiffness matrix $\mathbf{c}$ for orthotropic materials (e.g. uniaxially stretched PVDF films) in a plane stress state reduces to the form

$$\mathbf{c} = \begin{bmatrix} c_{11} & c_{12} & 0 \\ c_{12} & c_{22} & 0 \\ 0 & 0 & c_{66} \end{bmatrix} \tag{2.4}$$

where the matrix components depend on elastic constants: the Young moduli in the 1 and 2 directions E^p_{11} and E^p_{22}, the shear modulus G^p_{12}, the Poisson ratio ν_{12}^p, and are given by the well known relations

$$\begin{aligned}
c_{11} &= \frac{E_{11}^p}{1-\nu_{12}^p\nu_{21}^p} & c_{22} &= \frac{E_{22}^p}{1-\nu_{12}^p\nu_{21}^p} & c_{66} &= G_{12}^p \\
c_{12} &= \frac{\nu_{12}^p E_{22}^p}{1-\nu_{12}^p\nu_{21}^p} & \nu_{21} &= \frac{\nu_{12}^p E_{22}^p}{E_{11}^p}
\end{aligned} \tag{2.5}$$

In the analysed case the piezoelectric element natural axes 1, 2 are not coincident with the plate axes x, y. Constitutive relations Eqs (2.3) transformed to the reference axes of the plate, and after replacing the piezoelectric stress/charge coefficients $\mathbf{e}$ by the strain/charge coefficient matrix $\mathbf{d}$ according to the relation $\mathbf{e} = \mathbf{cd}$, can be written as

$$\begin{aligned}
\left[\begin{array}{c} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{array}\right] &= \overline{\mathbf{c}} \left[\begin{array}{c} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{array}\right] - E_3 \mathbf{T}^{-1}\mathbf{c} \left[\begin{array}{c} d_{31} \\ d_{32} \\ d_{36} \end{array}\right] \\
D_3 &= [d_{31}, d_{32}, d_{36}]\mathbf{c}\mathbf{T}^{-\top} \left[\begin{array}{c} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{array}\right] + \in_{33} E_3
\end{aligned} \tag{2.6}$$

where

$\overline{\mathbf{c}}$ – stiffness matrix of the piezoelectric material with respect to the plate reference axes

$$\overline{\mathbf{c}} = \mathbf{T}^{-1}\mathbf{c}\mathbf{T}^{-\top} \tag{2.7}$$

$\mathbf{T}$ – transformation matrix

$$\mathbf{T} = \left[\begin{array}{ccc} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2-n^2 \end{array}\right] \tag{2.8}$$

with $m = \cos\theta$, $n = \sin\theta$, where θ denotes the skew angle between the plate axes and the natural axes of the piezoelectric material.

2.2. Constitutive equations of the laminate

The displacement field of the considered thin laminated plate is approximated due to the Kirchhoff hypothesis and given by

$$u = u_0 - zw_{,x} \qquad v = v_0 - zw_{,y} \qquad w = w_0 \tag{2.9}$$

where

u, v, w	–	displacement components in the x, y and z directions, respectively
u_0, v_0, w_0	–	in-plane and transverse displacements of the midplane point, respectively.

The constitutive equations are formulated for midplane symmetric laminates, which are composed of n orthotropic layers and show specially orthotropic behaviour. The above statements yield the following relations that describe the uncoupled stretching and bending effects (cf Whitney, 1987)

$$\boldsymbol{N} = \mathbf{A}\boldsymbol{\varepsilon}_0 \qquad\qquad \boldsymbol{M} = \mathbf{D}\boldsymbol{\kappa} \tag{2.10}$$

where

$\boldsymbol{N}$	–	stress resultant, $\boldsymbol{N}^\top = [N_x, N_y, N_{xy}]$
$\boldsymbol{M}$	–	moment resultant, $\boldsymbol{M}^\top = [M_x, M_y, M_{xy}]$
$\boldsymbol{\varepsilon}_0$	–	midplane strain, $\boldsymbol{\varepsilon}_0^\top = [\varepsilon_x, \varepsilon_y, \varepsilon_{xy}]$
$\boldsymbol{\kappa}$	–	midplane curvature, $\boldsymbol{\kappa}^\top = [\kappa_x, \kappa_y, \kappa_{xy}]$
$\mathbf{A}, \mathbf{D}$	–	laminate in-plane stiffness matrix and bending stiffness matrix, respectively

$$(A_{ij}, D_{ij}) = \sum_{k=1}^{n} \overline{c}_{ij}^{(k)}(1, z^2)\, dz \qquad\qquad i,j = 1,2,6 \tag{2.11}$$

and $\overline{c}_{ij}^{(k)}$ is the stiffness matrix component for the kth layer with respect to the laminate reference axes.

Viscoelastic properties of the laminate are approximated by the Kelvin-Voigt model. Therefore, the stiffness moduli of the orthotropic layer can be expressed as the following functions of the differential operator

$$\widetilde{E}_{ii} = E_{ii}\Big(1 + \mu_{ii}\frac{\partial}{\partial t}\Big) \qquad (i = 1,2) \qquad\qquad \widetilde{G}_{12} = G_{12}\Big(1 + \mu_{12}\frac{\partial}{\partial t}\Big) \tag{2.12}$$

where μ_{ii} and μ_{12} denote the retardation times for tension/compression and in-plane shear, respectively. It should be noticed, that this simple rheological model with the constant and independent of the frequency parameters is acceptable for the limited group of composite materials demonstrating a relatively week passive damping effect (cf Schultz et al., 1969).

2.3. Actuator equation

The analysis concerns only a stretching/compressing effect of the activated piezoelectric actuator and its influence on the dynamic behaviour of the system. The tangential forces produced by the actuator are transmitted to the main structure by stresses and moments. Since the actuator patch is thin comparing with the plate, the stresses are imposed to be uniformly distributed in the actuator cross-section. Due to the perfect bonding assumption and the static coupling model (mass of the actuator is neglected) the moment resultants acting along the actuator edges can be approximated by the simple formula (cf Lee, 1990)

$$\begin{bmatrix} m_x \\ m_y \\ m_{xy} \end{bmatrix} = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} t_a z_a^0 \tag{2.13}$$

where

- z_a^0 – distance of the actuator from the laminate midplane, $z_a^0 = (t_a + t_l)/2$
- t_a – actuator thickness
- t_l – total laminate thickness.

Substituting the stress matrix from governing relation $(2.6)_1$, the moment resultant matrix becomes

$$\begin{bmatrix} m_x \\ m_y \\ m_{xy} \end{bmatrix} = \Big(\overline{\mathbf{c}} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} - E_3 \mathbf{T}^{-1}\mathbf{c} \begin{bmatrix} d_{31} \\ d_{32} \\ d_{36} \end{bmatrix}\Big) t_a z_a^0 \tag{2.14}$$

The strains components ε_x, ε_y and γ_{xy} can be formulated by considering pure tension or compression of the plate as a result of the actuator electrical activation. For the perfectly glued piezoelectric patch the equilibrium condition of the stress resultants in the actuator and the plate cross-sections gives the relation

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \Big(t_a \mathbf{d}_a \overline{\mathbf{c}} + \mathbf{d}_l \mathbf{A}\Big)^{-1} \mathbf{d}_a \mathbf{T}^{-1}\mathbf{c} \begin{bmatrix} d_{31} \\ d_{32} \\ d_{36} \end{bmatrix} t_a E_3 \tag{2.15}$$

where $\mathbf{d}_a$ and $\mathbf{d}_l$ are the actuator and plate dimension matrices, respectively

$$\mathbf{d}_a = \mathrm{diag}\Big[x_2 - x_1, y_2 - y_1, \frac{1}{2}(x_2 - x_1 + y_2 - y_1)\Big]$$

$$\mathbf{d}_l = \mathrm{diag}\Big[a, b, \frac{1}{2}(a + b)\Big]$$

with

- x_i, y_i – actuator co-ordinates, $i = 1, 2$
- a, b – midplane dimensions of the plate.

In the above equation it is assumed that the average deformation along the plate cross-section is uniform.

Using Eq (2.14) in conjunction with Eq (2.15), the actuator pattern function and the electric field relation $E_3 = V(t)/t_a$ yields

$$\begin{bmatrix} m_x \\ m_y \\ m_{xy} \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_6 \end{bmatrix} z_a^0 \lambda(x,y)V(t) \tag{2.16}$$

where C_i $(i = 1,2,6)$ are the actuator constants defined as

$$\begin{bmatrix} C_1 \\ C_2 \\ C_6 \end{bmatrix} = \Big[t_a\overline{\mathbf{c}}\Big(t_a\mathbf{d}_a\overline{\mathbf{c}} + \mathbf{d}_l\mathbf{A}\Big)^{-1}\mathbf{d}_a - \mathbf{1}\Big]\mathbf{T}^{-1}\mathbf{c}\begin{bmatrix} d_{31} \\ d_{32} \\ d_{36} \end{bmatrix} \tag{2.17}$$

where $\lambda(x,y)$ is the actuator pattern function, which for rectangular actuators, is

$$\lambda(x,y) = [H(x-x_1) - H(x-x_2)][H(y-y_1) - H(y-y_2)] \tag{2.18}$$

with $H(x)$ being the Heaviside step function.

Equation (2.16) determines the resultant moment generated by the actuator along its edges in relation to the applied voltage $V(t)$ and the electromechanical and geometric parameters of the system (Eq (2.17)), and is called the actuator equation.

2.4. Sensor equation

Mechanical deformation of the piezoelectric sensor generates an electric displacement due to the direct piezoelectric effect described by constitutive equation Eq $(2.1)_2$ or Eq $(2.6)_2$. The closed circuit charge Q stored on the surface electrodes is given by the integral over the effective surface electrode S and can only be a time-function

$$Q = \int\limits_S D_3\, ds \tag{2.19}$$

Substituting Eq $(2.6)_2$ into Eq (2.19), after setting $E_3 = 0$ the general form of the sensor equation is obtained

$$Q = [d_{31}, d_{32}, d_{36}]\ \mathbf{c}\mathbf{T}^{-\top} \int\limits_S \boldsymbol{\varepsilon}\, ds \tag{2.20}$$

In the case when the sensor patch is perfectly glued to the plate, the strains $\boldsymbol{\varepsilon}$ in Eq (2.20) are given by the relation

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + z_s^0 \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \tag{2.21}$$

where

z_s^0 – distance of the sensor from the laminate midplane, $z_s^0 = (t_s + t_l)/2$

t_s – sensor thickness.

Taking into account only the bending effect $(\varepsilon_x^0 = \varepsilon_y^0 = \varepsilon_{xy}^0)$ and combining Eq (2.20) with the strain-displacement relations due to the linear elasticity, and after using the standard equation for capacitance, the voltage $V_s(t)$ produced by the sensor is as follows

$$V_s(t) = -\frac{z_s^0 t_s}{\in_{33} S}[d_{31}, d_{32}, d_{36}]\mathbf{cT}^{-\top} \int\limits_0^a \int\limits_0^b \begin{bmatrix} w_{,xx} \\ w_{,yy} \\ 2w_{,xy} \end{bmatrix} \lambda_s(x,y)\, dxdy \tag{2.22}$$

where

$\in_{33}$ – dielectric permittivity of the piezoelectric material

$\lambda_s(x,y)$ – effective electrode pattern.

The comma followed by an index denotes partial differentiation with respect to the co-ordinate associated with the index.

If the entire piezoelectric patch is covered by electrodes on both sides the function $\lambda_s(x,y)$ serves as the collocated sensor/actuator pattern $\lambda(x,y)$ (Eq (2.18)) and the effective surface electrode is $S = (x_2 - x_1)(y_2 - y_1)$.

2.5. Equation of motion and solution

In the considered case of symmetrically laminated, specially orthotropic plates which are excited by the external loading $q(x,y,t)$ the transverse motion $w(x,y,t)$ controlled by the piezoelectric device can be described by the following equation

$$D_{11}w_{,xxxx} + 2(D_{12} + 2D_{66})w_{,xxyy} + D_{22}w_{,yyyy} + \rho t_l w_{,tt} = q(x,y,t) - p(x,y,t) \tag{2.23}$$

where

D_{ij}	–	elements of the complex stiffness matrix for the laminate viscoelastic material determined from Eq (2.11) combined with Eq (2.12), $(i,j=1,2,6)$
ρ	–	equivalent density, $\rho = t_l^{-1}\sum_{k=1}^{N}\rho_k t_k$
$q(x,y,t)$	–	external loading distribution
$p(x,y,t)$	–	loading produced by the control system.

The loading $p(x,y,t)$ is determined by the resultant moments generated along the actuator edges and is given in the form

$$p(x,y,t) = m_{x,xx} + m_{y,yy} + 2m_{xy,xy} \tag{2.24}$$

After substituting the moment components from Eq (2.14) and differentiating the pattern functions, the actuator loading can be expressed as

$$\begin{aligned}
p(x,y,t) = \Big\{ & C_1[H(y-y_1)-H(y-y_2)][\delta'(x-x_1)-\delta'(x-x_2)] + \\
& +C_2[H(x-x_1)-H(x-x_2)][\delta'(y-y_1)-\delta'(y-y_2)] + \\
& +2C_6[\delta(x-x_1)-\delta(x-x_2)][\delta(y-y_1)-\delta(y-y_2)]\Big\} z_a^0 V(t)
\end{aligned} \tag{2.25}$$

where $\delta(x)$ and $\delta'(x)$ are the Dirac delta function and its first derivative, respectively.

Due to the simple control strategy the actuator and the sensor are electrically coupled with a velocity feedback. Therefore, the voltage applied to the actuator is proportional to the time derivative of the voltage induced by the sensor (Eq (2.22))

$$V(t) = k_d \frac{dV_s(t)}{dt} \tag{2.26}$$

where k_d is the gain factor of the controller.

Finally, the loading produced by the piezoelectric control system is given by Eq (2.25) in conjunction with the sensor voltage signal, Eq (2.22), transformed according to the control function, Eq (2.26).

The dynamic analysis concerns the steady-state behaviour of the plate. Therefore, the external loading is assumed to be a uniformly distributed and harmonic in time single frequency function of the intensity q_0

$$q(x,y,t) = q_0\lambda_q(x,y)\exp(\mathrm{i}\omega t) \tag{2.27}$$

where $\lambda_q(x,y)$ defines the field of the external disturbance

$$\lambda_q(x,y) = [H(x-x_{q1})-H(x-x_{q2})][H(y-y_{q1})-H(y-y_{q2})] \tag{2.28}$$

On the above assumption the response of the uncontrolled as well as controlled system can be predicted as harmonic with the same frequency as the excitation

$$w(x,y,t) = W(x,y)\exp(\mathrm{i}\omega t) \tag{2.29}$$

By solving governing equation (2.23) with the boundary conditions related to the simply supported edges the transverse displacements of the plate are obtained and expressed in terms of frequency characteristics.

3. Results

Calculations are performed for the simply supported, cross-ply laminate of dimensions $a = b = 0.4\,\mathrm{m}$ and the total thickness $t_l = 1.5\,\mathrm{mm}$, composed of five graphite-epoxy layers with the stacking sequence $[0°/90°/0°/90°/0°]$ and the equivalent mass density $\rho = 1600\,\mathrm{kg/m^3}$. A relatively small isotropic internal damping is applied (retardation times $\mu_{11} = \mu_{22} = \mu_{12} = 10^{-5}\,\mathrm{s}$). The control system consists of the PVDF-polymer sensor and actuator bonded symmetrically to both sides of the laminate. Their location and the mid-plane dimensions are determined by the co-ordinates $x_1 = y_1 = 0.1\,\mathrm{m}$ and $x_2 = y_2 = 0.2\,\mathrm{m}$. The thickness of the piezoelecric patches is $t_a = t_s = 0.4\,\mathrm{mm}$. The stiffness parameters of the graphite-epoxy material are as those used by Ha et al. (1992): $E_{11} = 15 \cdot 10^{10}\,\mathrm{Pa}$, $E_{22} = 9 \cdot 10^9\,\mathrm{Pa}$, $G_{12} = 7.1 \cdot 10^9\,\mathrm{Pa}$, $\nu_{12} = 0.3$. The piezoelectric patches PVDF with the following electromechanical parameters are used (Atochem Sensors, INC. – Technical Notes, 1987): $E_{11}^p = E_{22}^p = 2{\cdot}10^9\,\mathrm{Pa}$, $\nu_{12}^p = 0.3$, $d_{31} = 2.3{\cdot}10^{-11}\,\mathrm{m/V}$, $d_{32} = 0.3{\cdot}10^{-11}\,\mathrm{m/V}$, $d_{36} = 0$. Computations are carried out for a harmonic load of the intensity $q_0 = 1\,\mathrm{N/m^2}$.

In the first case the external load is uniformly distributed on the surface limited by $x_{q1} = y_{q1} = 0.2\,\mathrm{m}$ and $x_{q2} = y_{q2} = 0.3\,\mathrm{m}$. The amplitude-frequency characteristic referring to the uncontrolled vibration calculated at the piezoelectric patch field point $x = y = 0.1\,\mathrm{m}$ is shown in Fig. 2.

All resonance picks can be observed within the frequency range. They occur at the frequencies corresponding with the natural modes $\omega_{11} = 293.8$, $\omega_{12} = 624$, $\omega_{21} = 971$, $\omega_{22} = 1175$, $\omega_{13} = 1257$, $\omega_{23} = 1673$, $\omega_{31} = 2130$, $\omega_{32} = 2280$, $\omega_{33} = 2645\,\mathrm{s^{-1}}$. Due to the applied internal damping the intensity of energy dissipation increases with the frequency so the amplitudes of higher modes are reduced significantly.

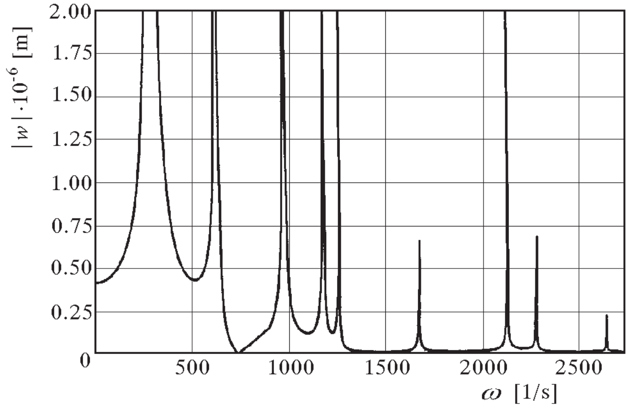

Fig. 2. Uncontrolled dynamic response in a wide range of harmonic excitation

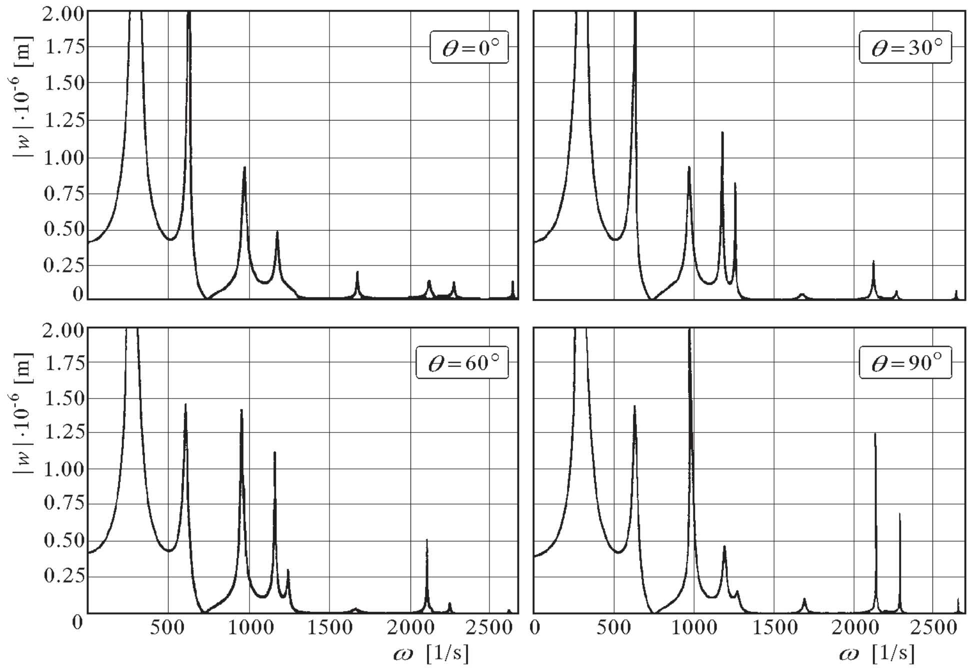

Fig. 3. Effects of variations in the skew angle on the active damping efficiency (external loading over the limited area)

In Figure 3 the dynamic responses of the actively damped plate are presented. To predict the influence of the skew angle between the piezoelectric element natural axes and the plate reference axes the calculations are performed for the following values of the angle $\theta = 0°$, $30°$, $60°$ and $90°$. It can be noticed that the resonance amplitudes depend on the orientation of the piezoelectric material axes. With changing the skew angle the controllability of some modes grows while the other become poorly damped. Therefore, the modal damping efficiency can be improved by adapting the proper skew angle. It is possible especially for PVDF films, which can be cut out in arbitrarily chosen directions. Of course, the observed effect is strongly coupled with the laminated plate geometry and the global stiffness properties.

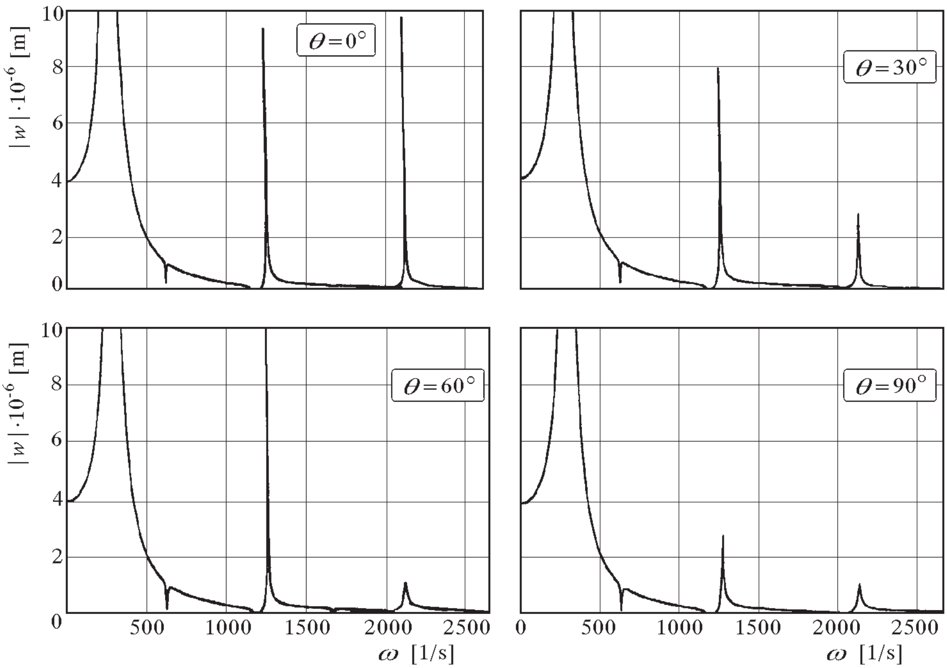

Fig. 4. Effects of variations in the skew angle on the active damping efficiency (external loading over the entire plate)

The above observations are confirmed by the frequency characteristics related to the excitation applied over the entire laminate, see Fig. 4. The location of the piezoelectric patches is the same as previously. In this case only symmetrical modes in relation to the plate axes are generated. The skew angles $\theta = 30°$ and $60°$ effect in suppression of $m = 3$, $n = 1$ vibration modes. The angle $\theta = 90°$ results in a significant reduction of both the $m = 3$, $n = 1$ and $m = 1$, $n = 3$ modes.

4. Conclusions

The dynamic model of a laminated plate with a collocated piezoelectric sensor/actuator system is introduced and discussed. The aim of the control system is active reduction of transverse vibrations. The voltage generated by the sensor is applied to the actuator following the control concept based on a constant-gain velocity feedback. Both the plate and piezoelectric elements are rectangular and oriented parallel to each other. The piezoelectric patches are perfectly bonded to the upper and lower surfaces of the main structure. Their influence on the global properties of the laminate is neglected. The orthotropic electromechanical properties of the piezoelectric material combined with two-dimensional piezoelectric effect develops the analysis. The governing equations of the system are formulated for a non-zero skew angle between the natural axes of the piezoelectric material and the plate reference axes. The theoretical analysis and presented numerical examples are focused on the influence of the skew angle on the efficiency of active damping corresponding to the harmonic excitation. The orientation of the piezoelectric element principal axes significantly effects the frequency response of transverse vibration by changing the controllability of particular modes. Therefore, the skewed piezoelectric sensors/actuators may be used in a segmented control system to improve the effectiveness of the modal damping of laminated structures.

References

1. Bailey T., Hubbard J.E., 1985, Distributed Piezoelectric-Polymer. Active Vibration Control of a Cantilever Beam, *Journal of Guidance, Control and Dynamics*, **8**, 605-611

2. Baz A., Chen T., 2000, Control of Axi-Symmetric Vibrations of Cylindrical Shells Using Active Constrained Layer Damping, *Thin-Walled Structures*, Elsevier, **36**, 1-20

3. Cady W.G., 1964, *Piezoelectricity*, vol. I. Dover, NY: Dover Publications, Inc.

4. Chandrashekhara K., Agarwal A.N., 1993, Active Vibration Control of Laminated Composite Plates Using Piezoelectric Devices: A Finite Element Approach, J. of Intell. Mater. Syst. And Struct., **4**, 4, 496-508

5. Clarc R.L., Fuller Ch.R., Wicks A., 1991, Characterization of Multiple Piezoelectric Actuators for Structural Excitation, *J. Acoust. Soc. Am.*, **90**, 346-357

6. Crawley E.F., de Luis J., 1987, Use of Piezoelectric Actuators as Elements of Intelligent Structures, *AIAA J.*, 25, 1373-1385

7. Dimitriadis E., Fuller C.R., Rogers C.A., 1991, Piezoelectric Actuators for Distributed Vibration Excitation of Thin Plates, *Journal of Applied Mechanics*, **113**, 100-107

8. Ha S.K., Keilers C., Chang F.-K., 1991, Analysis of Laminated Composites Containing Distributed Piezoceramic Sensors and Actuators, *J. of Intell. Mater. Syst. and Struct.*, **2**, 1, 59-71

9. Jie Pan, Hansen C.H., Snyder S.D., 1991, A study of the Response of a Simply Supported Beam to Excitation by a Piezoelectric Actuator, *Proceedings of the Conference on Recent Advances in Active Control of Sound and Vibration*, Virginia, 39-49

10. Lee C.K., 1990, Theory of Laminated Piezoelectric Plates for the Design of Distributed Sensors/Actuators. Part I: Governing Equations and Reciprocal Relationships, *J. Acoust. Soc. Am.*, **87**, 1144-1158

11. Pietrzakowski M., 1997, Dynamic Model of Beam-Piezoceramic Actuator Coupling for Active Vibration Control, *J. of Theoretical and Applied Mechanics*, **35**, 1, 3-20

12. Pietrzakowski M., 2000, Multiple Piezoelectric Segments in Structural Vibration Control, *J. of Theoretical and Applied Mechanics*, **38**, 1, 35-50

13. Schultz A.B., Tsai S.W., 1969, Measurements of Complex Dynamic Moduli for Laminated Fiber-Reinforced Composites, *J. of Composite Materials*, **3**, 434-443

14. Tylikowski A., 1993, Stabilization of Beam Parametric Vibrations, *J. of Theoretical and Applied Mechanics*, **31**, 3, 657-670

15. Tylikowski A., 1999, Stabilization of Beam Parametric Vibrations by means of Distributed Piezoelectric Elements, *J. of Theoretical and Applied Mechanics*, **37**, 2, 241-254

16. Tylikowski A., 2000, Two-Dimensional Piezoelectric Actuators for Vibration Excitation and Suppression of Thin Plates, *Machine Dynamics Problems*, **24**, 1, 199-207

17. Tzou H.S., Tzeng C.I.A., 1990, Distributed Piezoelectric Sensor/Actuator Design for Dynamic Measurement/Control of Distributed Parameter Systems: A Piezoelectric Finite Element Approach, *J. of Sound and Vibrations*, **138**, 1, 17-34

18. Wang B.-T., Rogers C.A., 1991, Modeling of Finite-Length Spatially-Distributed Induced Strain Actuators for Laminate Beams and Plates, *J. of Intell. Mater. Syst. and Struct.*, **2**, 1, 38-58

19. Whitney J.M., 1987, *Structural Analysis of Laminated Anisotropic Plates*, Technomic Publishing Co.Inc, Lancaster

Dowolnie zorientowane elementy piezoelektryczne w aktywnym tłumieniu drgań płyt laminowanych

Streszczenie

Praca dotyczy aktywnego tłumienia poprzecznych drgań lepkosprężystej, laminowanej płyty za pomocą rozłożonych elementów piezoelektrycznych o dowolnie zorientowanych kierunkach właściwości materiałowych. Rozpatrzono prostokątną płytę, swobodnie podpartą na brzegach, zbudowaną z symetrycznie ułożonych warstw ortotropowych i poddaną harmonicznemu wymuszeniu siłowemu. Układ sterowania stanowią prostokątne, piezoelektryczne elementy – pomiarowy (sensor) i wykonawczy (aktuator), umieszczone po obu stronach płyty, pracujące w pętli z prędkościowym sprzężeniem zwrotnym. Analizę dynamiczną oparto na klasycznej teorii płyt laminowanych i założeniu idealnego połączenia bezmasowych elementów piezoelektrycznych (model statyczny) z płytą. Zgodnie z przyjętym modelem oddziaływanie aktuatora sprowadzono do momentów równomiernie rozłożonych wzdłuż jego krawędzi. W przedstawionej analizie uwzględniono ortotropowe właściwości materiału sensora i aktuatora oraz dwukierunkowy efekt piezoelektryczny. Równania ruchu układu zostały sformułowane w przypadku dowolnie zorientowanych, w stosunku do osi płyty, głównych kierunków właściwości materiału piezoelektrycznego. Wyniki obliczeń przedstawiono w formie charakterystyk amplitudowo-częstotliwościowych ilustrując wpływ orientacji osi materiałowych na skuteczność aktywnej redukcji drgań.

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