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THIN-WALLED BEAM WITH OPEN CROSS-SECTIONS AS A TIMOSHENKO BAR[1]

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The principal aim of the present paper is to analyse influence of shear stress on deformation of thin-walled beams with open cross-sections. In particular, the text presents critical remarks on the hitherto existing attempts to consider the shear effect by substituting function defining warping of a beam within Vlasov's theory with a new function $\theta(x)$. The paper is also aimed at a quantitative analysis of influence of shear-force on the displacement field which fulfils Bernoulli's assumptions. Contrary to solid bars, where the quantitative analysis shows that the influence of the shear forces on the deformation is negligible, this influence can be significant in thin-walled beams.

key words: thin-walled beam, Timoshenko bar, warping

1. Introduction

The problem of recognising the influence of shear-force on the deflection of the centre line of a solid bar was presented by Timoshenko (Gere and Timoshenko, 1984). The analysis of the so-called Timoshenko bar and numerical examples are given by Piechnik (1999).

In the papers by Gunnlaugsson and Pedersen (1982), Chen and Blandford (1989), Kim et al. (1994), where thin-walled beams are considered, the displacement field, which takes into account the shear effect on bars deflection and displacement in the direction of their centre lines is used. Gunnlaugsson and Pedersen (1982) and Chen and Blandford (1989) focus on building a finite

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element for a thin-walled beam, which makes them ignore Bernoulli's law, like in the case of Timoshenko's solid bar, and establish a new form of the function describing beam warping in Vlasov's theory. The second idea consists in substituting the expression $\alpha'(x)\omega(s)$ with $\theta(x)\omega(s)$. The authors of the present paper consider this idea in question to be of doubtful value, because it was introduced without a physical analysis of the new quantity $\theta(x)$, which is the seventh degree of freedom. Kim et al. (1994) analysed spatial stability and free vibration of thin-walled beams, but these problems permit us to oust the new quantity, which makes the analysis of $\theta(x)$ unnecessary.

The present paper is based on the work by Piechnik (2000), which is a source of notations of kinematics and static quantities as well as of auxiliary relations referring to thin-walled beams with open cross-sections. Gunnlaugsson and Pedersen (1982) provides a basis for the notations of new quantities resulting from the recognising of the shear effect on a thin-walled beam deformation, while paper Chen and Blandford (1989) provides a basis for the algorithm allowing one to consider the skew symmetric shear stresses, the influence of which in thin-walled beams with open cross-sections is significant.

2. Basic assumptions

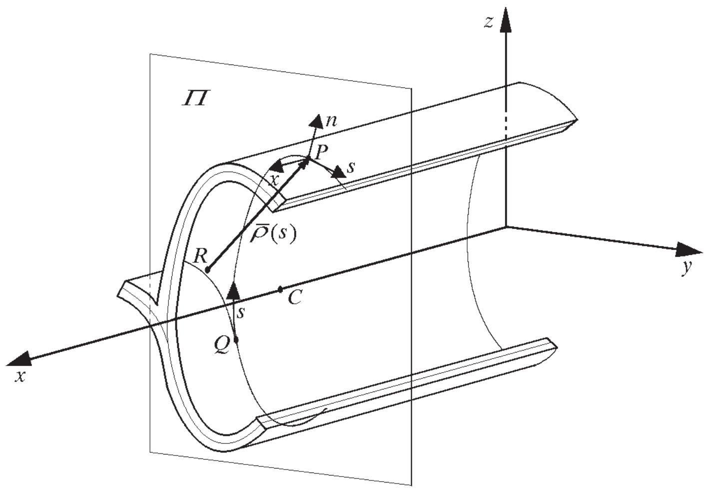

Fig. 1.

Let us consider a thin-walled prismatic beam of any open cross-section as shown in Fig. 1.

Let us assume the following co-ordinate systems: global xyz and local xsn. The x-axis of the global system is the beam centre line, while the y and z-axes are the principal axes of inertia. The co-ordinate system xsn can be defined at any point of the middle surface P – it consists of the x-axis parallel to the beam centre line, the s-axis tangential to the middle line, and the n-axis perpendicular to the x and s-axes. C denotes the cross-section centre of gravity, R denotes the shear centre and Q is the origin of the curvilinear co-ordinate s.

3. Displacement fields

Displacement fields – the foundamental concept of the present paper – are modifications of the field of Vlasov's theory. The displacement field Vlasov's theory is defined by relations

$$\begin{aligned} u_x(x,s) &= \alpha'(x)\omega(s) - v'(x)y(s) - w'(x)z(s) + u_0(x) \\ u_s(x,s) &= v(x)\dot{y}(s) + w(x)\dot{z}(s) - \alpha(x)\rho_n(s) \\ u_n(x,s) &= -v(x)\dot{z}(s) + w(x)\dot{y}(s) + \alpha(x)\rho_s(s) \end{aligned} \tag{3.1}$$

Equations (3.1) are related to the local co-ordinate system. $\alpha(x)$ is a function of the cross-section rotation about the shear centre. The quantities $v(x)$ and $w(x)$ are displacements of the shear centre in the directions y and z, while the function $u_0(x)$ is the displacement of the neutral axis in the x-direction. In the case when the shear influence is recognised only in the beam deflection, the displacement field is built by substituting the quantities $w'(x)$ and $v'(x)$ by the functions $\varphi(x)$ and $\vartheta(x)$, respectively. This substitution is equivalent to ignoring Bernoulli's law. It leads to the following new displacement field

$$\begin{aligned} u_x(x,s) &= \alpha'(x)\omega(s) - \vartheta(x)y(s) + \varphi(x)z(s) + u_0(x) \\ u_s(x,s) &= v(x)\dot{y}(s) + w(x)\dot{z}(s) - \alpha(x)\rho_n(s) \\ u_n(x,s) &= -v(x)\dot{z}(s) + w(x)\dot{y}(s) + \alpha(x)\rho_s(s) \end{aligned} \tag{3.2}$$

The displacement field desribed by Gunnlaugsson and Pedersen (1982), Chen and Blandford (1989) and Kim et al. (1994) originates from an additional operation, namely the substitution of the quantity $\alpha'(x)$ with a more general function $\theta(x)$. Thus, the second approach takes into account the shear effect also on the warping, which is defined according to Vlasov's theory. In this case, the displacement field takes the form

$$
\begin{aligned}
u_x(x,s) &= \theta(x)\omega(s) - \vartheta(x)y(s) + \varphi(x)z(s) + u_0(x) \\
u_s(x,s) &= v(x)\dot{y}(s) + w(x)\dot{z}(s) - \alpha(x)\rho_n(s) \\
u_n(x,s) &= -v(x)\dot{z}(s) + w(x)\dot{y}(s) + \alpha(x)\rho_s(s)
\end{aligned}
\tag{3.3}
$$

The quantities $\dot{y}(s)$ and $\dot{z}(s)$ in relations (3.1) $\div$ (3.3) are derivatives with respect to the s co-ordinate of the functions $y(s)$ and $z(s)$, which define the parametric equation of the middle line.

4. Solution of the problem when Bernoulli's law is disregarded

Strains at points of the middle surface are calculated from equations of displacement field (3.2) by means of Cauchy's equations. These strains assume the following form

$$
\begin{aligned}
\varepsilon_x(x,s) &= \alpha''(x)\omega(s) - \vartheta'(x)y(s) + \varphi'(x)z(s) + u_0'(x) \\
\gamma_{xs}(x,s) &= \gamma_{sx}(x,s) = \Big(v'(x) - \vartheta(x)\Big)\dot{y}(s) + \Big(w'(x) + \varphi(x)\Big)\dot{z}(s)
\end{aligned}
\tag{4.1}
$$

Stresses at points of the middle line are calculated from (4.1) by means of Hooke's equations. It is assumed that the longitudinal stress does not change with thickness, and the shear stress changes linearly with the thickness, which permits us to write: $\tau_{xs}(x,s,n) = \tau_\omega(x,s) + \tau_s(x,s,n)$, where $\tau_\omega(x,s)$ is the average stress and $\tau_s(x,s,n)$ is the linear skew symmetric stress (Saint-Venant's stress). Then, it is also assumed that $\tau_{xn}(x,s,n) \equiv 0$. So the stresses take the form

$$
\begin{aligned}
\sigma_x(x,s,n) &= \widetilde{E}\Big[\alpha''(x)\omega(s) - \vartheta'(x)y(s) + \varphi'(x)z(s) + u_0'(x)\Big] \\
\tau_{xs}(x,s,n) &= G\Big[\Big(v'(x) - \vartheta(x)\Big)\dot{y}(s) + \Big(w'(x) + \varphi(x)\Big)\dot{z}(s) + 2\alpha'(x)n\Big]
\end{aligned}
\tag{4.2}
$$

where

$$
\widetilde{E} \stackrel{def}{=} \frac{E}{1-\nu^2}
$$

The equations governing the problem in question are calculated with the help of the stationary potential energy theorem; i.e. Lagrange's theorem. The total potential energy for a beam is determined by the following formula

$$\mathit{\Pi} = U - L$$

where U is the internal energy and L is the external one.

The formula determining the internal energy that is used in the present procedure, has the form

$$U = \frac{1}{2} \iiint\limits_V \Big(\frac{\sigma_x^2(x,s)}{\widetilde{E}} + \frac{\tau_{xs}^2(x,s,n)}{G} \Big)\, dV \tag{4.3}$$

Substituting stresses (4.2) into Eq (4.3) and transforming this equation, we obtain

$$\begin{aligned} U &= \frac{\widetilde{E}}{2} \int\limits_0^l \Big\{ [\alpha''(x)]^2 I_\omega + [\vartheta'(x)]^2 I_z + [\varphi'(x)]^2 I_y + [u_0'(x)]^2 A \Big\}\, dx + \\ &+ \frac{G}{2} \int\limits_0^l \Big\{ [v'(x) - \vartheta(x)]^2 I_{pp} + 2[v'(x) - \vartheta(x)][w'(x) + \varphi(x)] I_{pq} + \\ &+ [w'(x) + \varphi(x)]^2 I_{qq} + [\alpha'(x)]^2 I_s \Big\}\, dx \end{aligned} \tag{4.4}$$

In Eq (4.4), apart from the cross-sectional property constants well-known from Vlasov's theory, there are new cross-sectional property constants:

— shear areas in y and z directions

$$I_{pp} \stackrel{def}{=} \int\limits_d \dot{y}^2(s)\delta(s)\, ds \qquad\qquad I_{qq} \stackrel{def}{=} \int\limits_d \dot{z}^2(s)\delta(s)\, ds$$

— mixed shear area

$$I_{pq} \stackrel{def}{=} \int\limits_d \dot{y}(s)\dot{z}(s)\delta(s)\, ds$$

The formula that determines the external energy (the work of the external forces: b_ω, m_z, m_y, q_x, q_y, q_z, m_{Rx}, B_ω, M_z, M_y, F_x, F_y, F_z, M_{Rx} on the displacements: α', ϑ, φ, u_0, v, w, α) has the following form

$$
\begin{gathered}
L=\int\limits_0^l \Big[\alpha'(x)b_\omega 9x)+\vartheta(x)m_z(x)+\varphi(x)m_y(x)+u_0(x)q_x(x)+v(x)q_y(x)+\\
+w(x)q_z(x)+\alpha(x)m_{Rx}(x)\Big]\,dx-\alpha'(0)B_\omega(0)-\vartheta(0)M_z(0)-\varphi(0)M_y(0)+\\
-u_0(0)F_x(0)-v(0)F_y(0)-w(0)F_z(0)-\alpha(0)M_{Rx}(0)+\alpha'(l)B_\omega(l)+\\
+\vartheta(l)M_z(l)+\varphi(l)M_y(l)+u_0(l)F_x(l)+v(l)F_y(l)+w(l)F_z(l)+\alpha(l)M_{Rx}(l)
\end{gathered}
\tag{4.5}
$$

The static quantities in relation (4.5) indicated by small letters are the distributed line forces, while the capital letters are used to indicate the concentrated forces acting on the ends of the beam. If we combine relations (4.4) and (4.5) and then calculate the variation and equate it to zero, we will obtain a system of six ordinary differential equations and boundary conditions creating a solution of the problem of a thin-walled beam with open cross-sections. The resultant system of the equations consists of the following:

— the equations related to the condition that the expression with the multiplier $\delta\Big(\alpha'(x)\Big)$ is zero

$$
\begin{gathered}
\widetilde{E}I_\omega\alpha'''(x)-GI_s\alpha'(x)+b_\omega(x)+M_{Rx}(x)=0\\
\widetilde{E}I_\omega\alpha''(0)=B_\omega(0) \qquad \widetilde{E}I_\omega\alpha''(l)=B_\omega(l)
\end{gathered}
\tag{4.6}
$$

— the equations related to the condition that the expression with the multiplier $\delta\varphi(x)$ is zero

$$
\begin{gathered}
\widetilde{E}I_y\varphi''(x)-GI_{pq}[v'(x)-\vartheta(x)]-GI_{qq}[w'(x)+\varphi(x)]+m_y(x)=0\\
\widetilde{E}I_y\varphi'(0)=M_y(0) \qquad \widetilde{E}I_y\varphi'(l)=M_y(l)
\end{gathered}
\tag{4.7}
$$

— the equations related to the condition that the expression with the multiplier $\delta\vartheta(x)$ is zero

$$
\begin{gathered}
\widetilde{E}I_z\vartheta''(x)+GI_{pp}[v'(x)-\vartheta(x)]+GI_{pq}[w'(x)+\varphi(x)]+m_z(x)=0\\
\widetilde{E}I_z\vartheta'(0)=M_z(0) \qquad \widetilde{E}I_z\vartheta'(l)=M_z(l)
\end{gathered}
\tag{4.8}
$$

— the equations related to the condition that the expression with the multiplier $\delta u_0(x)$ is zero

$$
\widetilde{E}Au_0''(x)+q_x(x)=0 \qquad \widetilde{E}Au_0'(0)=F_x(0) \qquad \widetilde{E}Au_0'(l)=F_x(l)
\tag{4.9}
$$

— the equations related to the condition that the expression with the multiplier $\delta v(x)$ is zero

$$
\begin{aligned}
&GI_{pp}[v'(x) - \vartheta(x)]' + GI_{pq}[w'(x) + \varphi(x)]' + q_y(x) = 0 \\
&GI_{pp}[v'(0) - \vartheta(0)] + GI_{pq}[w'(0) + \varphi(0)] = F_y(0) \\
&GI_{pp}[v'(l) - \vartheta(l)] + GI_{pq}[w'(l) + \varphi(l)] = F_y(l)
\end{aligned}
\tag{4.10}
$$

— the equations related to the condition that the expression with the multiplier $\delta w(x)$ is zero

$$
\begin{aligned}
&GI_{pq}[v'(x) - \vartheta(x)]' + GI_{qq}[w'(x) + \varphi(x)]' + q_z(x) = 0 \\
&GI_{pq}[v'(0) - \vartheta(0)] + GI_{qq}[w'(0) + \varphi(0)] = F_z(0) \\
&GI_{pq}[v'(l) - \vartheta(l)] + GI_{qq}[w'(l) + \varphi(l)] = F_z(l)
\end{aligned}
\tag{4.11}
$$

By solving the system of the equations, we obtain the following relations that determine the desired functions of x in displacement field (3.2)

$$
\begin{aligned}
u_0(x) &= -\int_0^x \Big(\int_0^\eta \frac{q_x(\xi)}{\widetilde{E}A}\,d\xi\Big)\,d\eta + C_1x + C_2 \\
\alpha(x) &= C_3\mathrm{e}^{kx} + C_4\mathrm{e}^{-kx} + C_5 + \alpha_p(x) \\
\varphi(x) &= -\int_0^x \Big[\int_0^\psi \Big(\int_0^\eta \frac{q_z(\xi) + m_y'(\xi)}{\widetilde{E}I_y}\,d\xi\Big)\,d\eta\Big]\,d\psi + C_6\frac{x^2}{2} + C_7x + C_8 \\
\vartheta(x) &= \int_0^x \Big[\int_0^\psi \Big(\int_0^\eta \frac{q_y(\xi) - m_z'(\xi)}{\widetilde{E}I_z}\,d\xi\Big)\,d\eta\Big]\,d\psi + C_9\frac{x^2}{2} + C_{10}x + C_{11} \\
v(x) &= \int_0^x \Big(\int_0^\eta \frac{\det \mathbf{K}_1}{\det \mathbf{K}_3}\,d\xi\Big)\,d\eta + C_{12}x + C_{13} \\
w(x) &= \int_0^x \Big(\int_0^\eta \frac{\det \mathbf{K}_2}{\det \mathbf{K}_3}\,d\xi\Big)\,d\eta + C_{14}x + C_{15}
\end{aligned}
\tag{4.12}
$$

where $k = \sqrt{GI_s/(\widetilde{E}I_\omega)}$, $\alpha_p(x)$ is the particular integral of the heterogeneous equation, and

$$
\mathbf{K}_1 = \begin{bmatrix} GI_{pp}\vartheta'(\xi) - GI_{pq}\varphi'(\xi) - q_y(\xi) & GI_{pq} \\ GI_{pq}\vartheta'(\xi) - GI_{qq}\varphi'(\xi) - q_z(\xi) & GI_{qq} \end{bmatrix}
$$

$$\mathbf{K}_2 = \begin{bmatrix} GI_{pp} & GI_{pp}\vartheta'(\xi) - GI_{pq}\varphi'(\xi) - q_y(\xi) \\ GI_{pq} & GI_{pq}\vartheta'(\xi) - GI_{qq}\varphi'(\xi) - q_z(\xi) \end{bmatrix}$$

$$\mathbf{K}_3 = \begin{bmatrix} GI_{pp} & GI_{pq} \\ GI_{pq} & GI_{qq} \end{bmatrix}$$

The constants in (4.12) are determined from static boundary conditions $(4.6)_2 \div (4.11)_2$ and from the kinematic boundary conditions dependent on constraints. By finding the quantities defined by (4.12), we obtain the displacement field, and thus the strain and stress fields. Before considering the possibility of recognising the effect of shear forces also by substituting $\alpha'(x)$ with the new unknown function $\theta(x)$, which is presented by Gunnlaugsson and Pedersen (1982), Chen and Blandford (1989) and Kim et al. (1994), we will first show a numerical example where the quantitative influence of the shear forces without the above substitution is analysed.

5. Numerical example

Let us consider a channel section cantilever beam, which is loaded as shown in Fig. 2. The analysis will quantitatively show the shear effect on the deformation of a thin-walled beam with open cross-sections.

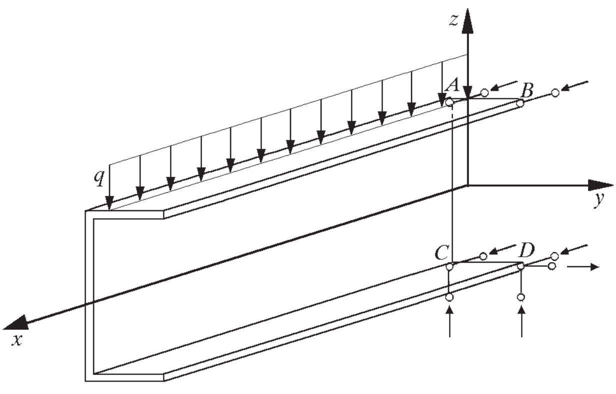

Fig. 2.

Let us use the following numerical data: length of beam $l = 2.0\,\mathrm{m}$, load $q = 1.0\,\mathrm{kN/m}$, material constants $E = 205\,\mathrm{GPa}$, $G = 80\,\mathrm{GPa}$, $\nu = 0.3$, height of the cross-section $h = 180\,\mathrm{mm}$, width of the cross-section $b = 70\,\mathrm{mm}$, its thickness $\delta = 6\,\mathrm{mm}$.

Relations resulting from the equations of constraint and the boundary conditions for (4.12), take the following forms

$$
\begin{aligned}
&u_x(0,s_i) = \alpha'(0)\omega(s_i) - \vartheta(0)y(s_i) + \varphi(0)z(s_i) + u_0(0) = 0 \qquad i = A,B,C,D\\
&u_y(0,y_D,z_D) = v(0) - \alpha(0)(z_D - z_R) = 0\\
&u_z(0,y_C,z_C) = w(0) + \alpha(0)(y_C - y_R) = 0\\
&u_z(0,y_D,z_D) = w(0) + \alpha(0)(y_D - y_R) = 0
\end{aligned}
$$

In order to simplify the analysis, the equations determining the displacements in the cross-section plane are written in the global co-ordinate system. The above system of seven equations contains seven unknown quantities, so the boundary conditions required for solving the problem must be calculated at the some. The remaining constants in (4.12) can be determined from the static boundary conditions, which contain the following static quantities: $F_x(l)$, $B_\omega(l)$, $M_y(0)$, $M_y(l)$, $M_z(0)$, $M_z(l)$, $F_y(0)$ and $F_z(0)$.

As can be seen from the above general solution, only the functions $w(x)$ and $v(x)$ can differ from those in Vlasov's theory. So, the shear forces effect only these functions. In the above example, only the functions $w(x)$ will differ in both theories. Let us denote by 1 the results of Vlasov's theory and by 2 the results of the new theory. The solutions in both cases have the following forms

$$
w_1(x) = -\frac{q}{24\widetilde{E}I_y}x^4 + \frac{ql}{6\widetilde{E}I_y}x^3 - \frac{ql^2}{4\widetilde{E}I_y}x^2
$$

$$
w_2(x) = -\frac{q}{24\widetilde{E}I_y}x^4 + \frac{ql}{6\widetilde{E}I_y}x^3 + \Big(-\frac{ql^2}{4\widetilde{E}I_y} + \frac{q}{2\widetilde{G}I_{qq}}\Big)x^2 - \frac{ql}{GI_{qq}}x
$$

As can be seen, both relations differ in terms of the new stiffness GI_{qq}. After substituting the numerical values, both deflection functions assume the forms mentioned below. The dependent variables of these functions (at five points) and proportional differences are presented in the table:

$$
w_1(x) = -1.905\cdot 10^{-5}x^4 + 1.524\cdot 10^{-4}x^3 - 4.572\cdot 10^{-4}x^2
$$

$$
w_2(x) = -1.905\cdot 10^{-5}x^4 + 1.524\cdot 10^{-4}x^3 - 4.515\cdot 10^{-4}x^2 - 2.315\cdot 10^{-5}x
$$

$$
r(x) = \frac{w_2(x) - w_1(x)}{w_1(x)}
$$

x [m]	$w_1(x)$ [m]	$w_2(x)$ [m]	$r(x)$ [%]
0.0	0	0	–
0.5	$-9.644 \cdot 10^{-5}$	$-1.066 \cdot 10^{-4}$	10.525
1.0	$-3.239 \cdot 10^{-4}$	$-3.413 \cdot 10^{-4}$	5.388
1.5	$-6.108 \cdot 10^{-4}$	$-6.327 \cdot 10^{-4}$	3.586
2.0	$-9.144 \cdot 10^{-4}$	$-9.379 \cdot 10^{-4}$	2.570

The increase of the deflection is not negligible after taking into account the shear effect. It equals 10.5%.

6. Second approach

In this section the authors present another approach which takes into account the shear effect on the beam deflection and warping measure.

If we use relations (3.3) instead of (3.2) in the algorithm for deriving the equations governing the considered problem, we will obtain a system of seven differential equations. These equations will not be presented but discussed, because the analysed approach is incorrect – which will be demonstrated – so these equations have no practical application. The considered differential equations – except for the equation determining the quantity $u_0(x)$ – are coupled. When this system of equations is uncoupled, it appears that the quantities $\vartheta(x)$ and $\varphi(x)$ are determined by third order differential equations, while the quantities $\alpha(x)$, $v(x)$, $w(x)$ and $u_0(x)$ are determined by second order differential equations. The constants occurring in the solutions of the above-mentioned differential equations are calculated from the static boundary conditions obtained from Lagrange's theorem, and from the kinematic boundary conditions resulting from the applied constraints.

The last desired function is $\theta(x)$. The uncoupled differential equation involving this function has the form

$$
\begin{aligned}
&-\widetilde{E}I_\omega\Big(\det \mathbf{K}_3 + \frac{\det \mathbf{K}_4}{GI_s}\Big)\theta'''(x) + \det \mathbf{K}_4\theta'(x) = \\
&= \det \mathbf{K}_4 \frac{b'_\omega(x) - m_{Rx}(x)}{GI_s} - \det \begin{bmatrix} GI_{pq} & GI_{pr} \\ GI_{qq} & GI_{qr} \end{bmatrix} q_y(x) + \\
&+ \det \begin{bmatrix} GI_{pp} & GI_{pr} \\ GI_{pq} & GI_{qr} \end{bmatrix} q_z(x) + \det \begin{bmatrix} GI_{pp} & GI_{pq} \\ GI_{pq} & GI_{qq} \end{bmatrix} b'_\omega(x)
\end{aligned}
\tag{6.1}
$$

where

$$
\mathbf{K}_4 = \begin{bmatrix} GI_{pp} & GI_{pq} & GI_{pr} \\ GI_{pq} & GI_{qq} & GI_{qr} \\ GI_{pr} & GI_{qr} & GI_{rr} \end{bmatrix}
$$

The stiffnesses GI_{pr}, GI_{qr} and GI_{rr} in (6.1) result from the determination of the shear effect on the warping measure. It is clear that the order three of (6.1) implies three independent boundary conditions to be set forth before finding the function $\theta(x)$. Two conditions result from Lagrange's theorem; they have static character and are the consequence of knowledge of the bimoments at the beam ends. The third condition – as the structure of (6.1) prompts – needs knowing the dependent variable of the function $\theta(x)$ at one point, so it has kinematic character. As it was mentioned in the introduction, we do not have a physical interpretation of $\theta(x)$, so we cannot formulate the required boundary condition. Although we can determine the dependent variable of the function $\theta(x)$ for $x = 0$ with the constraint equations from the numerical example presented above[2], we can do so at the cost of not knowing the hyperstatic reaction and not knowing the bimoment at the beam end. If we consider the above example as statically determined, i.e. if we know all forces, which are applied to the beam ends, we cannot have the above-mentioned kinematic boundary condition (we cannot determine seven unknowns from six

[2]The constraint equations for the considered approach differ from the equations, which were presented in the previous theory, only in the occurrence of the quantity $\theta(0)$ instead of $\alpha'(0)$, i.e. only in the notation of the unknown, which does not affect the solution.

constraint equations). So, regardless of type of example, we cannot determine the function $\theta(x)$, because we lack one boundary condition in each case.

The conclusion of the analysis is that a mistake was made at the beginning of the procedure of the incorporated kinematic approach, therefore it was made during the formulation of the supposed displacement functions. The mistake consists in using an excessive number of unknowns and no physical interpretation of $\theta(x)$. Since it is impossible to determine the function $\theta(x)$, we cannot calculate the displacement, strain and stress fields. Thus, it is impossible to find the complete solution.

7. Conclusions

The paper presents a scheme of an algorithm for deriving equations which govern the problem of shear effect on static and on kinematic quantities in a thin-walled beam with open cross-sections by omitting Bernoulli's law and solving a system of differential equations. The algorithm shows an original manner of transforming the constraint equations into the boundary conditions, necessary to solve the problem. The analysis of results of a numerical example shows that the contribution of deformation due to shear is significant.

The important result of this paper consists in proving that the approach existing so far – which takes into account the shear effect on the deflection and also on warping measure – is incorrect. Since it is impossible to determine the required boundary conditions in this case, the replacement of the quantity $\alpha'(x)$ with the function $\theta(x)$ – the fundamental one in the considered approach – seems to be an inappropiate idea.

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Pręt cienkościenny otwarty jako belka Timoszenki

Streszczenie

Celem pracy jest analiza wpływu sił poprzecznych na deformację prętów cienkościennych o profilu otwartym. W artykule zaprezentowano szkic algorytmu wprowadzania wzorów rządzących problemem wpływu sił poprzecznych na wielkości statyczne i kinematyczne belki cienkościennej o profilu otwartym, czyniąc jedynie odstępstwo od zasady Bernoulliego oraz przedstawiono rozwiązanie otrzymanego układu równań różniczkowych. Szczególną uwagę skupiono na krytycznej analizie dotychczasowego podejścia uwzględniającego wpływ ścinania nie tylko na ugięcie belki, ale także na miarę spaczenia. Brak możliwości określenia potrzebnych warunków brzegowych w tym przypadku powoduje, iż podejście to jest niepoprawne. W pracy przeprowadzono także analizę jakościową wpływu sił poprzecznych na wartości pola przemieszczeń, które spełnia założenia Bernoulliego. W przeciwieństwie bowiem do belek litych, w których analiza ilościowa wskazuje na możliwość pominięcia tych sił przy obliczeniu deformacji, belki cienkościenne w wielu przypadkach mogą doznawać znacznych deformacji wywołanych naprężeniami ścinającymi.

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