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DYNAMIC ELASTICITY CHARACTERISTICS OF SOIL LAYERS NOT DEEPLY SITUATED UNDER THE EARTH SURFACE

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If a simplified elasticity model of the soil is considered sufficiently precise for practical use, the problem of values of elasticity modulus, Poisson ratio and wave velocities so as their change with depth in the shallow soil layers should be determined. The paper presents results of investigations of the above mentioned characteristics in situ for cohesive soil. The changes with depth are fundamental for practical applications. More details of the conducted investigations the Reader can find in the monography Ciesielski et al. (1999).

Key words: soil dynamics, wave velocities in soil, elasticity characteristics of soil

1. Introduction

For simplified models of soil, treated as a solid, elastic, and homogeneous body, a solution representing the response of the soil (deformation, stress and strain) to a static load transferred from the surface was presented a long time ago. Description of the soil behaviour under dynamic loads is a more troublesome problem. A model classical case of elastic soil deformation distribution under vertical, harmonic time dependent load action (elastic half-space, elastic half-plane) was resolved by Rayleigh (1885), Timoshenko (1959), Biot (1962).

Description of propagation of the induced vibration is based on the theory of wave motion and, in practice, the wave equations were used for a linear elastic medium. Considering wave motion in the soil approximate results were obtained which, however, when compared with measurements in situ evoke numerous doubts. It concerns particularly spatial distribution of displacements and deformation in vertical sections of the soil and vibration propagation not only on the surface but also in subsurface layers.

These data are essential for consideration of the soil-structure interaction and at establishing dynamic loads acting on embedded structures connected directly with static and dynamic parameters of the adopted soil model.

2. Static properties of soil

It is known the literature (cf, e.g. Snitko, 1963; Fajklewicz, 1972) that the values of static parameters of the soil, commolnly used in engineering practice, differ from its dynamic parameters. That must be taken into account in analysis of vibration propagation in the soil. In both cases, however, use is generally made of identical descriptions based on the theory of solid linear-elastic body.

Experimental investigations conducted by a number of authors have showed that dynamic values of modulus of elasticity are higher than the static modulus $E_d > E_s$, where, as it results from the measurements made by Thiel, the ratio E_d/E_s is $2.07 \div 8.25$ (Fajklewicz, 1972). As Russian experiments showed (among others Snitko's, 1963) the values of the Poisson ratio ν under dynamic load increases slightly in relation to the static ones and could be adopted as:

— for sands: $\nu = 0.30 \div 0.35$

— for clays: $\nu = 0.40 \div 0.50$.

The static and dynamic values E and ν differ because the soil medium generally departs from the Hooke body model, thus the assumptions of the classical theory of elasticity can be accepted only as great approximation.

Proper predicting of soil vibration must be based on knowledge of dynamic properties of elasticity of soils arising in the process of wave propagation. Their values may be determined on the basis of:

- knowledge of geotechnical structure and data given in the literature
- laboratory dynamic investigations of elasticity of the soil sample
- measurements of dynamic properties of soil elasticity taken in situ.

The first two ways give only approximate information whereas, the results obtained by means of investigations in situ – including the so called parallel passage of the seismic wave through soil levels correspond best with real conditions.

This method consists in inducting seismic waves on the one side of the examined section of the soil and recording of time it takes them to pass to the other side. Direct waves (longitudinal V_P and shear V_S) which not undergo any deflections or refractions and reach the receiver are recorded. When the distance and time of the wave passage between the points of inducting and receiving is known the wave velocity V_P and V_S can be determined. In turn, the values of velocity are used in determination of dynamic properties of elasticity basing upon the given below relations used in geophysics (Fajklewicz, 1972).

– Modulus of Young linear elasticity

$$E = \frac{\rho(3V_P^2V_S^2 - 4V_S^4)}{V_P^2 - V_S^2} \tag{2.1}$$

– Poisson ratio

Introducing the formula from theory of elasticity (Ciesielski and Maciąg, 1990)

$$V_S = \sqrt{\frac{E}{\rho}\frac{1}{2(1+\nu)}} \tag{2.2}$$

the equation

$$\nu = \frac{3V_P^2 - 4V_S^2}{2(V_P^2 - V_S^2)} - 1 \tag{2.3}$$

is obtained.

– Rayleigh's velocity of the surface wave (in approximation) after Knopoff (1952)

$$V_R = V_S\frac{0.87 + 1.12\nu}{1+\nu} \tag{2.4}$$

where

E – Young modulus [Pa]
ρ – soil mass density [kg/m^3]
V_P – longitudinal wave velocity [m/s]
V_S – shear wave velocity [m/s]

The above data are used for calculations of vibration propagation in the soil.

3. Description of the investigated area

In order to obtain the real values of static and dynamic parameters of chosen characteristics of elasticity, the soil experimental investigations in situ and laboratory ones were carried out. For these investigations a relatively homogenous soil located in the airfield at Balice near Cracow was chosen. This soil consists of thick layer of clayey sand deposited at depth of $5 \div 25$ m under the soil level superposed with the layer of cohesive soil of 5 m in depth on which a thin layer of soil is deposited. The soil water level was found at the depth of 5.5 m.

4. Static soil parameters

The soil samples taken (every 0.25 m to the depth 5.5 m) were subjected to laboratory tests and on this basis the static parameters of the soil were determined. The analysis showed that it was a homogenous soil formed of plastic and stiff plastic cohesive soil. Fig.1 shows distributions of the investigated soil, determined by the use of laboratory methods.

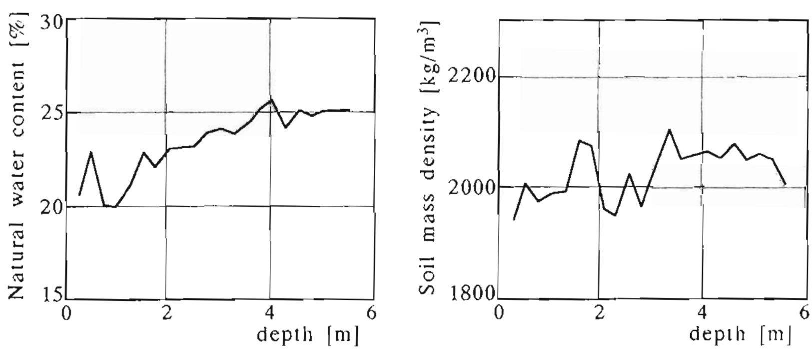

Fig. 1. Static soil parameters versus depth determined by the use of conventional laboratory methods

On the basis of the above given values of static parameters of each considered soil level, based on the equations after Motak (1990) and the Polish Soil Code [11], were determined. Their courses versus depth are presented in Fig.2.

The presented distributions of the chosen static parameters of the investigated soil show that the adoption of a constant value of given parameters in

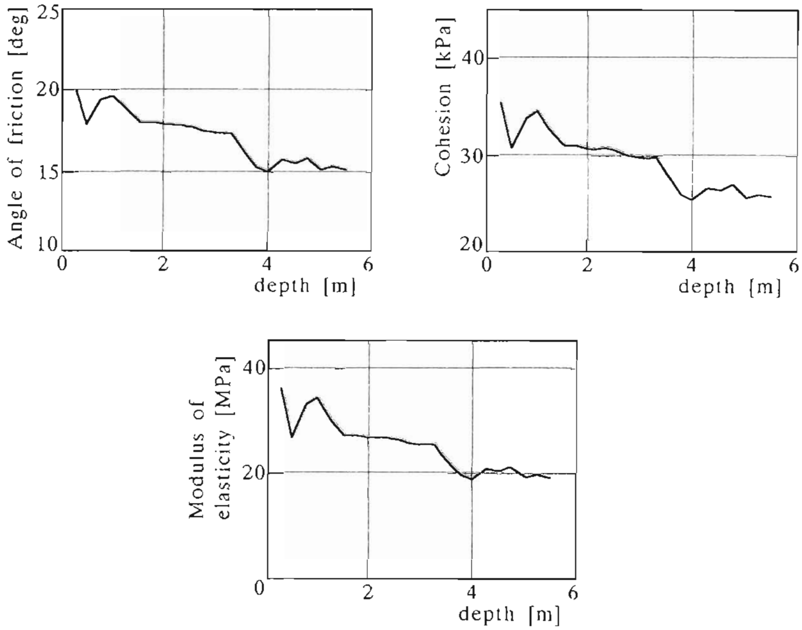

Fig. 2. Soil static parameters versus depth determined on the basis of the equations after Motak (1990) and the Polish Soil Code [11]

engineering calculations leads to significant inaccuracies in presentation of the behaviour of the soil. In this case, the mean constant values of the considered soil parameters (of plastic and stiff plastic cohesive soil) given by the Polish Soil Code are:

Natural water content	20% (25% for stiff plastic cohesive soil)
Soil mass density	2100 kg/m^3 (2000 kg/m^3 for stiff plastic cohesive soil)

and calculated on the plasticity ratio I_L:

Angle of friction	17°
Cohesion	29.5 kPa
Modulus of elasticity	25 kPa

The soil parameters variation as a function of depth is very important for calculations of structures embedded in the soil.

5. Dynamic soil parameters

In the Balice area the dynamic investigations of seismic wave propagation in the soil, called parallel passage of the seismic wave through soil levels down to the depth of 4.5 m were carried out.

Paraseismic excitation was generated by the use of an electro-hydrodynamic generator of seismic wave with speak discharge, type EHD-3200/04 of parameters 1600 J/8 kV used in geophysical investigations. This excitation is of impulse character. This inductor works in a stable way what assures a very good repeatability of the vibration source signals.

The seismic impulse is generated by the EHD generator in the way of conversion of electrical energy stored in the condenser at high voltage into mechanical energy of the pressure shock wave. It is formed during spark discharge between electrodes immersed in water. In order to assure proper operational conditions for the equipment a hole of 80 mm in diameter drilled in the soil under impulse excitation is filled with water. Then the head of the generator is placed there at a regained depth and the impulse is sent.

The generator and its head are presented in Fig.3.

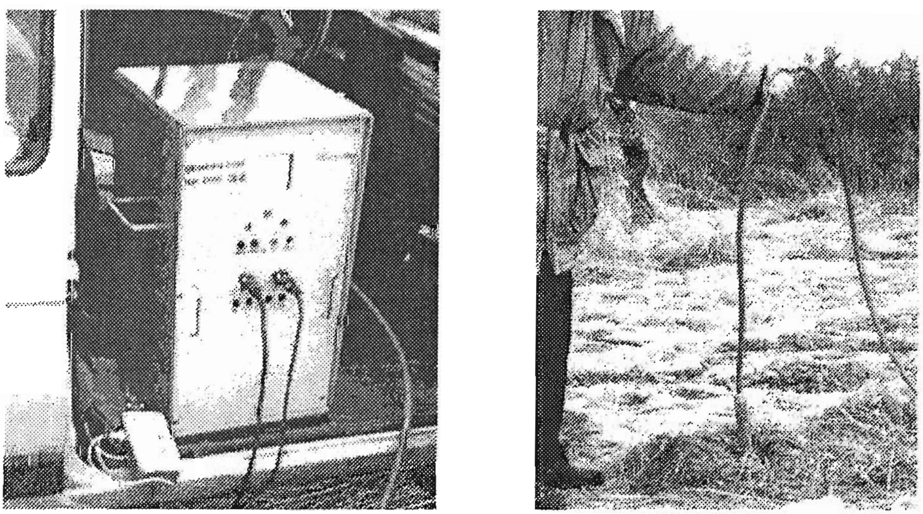

Fig. 3. EHD inducer with spark head

For velocity the measurement of the longitudinal and shear (vertical and horizontal) waves three direction geophones, serving for recording soil vibration velocity in geophysical works, were used (Fig.4).

Fig. 4. Depth tree direction geophones

The signals obtained from the geophones were recorded by the use of 24-canal digital seismic apparatus GEOMERICS.

The recorded times of transition of waves alowed us to determine their propagation velocity in the soil. Distribution over the depths of the values of propagation velocity of P, SV and SH waves on given depth level are presented in Fig.5.

The measurements performed showed that the velocities of P and SV waves in the soil increases approximately in a linear way with depth. This is caused, to a great extend, by increase in the overburden pressure with depth. Significant influence can be exerted by the change with depth of soil water content.

The fact that, velocity of SH wave remains unchanged with depth is an interesting phenomenon. The factors essential for P and SV waves (overburden pressure, soil water content) do not influence the velocity value of SH wave. The approximately constant value with depth of SH wave velocity (Fig.5) is connected, most probably, with a given kind of soil (in a homogenous soil practically no changes of this value with depth are observed). The parameters defined in Eqs (2.1), (2.3) and (2.4) are the dynamic soil properties calculated on the basis of results obtained from the parallel passage of the seismic wave through soil levels. Their values depend, to the greatest extent, on the velocity of the longitudinal and shear waves.

Analysing the distribution of velocity in Fig.5, a conclusion can be easily

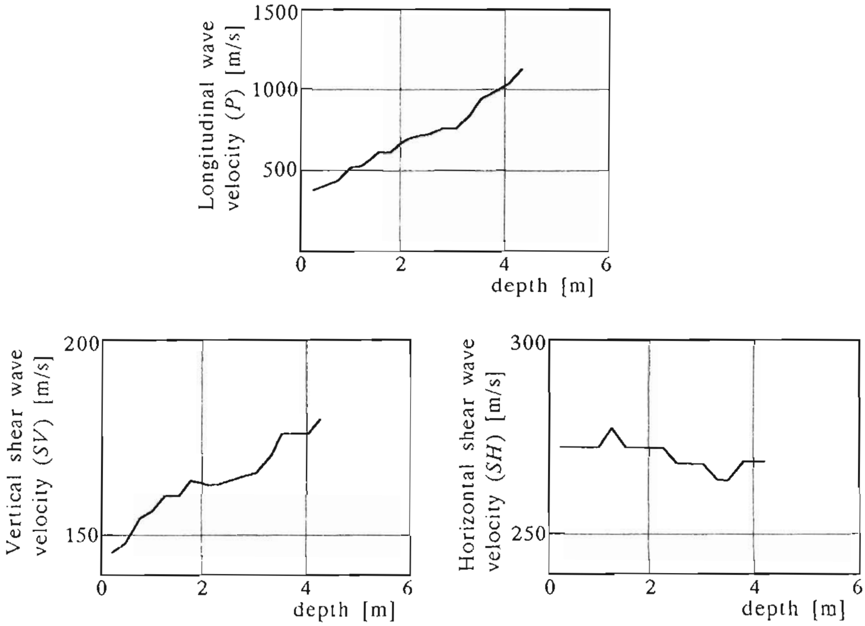

Fig. 5. Distribution of the values of P, SV, SH wave propagation velocity with regard to depth

drawn that the calculated soil parameters are not constant, but change with depth.

However, in the adopted formulae, based upon the linear theory of elasticity it is not determined what kind of shear wave velocity should be taken for the calculations. Since the velocity distributions for the waves SV and SH differ from each other both by their values as well as their shape and distribution over the depth. When determining dynamic soil parameters in this way their different properties for the vertical and horizontal directions should be distinguished. This is a quite new approach in view of the mean values of the shear wave adopted for calculations so far.

In order to consider this dissimilarity, the calculations and analysis based on Eqs (2.1), (2.2) and (2.4) were performed adopting separately the values for SV and SH waves. Basing upon these values the mean values were also determined. The results of analysis are presented in Fig.6, Fig.7 and Fig.8 for the three cases taken for calculations.

When comparing the static and dynamic Young moduli of the analysed soil considerable discrepancies were noticed. These occur both in the field of values and shape distribution over the depth.

The value of the static modulus (independent of the direction – as treated in [11]) changes with the range $20 \div 40$ MPa, whereas the dynamic modulus depends on the given kind of shear wave (the considered direction is imported) adopted for calculations. For the wave SV (vertical direction) its values are within the range $100 \div 200$ MPa, while for the wave SH (shear vibration) $250 \div 350$ MPa.

Unfortunately, it is impossible to determine directly that the static modulus is smaller (by a certain constant value) than the dynamic one. This is accounted for by the shape of distribution of this moduli versus depth. The static modulus decreases almost linearly with depth from about 40 MPa on the surface to 20 MPa at the depth of 4 m, and then constant values are adopted (Fig.2). The dynamic modulus behaves in quite a different way. The vertical modulus increases almost lineary with depth from the value of about 100 MPa on the surface to 200 MPa at the depth of 4.5 m. The horizontal modulus increases sharply from about 300 MPa on the surface to 420 MPa at the depth of 1.8 m and reaches then the value of about 440 MPa, at the depth of 4.5 m (Fig.6). Average value of the dynamic modulus of elasticity (A), calculated according to Eq (2.1) on the basis of the mean value of shear wave velocities (SH and SV waves), is presented in Fig.6.

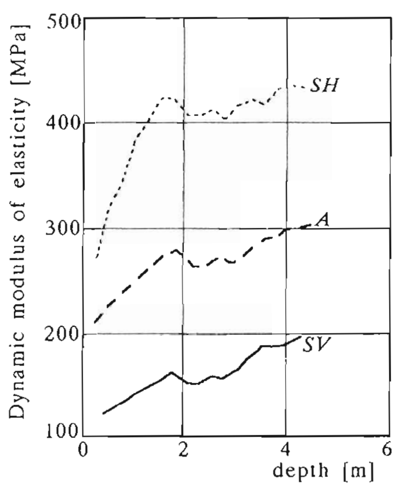

Fig. 6. Distribution of the values of dynamic modulus of elasticity over the depth calculated (Eq (2.1)) on the basis of propagation velocity of SV, SH waves and the calculated average value of shear wave velocities (A)

A similar distributions with depth are observed for a calculated distribution of the Poisson ratio at horizontal vibrations. In this case the values obtained experimentally on the surface close to zero are disputable and require more detailed explication. On the other hand, the Poisson ratio distribution with depth for the vertical vibration (as value is concerned) is closer to that found in the literature (Fig.7).

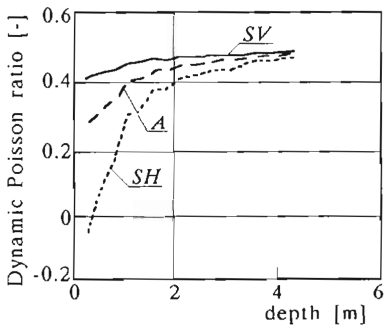

Fig. 7. Distribution of the Poisson ratio values over the depth calculated (Eq (2.3)) on the basis of propagation velocity of SV, SH waves and the calculated average value of shear wave velocities (A)

The Young modulus and Poisson ratio are adopted in numerical calculations as basic data. The routine adoption of static values in dynamic calculation, common in engineering practice, leads to far going discrepancies in presentation of the real behaviour of embedded structures interacting with the soil subject to the dynamic action.

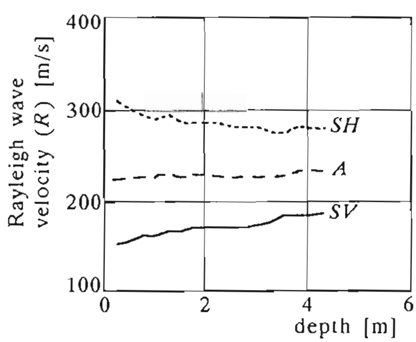

Fig. 8. Distribution of the values of surface wave propagation velocity over the depth calculated (Eq (2.4)) on the basis of propagation velocity of waves SV, SH and the calculated average value of shear wave velocities (A)

The distribution of propagation velocity with respect to depth of the Rayleigh surface wave (R) for vertical and horizontal vibrations, presented in Fig.8, confirms the behaviour of the surface wave (movement of particles) in real behaviour. Distribution of the surface wave velocity calculated (Eq (2.4)) on the basis of the SV wave velocity gives smaller values of velocity on the surface than in deeper layers of the soil. This relation is in agreement with the ellipsoidal direction of the movement of Rayleigh surface wave particles in the vertical plane. On the other hand, the curve of surface wave velocity distribution calculated (Eq (2.4)) on the basis of the SH wave velocity (the velocity value decreases with depth) is in agreement with the displacement of soil particles under the influence of the passing Love surface wave (Q). The characteristic motions of P, SV, SH, R and Q waves presents Fig.9.

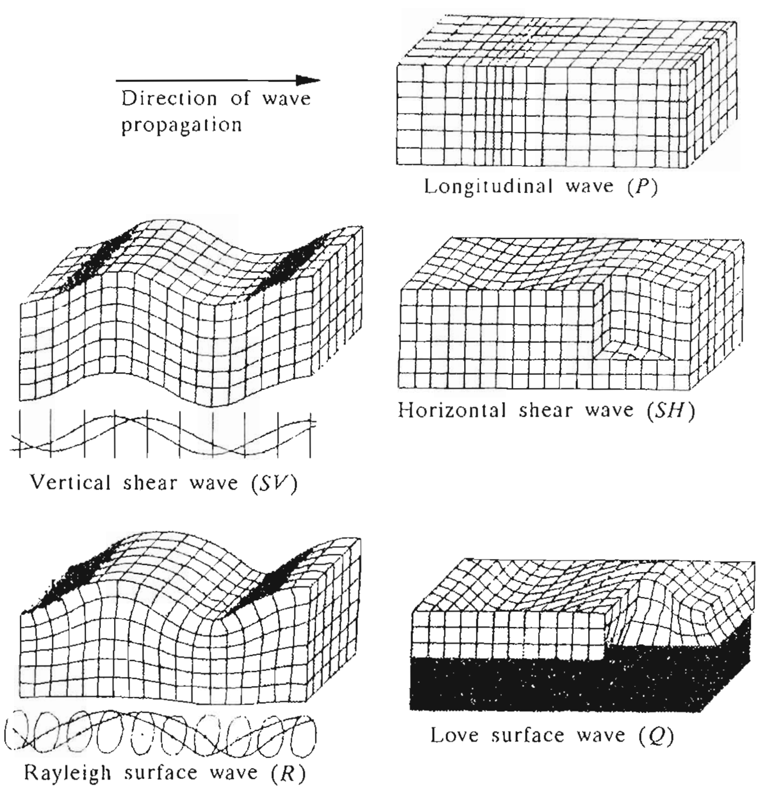

Fig. 9. Characteristic motions of P, SV, SH, R and Q waves (comp. Stenz and Mackiewicz, 1964)

Higher values of the Love wave velocity (calculated on the basis of wave SH velocity acc. Eq (2.4)) related to the Rayleigh wave velocity (calculated on the basis of wave SV velocity acc. Eq (2.4)) – presented in Fig.8 – is confirmed in the literature (Stenz and Mackiewicz, 1964) and nature.

The distributions of mean values over the depth presented in Fig.6, Fig.7 and Fig.8 could probably be adopted for calculations in simplified models. However, a more precise approach to dynamic problems in the soil should take into account independently the behaviour of the soil in the vertical and horizontal plane. variation with depth of elastic soil parameters is also essential.

6. Conclusions

If we decide creating the soil models in which the properties of materials comply with the laws of the theory of elasticity, then our investigations will confirm the opinion on difference of dynamic moduli of elasticity in horizontal and vertical directions. Their values are considerably bigger in the horizontal direction then in the vertical one. At the beginning the dynamic moduli increase with depth intensively – their traces of changes being close to linear – and then less intensively and non-linearly. It is characteristic that this increase with depth does not concern the static moduli of elasticity adopted generally (e.g. in standards) as a constant value. It is to mention that taking into account of the difference between dynamic properties of moduli of elasticity is purposeful in the solution of important practical engineering cases.

Considering the behaviour of the structure in the soil both change of soil parameters (static and dynamic) with depth and the difference between the values dynamic and static parameters should be considered. One should also be aware of differences of dynamic properties of the soil between the vertical and horizontal directions in the cross section of the soil.

The presented investigations showed what a complex problem is there to be dealt with. It requires further laboratory and in situ investigations. Into case of latter ones the used method has proved to be very helpful.

Of course, other kinds of soil, different from the investigated ones require similar experimental investigations in situ.

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Dynamiczne charakterystyki sprężystości warstw gruntu występujących płytko pod powierzchnią terenu

Streszczenie

Przyjęcie w praktyce inżynierskiej, jako wystarczająco precyzyjne, uproszczonego sprężystego modelu podłoża gruntowego pociąga za sobą konieczność określenia rzeczywistej wartości modułów sprężystości, współczynnika (liczby) Poissona oraz prędkości rozchodzenia się fali w gruncie płytko pod powierzchnią terenu. Wartości tych parametrów sprężystych są zmienne wraz ze wzrostem głębokości. Niniejsza praca prezentuje wyniki badań terenowych "in situ" określających wyżej wspomniane cechy sprężyste dla podłoża gliniastego. Zmiany z głębokością wspomnianych parametrów sprężystych są znaczące i istotne dla praktycznych zastosowań. Więcej informacji dotyczących przeprowadzonych badań można znaleźć w monografii Ciesielski et al. (1999).

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