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TWO METHODS OF MODELLING OF PERIODIC NONHOMOGENEOUS ELASTIC PLATES

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The effect of a sequence of application of the two methodes, i.e. approximation by constraints and nonstandard homogenisation has been studied in the paper. The author has also tried to find out wheather the results yielded by the homogenisation with respect to successive co-ordinates, i.e. successive sides of the representative element, are the same as those obtained from the spatial homogenisation, i.e. performed over the whole 3D representative element simultaneously.

Key words: theory of plates, nonstandard homogenization, approximation by constraints

1. Introduction

Two methods of modelling usually employed when dealing with the elasticity problems are presented in the paper. When applying the first method one obtains the models resulting from approximation by constraints, while in the second approach nonstandard homogenisation is used.

In a model due to approximation by constraints the strain and stress state is subject to the constraints determined by certain functions depending on co-ordinates of the points which belong to the reference configuration of the body, (cf Woźniak, 1973, 1988). These models are dealt with in theories of shells, plates, bars, etc. In the present contribution we confine ourselves to 2D models of plates, (cf Nagórko, 1989).

A model of nonhomogeneous (anisotropic, in general) body due to nonstandard homogenisation emerges from application of the nonstandard analysis approach to approximate description of nonhomogeneity, (cf Woźniak, 1986, 1995).

When solving a variety of mechanical problems one has to use both the aforementioned ways of modelling simultaneously. Therefore, in the present contribution we study the effect of changing the application sequence of these methods on basic relations of the model. We also try to find out whether the results obtained after applying the step-wise homogenisation, i.e. with respect to successive co-ordinates (successive sides of the representative element) are different from those resulting from the spatial homogenisation, i.e. applied to the whole representative element simultaneously.

Generally, it turns out that having the some weight functions one cannot replace the spatial homogenisation with the step-wise one made with respect to co-ordinates. If that homogenisation is possible for each co-ordinate also the 3D homogenisation is possible. The sequence of using the modelling methods, i.e. approximation by constraints and nonstandard analysis approach is also of crucial importance and may affect the results. The conditions under which this sequence does not matter are given in the work.

2. Approximation by constraints

Consider nonhomogeneous anisotropic elastic bodies, the reference configuration of which we denote as Ω, $\Omega \subset R^3$, while $v = (v_k)$ represent the displacements being of adequate regularity, where $v_k : \Omega \to R^1$, $k = 1,2,3$. The space of vector functions v is denoted by V. The body forces $b = (b_k)$, $b_k : \Omega \to R^1$ and the surface ones $p = (p_k)$ are acting upon the body, $p_k : \partial_1\Omega \to R^1$ where $\partial_1\Omega$ is a part of the boundary of the body $\partial_1\Omega \subset \partial\Omega$. Elastic properties of the body are represented by the scalar functions $B_{klmn} : \Omega \to R^1$, $k,l,m,n = 1,2,3$ satisfying the conditions $B_{klmn} = B_{lkmn} = B_{klnm} = B_{mnkl}$. Let $s = (s_{kl})$ represent stresses in the body and have the following form

$$s = (s_{kl}) = \Big(\frac{1}{2}B_{klmn}(v_{m,n} + v_{n,m})\Big) \tag{2.1}$$

Since S represents the space of vector functions s, therefore, Eq (2.1) defines the operator $M^1 : V \to S$.

Local static relations of the theory of elasticity have the form

$$\operatorname{div} s + b = 0 \qquad s\Big|_{\partial_1\Omega} n = p \qquad v\Big|_{\partial_2\Omega} = u^0 \tag{2.2}$$

where

n – outward normal unit vector to the boundary $\partial_1\Omega$

u^0 – given displacement on the remaining part of the boundary $\partial_2\Omega$.

Eqs $(2.2)_{1,2}$ define the operator $M^2 : S \to F$, where F represents the space of pairs (b,p) of body and surface forces, respectively. The static model under consideration of the theory of linear elasticity may be represented by the following scheme

$$V \overset{M^1}{\to} S \overset{M^2}{\to} F$$

Within the framework of this model, most frequently, the following problem can be solved:

for a given element $f = (b,p) \in F$ *find the element* $u \in V$ *satisfying the condition* $(2.2)_3$ *and* $\sigma \in S$ *so that*

$$M^1(u) = \sigma \qquad \text{and} \qquad M^2(\sigma) = f \tag{2.3}$$

The set of static relations of the theory of linear elasticity given above is usually, for various reasons, simplified. A 2D model of a plate may be presented as an example of such simplification.

Let the body under consideration have the reference configuration $\Omega = \Pi \times (-h/2, h/2)$. In this case, *the approximation by constraints* is a very convenient method for construction of a 2D model.

In mechanics there are many accepted meanings of the term "constraints". Here, we consider the constraints as a limitation imposed on the set of all displacements or stresses bounding it to the proper subset.

In the present contribution we define the constraints using the space Y of the functions defined on Π in the way ensuring that they are the *generalised co-ordinates* for V. Assume that the operator $A^1 : Y \to V$ such that $\emptyset \neq A^1(Y) \subset V$, and $A^1(Y) \neq V$ is defined on Y. This operator defines the disp;acement constraints. In a similar way we create the space of functions $t \in T$, which are defined on Π in the may ensuring that the operator $A^2 : T \to S$ establishes the stress constraints (Fig.1).

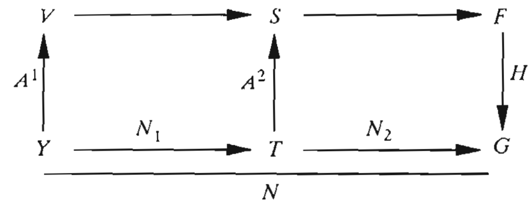


Fig. 1.

When introducing independently the displacement and stress constraints (it should be noted here, that strain constraints are not required when dealing with linear elasticity problems since they can be easily redefined in the form of displacement constraints) the following cases can be considered:

- only the displacement constraints are imposed on the body, i.e. the operator A^1, is given;
- only the stress constraints hold, i.e. the operator A^2 is specified;
- both the constraints are imposed, i.e. the two operators A^1 and A^2 are specified.

In the last case, however, it may sometimes turn out that when introducing independently the stress and displacement constraints, i.e. formulating separately the operators A^1 and A^2 one obtains contradictory results, what means that the sets of admissible displacements and stresses may be the disjoint ones.

Having the constraints introduced into the model one should construct *the relations* of a simplified model, i.e. the relations between displacements, strains and stresses defined only on the surface Π and generalised forces defined also only on Π and $\partial\Pi$. To this end we search for the generalised forces space G and the operators $H : F \to G$ and $N : Y \to G$, or $N^1 : Y \to T$, $N^2 : T \to G$, (Fig.1). Having these operators we can reformulate the problem represented by Eq (2.3) as follows

for a given element $f = (b, p) \in F$ *find* $y_0 \in Y$ *or* $y_0 \in Y$ *and* $t_0 \in T$ *thus that*

$$N(y_0) = H(f) \tag{2.4}$$

or

$$N^1(y_0) = t_0 \qquad\qquad N^2(t_0) = H(f) \tag{2.5}$$

The model represented by Eqs (2.4) and (2.5) is obtained due to approximation by constraints. Solution to the problem represented by Eq (2.4.) or (2.5) is the exact one in this model. This solution determines the elements $A^1(y_0) \in V$ and $A^2(t_0) \in S$. These are the approximated solutions to the problem (2.3) obtained basing on an approximated model, which in general, differ from the sought after solutions u, σ to the problem (2.3).

An attempt at assessement of differences between the exact solutions u, σ to the problem (2.3) and the approximated ones $A^1(y_0)$, $A^2(t_0)$ involves a lot of complicated problems and was undertaken many times (cf Nagórko, 1983). It can de proved that:

when an approximated model is constructed basing on the virtual work principle the approximated results obtained minimise the error measured in energetic norm, (cf Nagórko, 1983).

Coming back to the plate model, assume the operator A^1 in the following form

$$v_k(x_1,x_2,x_3) = w_k^0(x_1,x_2) + x_3 w_k^1(x_1,x_2) + x_3^2 w_k^2(x_1,x_2) \tag{2.6}$$

Each element of the space Y consists of certain nine adequately regular functions (w_k^0, w_k^1, w_k^2) defined on Π. The constraints (2.6) represent the well-known Kirchhoff hypothesis

$$\begin{aligned} w_k^0(x_1,x_2) &= \Big(0,0,w(x_1,x_2)\Big) \\ w_k^1(x_1,x_2) &= \Big(-w_{,1}(x_1,x_2), -w_{,2}(x_1,x_2), 0\Big) \\ w_k^2(x_1,x_2) &= (0,0,0) \end{aligned} \tag{2.7}$$

When applying the Kirchhoff hypothesis the plate deflection $w(x_1,x_2)$ is the only element of the space Y.

Most simplest stress constraints usually introduced for plates have the form $\sigma_{33} = 0$. Assuming a modified form of the constraints (2.7) and the stress constraints $\sigma_{33} = 0$, by virtue of the virtual work principle, we obtain the well-known local relations for anisotropic nonhomogeneous plates

$$(D_{\alpha\beta\gamma\delta} w_{,\gamma\delta})_{,\alpha\beta} = \int\limits_{-\frac{h}{2}}^{\frac{h}{2}} b \, dx_3 + p_+ + p_- \tag{2.8}$$

where $D_{\alpha\beta\gamma\delta}$, $\alpha,\beta,\gamma,\delta = 1,2$ are the elasticity moduli of the plate, while p_+ and p_- represent the loads acting upon the upper and bottom surfaces, respectively, of the plate, i.e. $\Pi \times \left\{\frac{h}{2}\right\}$ and $\Pi \times \left\{\frac{h}{2}\right\}$. The boundary conditions for displacements have the form of deflections specified on $\partial\Pi$.

The model of anisotropic nonhomogeneous plate given above is based on some simplifications which change material properties of the body. Thus, instead of the material functions B_{klmn} one should employ the elastic moduli $D_{\alpha\beta\gamma\delta}$ such that

$$D_{\alpha\beta\gamma\delta} = B_{\alpha\beta\gamma\delta} - \frac{B_{\alpha\beta33} B_{\gamma\delta33}}{B_{3333}} \tag{2.9}$$

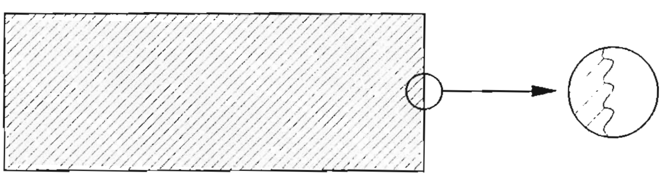
In the boundary-value problems, in which B_{klmn} are oscillating, stepwise functions, the functions represented by Eqs (2.9) are also oscillating and discontinuous. It is very inconvenient when one has be solve Eq (2.4), even applying a numerical approach. This problem appears e.g. when dealing with the periodic composites of a great number of periodicity cells. This case is considered below.

3. Nonstandard homogenisation

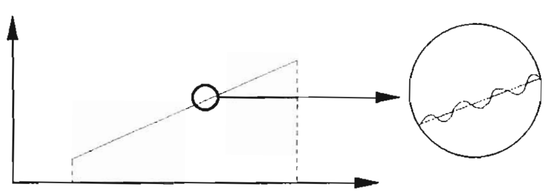
There are many problems to be dealt with in mechanics which can be simplified by means of neglecting some quantities in respect of the other ones. It can be done in certain reference scales. We consider the *macro* scale as that applicable to the quantities which cannot be neglected, while the *micro* refers to the negligible ones.

For example:

- in the macro scale the body configuration may be represented by the area determined by straight lines (the boundary is composed of segments), while in the micro scale each side of these segments may be a strongly irregular curve



- in the macro scale the displacement of the body may be represented by a linear function, while when turning to the micro scale it may oscillate about the linear function



- in the micro level the body may be assumed as a homogeneous one, while when going down to the micro level it may reveal some inclusions of a different material and therefore should be regarded as a nonhomogeneous one.

Application of both these scales, with the description constructed in many different ways is quite widespread in mechanics. A survey can be found in Woźniak (1993). It should be also noted that the term "macrofunction" emerges from the necessity for description of the body in both macro and micro scales.

Now, we apply the nonstandard analysis approach, (cf Robinson, 1966). When describing the body in the macro scale we use classic real numbers from the R space, while when using the micro scale one should apply the infinitensimal numbers, i.e. those of modules smaller than each number $r \in R_+$. The infinitensimal numbers belong, therefore, to the completely new number space, the so-called extension. which we denote by $^*R$.

This approach we use to obtain a model of the periodically nonhomogeneous body with the representative element of a very small size in the macro scale.

It should be emphasised that having the representative element of a definite size, even a very small one, the body can be described and investigated using certain relations of the linear theory of elasticity. These relations can assume the following global form:

for a given element $f = (b,p) \in F$ *find the element* $u \in V$ *satisfying condition* $(2.2)_3$ *such that*

$$\forall v \in V \quad \int_{\Omega} (B_{klmn} u_{k,l}\, v_{m,n} - b_m v_m)\, dv = \int_{\partial_1 \Omega} p_m v_m\, da \tag{3.1}$$

As it was mentioned above this description is practically useless since the material functions are periodic and step-wise in a very small domain.

This difficulty can be overcome using the homogenisation approach, in which the number of periodicity cells is of crucial importance. Which means that if the composite is composed of a sufficiently high number of periodicity cells, the exact number does not matter, i.e. it might be twice, three, or n times higher with no visible effect on a solution to the problem. Therefore a difference between solutions to the problem with nm cells and with n cells, respectively, should be negligibly small, they should approximate each other. The above reasoning can be formulated in a more formal way.

Let $\widetilde{B}_{klmn}$ represent material functions of the composite, in which the representative element size has been reduced n times (the present size is l/n). If the representative configuration of the composite Ω and the forces (b,p) remain unchanged the current problem differs from that represented by Eq (3.1) only in the form of material functions, i.e. one should put $\widetilde{B}_{klmn}$ into Eq (3.1). Denote by $\widetilde{u}$ the solution to this problem, which usually differs from the solution u_k to the problem (3.1). We require, however, that this difference be negligibly small, i.e. that

$$\widetilde{u}_k(x_1,x_2,x_3) - u_k(x_1,x_2,x_3) = \overline{u}_k(x_1,x_2,x_3) \tag{3.2}$$

where the functions $\overline{u}_k(x_1,x_2,x_3)$ take negligibly small values for every $x \in \Omega$. If it is true for every n then when considering the representative element of the infinitensimal characteristic dimension $l = \varepsilon$, the functions $\overline{u}_k$

take the infinitensimal values (denoted by u_k^ε), while the functions u_k become $^*u_k$ defined on $^*\Omega$ and taking the values from $^*R$, where $^*\Omega \subset {}^*R$ is the set of all $y_k = x_k + \varepsilon_k$, $(x_k) \in \Omega$ and ε_k assumes an arbitrary infinitensimal value, $k = 1,2,3$, thus

$$\widetilde{u}_k(x_1,x_2,x_3) = {}^*u_k(x_1,x_2,x_3) + u_k^\varepsilon(x_1,x_2,x_3) \tag{3.3}$$

The space of the functions defined by Eq (3.3) we denote by $\widetilde{V}$. Assume that the representative element is infinitesimal (a point in the macro description). Let this element, in the micro description, be a cube of the dimensions ε_1, ε_2, ε_3, composed of n elastic and homogeneous elements.

The problem (3.1) can be reformulated as follows:

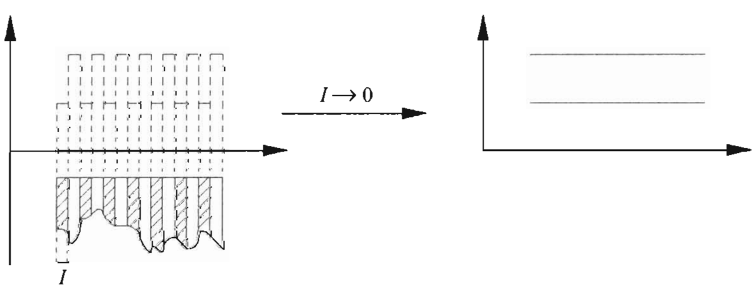
for a given element $f = (b,p) \in F$ *find the element* $\widetilde{u} \in \widetilde{V}$ *satisfying condition (3.2) such that*

$$\forall \widetilde{v} \in \widetilde{V} \qquad \int\limits_{^*\Omega} (\widetilde{B}_{klmn}\widetilde{u}_{k,l}\,\widetilde{v}_{m,n} - {}^*b_m\widetilde{v}_m)\;dv = \int\limits_{^*\partial_1\Omega} {}^*p_m\widetilde{v}_m\;da \tag{3.4}$$

where $^*b_k$, $^*p_k$ are the functions defined on $^*\Omega$ and taking the values from $^*R$. The functions $\widetilde{B}_{klmn}$ have now the infinitesimal period, thus

$$\widetilde{B}_{klmn}(x) = {}^*B_{klmn}(\overline{x}) \quad \text{where} \quad \overline{x} = \Big(\frac{x_1}{\varepsilon_1}, \frac{x_2}{\varepsilon_2}, \frac{x_3}{\varepsilon_3}\Big)$$

One of the disadvantages this approach suffers from consists in the fact that the material functions become multifunctions in the macro description, taking n values, e.g. for a two-component composite we have



The displacements $\widetilde{v}$ appearing in Eq (3.4) are arbitrary functions from $\widetilde{V}$. These functions may be subject to the constraints in the micro scale (micro constraints), i.e. the infinitensimal component u_k^ε may assume a certain required form. For example, we can take

$$u_k^\varepsilon = h_k^a\,{}^*q_k^a \tag{3.5}$$

where q_k^a are the sought after functions defined on R and taking the values from R, being, therefore, the standard ones (they are usually called microlocal parameters, see Woźniak (1987), while h_k^a are given infinitensimal functions obtained from the standard functions formulated for the representative element of definite dimensions by means of replacing those dimensions with the infinitensimal ones. These functions are constructed in a way similar to that followed when constructing weight functions in the finite element method. The functions h_k^a should satisfy the following condition

$$\int_{\Delta} h_{k,l}^a \, dv = 0 \tag{3.6}$$

The space of functions $q^a = (q_k^a)$ we denote by Q.

After applying the constraints (3.5) the variational principle for a homogenised periodically nonhomogeneous body takes the following form

$$\begin{aligned}\forall v \in V \quad \forall r^a \in Q \qquad & \int_{\Omega} B_{klmn}^A(\eta^A u_{k,l} v_{m,n} + \eta_{(m)n}^{Aa} u_{k,l} r_m^a + \\ & + \eta_{(k)l}^{Aa} q_k^a v_{m,n} + \eta_{(k)(m)ln}^{Aab} q_k^a r_n^b + b_k v_k) \, dv = \int_{\partial_1 \Omega} p_k v_k \, da\end{aligned} \tag{3.7}$$

where B_{klmn}^A represent material properties of the Ath component and

$$\begin{aligned}\eta^A &= \frac{\varepsilon_1^A \varepsilon_2^A \varepsilon_3^A}{\varepsilon_1 \varepsilon_2 \varepsilon_3} \qquad \Big(\eta_{(k)l}^{Aa} = \frac{1}{\varepsilon_1 \varepsilon_2 \varepsilon_3} \int_{\Delta^A} h_{(k)l}^a \, dv\Big)^{\circ} \\ \eta_{(k)(m)ln}^{Aab} &= \Big(\frac{1}{\varepsilon_1 \varepsilon_2 \varepsilon_3} \int_{\Delta^A} h_{(k)l}^a h_{(m)n}^b \, dv\Big)^{\circ}\end{aligned} \tag{3.8}$$

where $(\cdot)^{\circ}$ is the standard part of $(\cdot)$, Robinson (1966), and ε_k^A are the dimensions of the Ath component, while Δ^A denotes the area covered by the body.

It should be emphasised that almost all components appearing in Eq (3.7) are the standard ones, but the coefficients defined by Eq (3.8) are derivatives of nonstandard functions. However, it turns out that a proper choice of the functions h_k^a ensuring that their derivatives are integrable over the infinitensimal parts of the representative element allows the quantities (3.8) to take the real values.

The same approach can be followed when dealing with a 2D description of a plate in the following global form

$$\forall v \in V \quad \int\limits_{\Pi} (B_{\alpha\beta\gamma\delta} w_{,\alpha\beta}\, v_{,\gamma\delta} - bv)\, da = \int\limits_{\Pi_+ \cup \Pi_-} (p_+ + p_-) v\, ds \tag{3.9}$$

Then, we obtain

$$\forall v \in V \quad \forall r^a \in Q$$

$$\int\limits_{\Omega} [B^A_{\alpha\beta\gamma\delta}(\eta^A w_{,\gamma\delta}\, v_{,\alpha\beta} + \eta^{Aa}_{\gamma\delta} q^a v_{,\alpha\beta} + \eta^{Aa}_{\alpha\beta} w_{,\gamma\delta}\, r^a + \eta_{\alpha\beta\gamma\delta} q^a r^b) - pv]\, da = 0$$

The above formula represents a surface homogenised model of a periodically elastic plate. The macro constraints (2.6) or (2.7), in turn, which lead to a 2D model can be introduced to Eq (3.6) yielding a 2D surface model. Here, the question can be posed: is the process of modelling consisting of both the aforementioned methods commutative, i.e. can we obtain the same results despite the sequence of their application.

4. Commutativity of modelling

Consider first the homogenisation of one variable, e.g. x_1. Then for every $x \in \Pi$ relation (3.1) depends on one variable, with the representative elements having the form of the segment of the length ε.

The material functions are

$$\widetilde{B}_{klmn}(x_1, x_2, x_3) = {}^*B_{klmn}\Big(\frac{x_3}{\varepsilon}, x_2, x_3\Big)$$

In a way similar to the procedure represented by Eq (3.5) we impose on the displacements $\widetilde{u}$ the constraints of arbitrary shape functions, let us say, depending only on x_1. For the sake of simplicity, however, we can take on the same functions, which should, therefore, satisfy the condition (3.6) within the range in which y_0 represents the centre of representative element

$$\int\limits_{y_0 + \frac{\varepsilon}{2}}^{y_0 - \frac{\varepsilon}{2}} h^a_{k,l}\, dx_1 = 0 \tag{4.1}$$

Usually, it is not true. It can be, therefore, noted that the spatial homogenisation cannot be directly replaced with the homogenisation with respect to successive co-ordinates, having the same shape functions (and thus, the micro constraints). If the micro constraints have the form (3.5) and the homogenisation is possible for each co-ordinate x_k, $k = 1, 2, 3$ then the spatial homogenisation is also possible. If Eq (4.1) is true it implies that the triple integral (3.6) also equals zero. This conclusion is the same as that validated for asymptotic homogenisation, (cf Kohn and Vogelius, 1984).

The last problem consists in finding out if the modelling process is commutative, i.e. do the results obtained depend on the application sequence of modelling procedures (approximation by constraints and nonstandard homogenisation). Note, that approximation by constraints yields Eqs (2.6), which when extended take the form

$$\widetilde{u}_k(x_1, x_2, x_3) = {}^*w_k^0(x_1, x_2) + x_3 {}^*w_k^1(x_1, x_2) + \widetilde{f}(x_1, x_2, x_3) \tag{4.2}$$

where $\widetilde{f}$ is a nonstandard function assuming the infinitensimal values.

The homogenisation allows the following micro constraints to be imposed

$$\widetilde{u}_k(x_1, x_2, x_3) = {}^*u_k(x_1, x_2) + h_k^a(x_1, x_2, x_3) {}^*q_k^a(x_1, x_2) \tag{4.3}$$

Applying first the modelling with the constraints (4.2) and then the homogenisation with micro constraints of the form

$$\begin{aligned} w_k^0(x_1, x_2) &= {}^*v_k^0(x_1, x_2) + h_k^a(x_1, x_2) {}^*q_k^a(x_1, x_2) \\ w_k^1(x_1, x_2) &= {}^*v_k^1(x_1, x_2) + k_k^a(x_1, x_2) {}^*q_k^a(x_1, x_2) \end{aligned}$$

affects the form of functions h_k^a, k_k^a and q_k^a which appear in Eq (4.2), in particular, their dependence on x_3; thus the following condition should be satisfied

$$\begin{aligned} &h_k^0(x_1, x_2, x_3) {}^*q_k^a(x_1, x_2, x_3) = \\ &= h_k^a(x_1, x_2) {}^*q_k^a(x_1, x_2) + x_3 k_k^a(x_1, x_2) {}^*q_k^a(x_1, x_2) \end{aligned} \tag{4.4}$$

As can be easily seen from the above consideration the class of homogenisation problems (4.3) to be considered is considerably limited when first the approximation by constraints is applied followed by the homogenisation. The

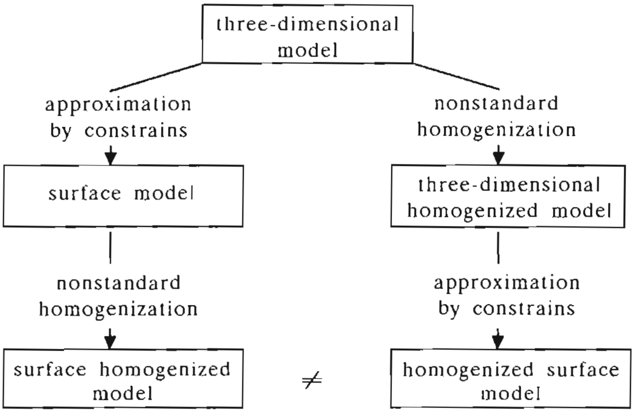


Fig. 2.

micro constraints (4.3) can be thus assumed only as depending explicitly on x_3 and satisfying the condition (4.4). If it is true the sequence of application of modelling techniques does not matter, Fig.2

From the above it follows that, generally speaking, these two methods of modelling are not commutative. This conclusion agrees with the results obtained for the asymptotic homogenisation by Kohn and Vogelius (1984).

In a certain singular case when the constraints and microconstraints have been chosen in the way ensuring the condition (4.4) to be satisfied, the modelling is commutative. However, Eq (4.4) represents the condition limiting the class of constraints, which additionally is a sufficient condition.

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Dwie metody modelowania niejednorodnych periodycznie płyt sprężystych

Streszczenie

W pracy rozważa się dwie metody modelowania w sprężystości. Pierwsza z nich prowadzi do modeli uproszczonych przez więzy a druga do modeli zhomogenizowanych niestandardowo.

W pracy badano czy podstawowe relacje modelu skonstruowanego przy użyciu metody więzów i metody homogenizacji niestandardowej zależą od kolejności ich stosowania. Zbadano też czy stosowanie homogenizacji etapami, względem kolejnych współrzędnych – kolejnych boków elementu reprezentatywnego – daje ten sam wynik co homogenizacja przestrzenna, tzn. dla całego trójwymiarowego elementu reprezentatywnego równocześnie.

Okazuje się, że ogólnie nie można zamieniać homogenizacji przestrzennej na homogenizację kolejno po współrzędnych przy tych samych funkcjach kształtu. Natomiast gdy homogenizacja jest możliwa dla każdej ze współrzędnych, to jest możliwa także homogenizacja przestrzenna. Także kolejność modelowania aproksymacyjnego przez więzy i homogenizacji przy użyciu analizy niestandardowej może dawać różne wyniki. W pracy określono warunki, przy których kolejność ta nie jest istotna.

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