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VORTEX CLOUD MODELLING OF SEPARATED FLOWS

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A method for numerical simulation of 2D incompressible high Reynolds number flows with laminar separation and big recirculation zones – regions of vortex formation and wake building – is presented. The paper is concerned with the case of separation from the surface, not from sharp edges. The calculation method draws on viscous splitting of the vorticity transport equation. At each time step, this equation is solved in three separate substeps – convection (of point vortices and vortex sheet elements with their local velocities), diffusion (by means of the random walk technique) and generation (by means of Martensen's concept of vortex sheet extended by Lewis). The method does not assume any separation criterion. Locations of the separation points can be roughly estimated by means of computer flow visualisation as the stations at which the calculation elements – particles of fluid – leave the vicinity of the profile and the fluid begins to recirculate. This method of modelling has been called the cloud vorticity method. The aerodynamic force and its moment are calculated from the balance of momentum and moment of momentum equations in the flow domain. The flow patterns for a circular cylinder and NACA profile operating under separating conditions are presented to illustrate the calculation method. The occurrence of secondary phenomena in separated flows is also considered in the paper.

Key words: vorticity transport equation, viscous splitting, random walk, vortex cloud

1. Introduction

Separation is by far one of the most important and interesting fluid flow phenomena. The location of separation points has a decisive effect on the profile lift and drag. Usually, the separation involves unsteadiness which can not be explained on the grounds of the steady flow theory. In fluid flow machinery the occurrence of separation changes the operating conditions resulting in efficiency drop, cavitation and stall in blade-to-blade channels.

Sharp edges and high curvatures are most evidently exposed to the danger of separation. However, separation occurs also from flat surfaces under the conditions of adverse pressure gradient. A separated shear layer makes the division between the recirculation zone and the domain of potential flow. In some cases the shear layer can reattach to the body at some point downstream, closing the recirculating zone of extension of the order of a few times the upstream boundary layer thickness. In other cases the separated shear layer never reattaches, mixing with the recirculating fluid. The extention of the recirculation zone is then comparable to the characteristic body dimension. This option is of strong unsteady nature, ending up with alternate shedding of vortex structures into the wake.

Despite its importance, the phenomenon of separation has not been satisfactorily explained so far. The weakest point in the separation theory is identification of the separation point location provided it is not a sharp edge. Zero wall shear stress and reverse flow at the wall downstream are known to be an adequate separation criterion only for steady laminar flow. In unsteady flow the separation point can wander distances upstream and downstream. Zero wall shear stress together with flow reversal can appear without breakaway as well as separation can be found without a trace of flow reversal at the wall. The velocity profiles of unsteady separation significantly differ from those of steady separated flow. For now, a universal criterion of the occurrence of separation weighty for all kinds of unsteady flows has not been found although much effort has gone into this question. The progress in understanding the phenomenon should be certainly attributed to experimental research, most of which done over the 1970s. This research activity has been suspended since then due to the lack of satisfactory results which could have paved the way for the formulation of the separation genesis. However, with birth of new flow visualisation techniques, it will be to nobody's surprise if one day the extensive experimental research is renewed. In the meantime let us recall some existing criteria of unsteady separation and the scope of their applicability.

- The Moore-Rott-Sears (MRS) criterion (Moore (1958), Rott (1956), Sears (1956))

  The criterion says that unsteady separation is associated with simultaneous vanishing of the shear stress and velocity in a coordinate system convected with the separation velocity, at a point within the boundary layer some distance away from the body. The criterion was positively verified experimentally for the separation point moving upstream or downstream, Ludwig (1964), Koromilas and Telionis (1980), however not in the case of oscillating boundary layers, Didden and Ho (1985). It should be also noted that the MRS criterion is far from being convenient due to its requirement of a priori knowledge of the separation velocity.

- The Sears-Telionis criterion (Sears and Telionis (1975))

  The criterion attributes the separation point to the so-called Goldstein singularity in the solution of boundary-layer equations. According to Sarpkaya (1990) the concept of this criterion is based on the reasoning that if at some location in the course of calculations the boundary-layer characteristics, for example the displacement thickness, show appreciable changes there must be also appreciable changes at that location in a corresponding real boundary layer. This criterion can not be verified experimentally as the singularities in this class of flows do not carry the physical meaning.

- The Despard-Miller criterion (Despard and Miller (1971))

  The criterion refers to the case of oscillating boundary layers and according to Telionis and Mathioulakis (1984) remains valid for fast oscillating boundary layers. The criterion says that, although the shear stress and other boundary layer characteristics follow the oscillations, the locations of the separation points remain stable and the recirculation zone begins at the most upstream location where flow reversal continues throughout the whole cycle of oscillations.

More light is shed on the problem of unsteady separation in review works of Telionis (1979), Williams (1977), Gad-el-Hak (1987), Gad-el-Hak and Bushnell (1991), Sarpkaya (1990). However, all the authors univocally acknowledge the lack of the separation criterion meaningful for all kinds of unsteady flows. This fact is of great importance for modelling the separated flows. The proper solving procedure must bypass the unsolved problem of the separation origin, giving at the same time a chance to illustrate the consequence of separation. It appears that the method put forward by Chorin (1973) is nothing short of the above mentioned assets. The method has been constantly modified by Chorin's followers, see Lewis (1986), Lewis and Porthouse (1983), Cheer (1989), Chorin (1978), Kudela (1992), Styczek (1987), and is richly represented in review works, for example Leonard (1980), Ghoniem and Shermann (1985), Shermann (1990). The method described in the present paper should be deemed a further modification of that of Lewis (1986), Lewis and Porthouse (1983). The fact that the calculations can be performed at a reasonably acceptable computer cost speaks in favour of the method. Although the method does not provide means for finding the boundary-layer characteristics, it allows rough estimation of the separation point location, gives the possibility of tracing the development of the separation, that is formation of vortex structures and wake, as well as enables evaluation of the aerodynamic force acting on the submerged bodies.

The present work is concerned with numerical solving of 2D high Reynolds number separated flows of incompressible fluids with large recirculation zones – regions of vortex formation. It is assumed that the flow remains laminar in the boundary layer and becomes turbulent no earlier than downstream of the separation. Only the case of separation from the surface, not from sharp edges, will be considered in the paper.

2. The cloud vortex method

2D incompressible flow is governed by the Navier-Stokes (here in terms of vorticity) and continuity equations

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{\omega} = \nu \nabla^2 \boldsymbol{\omega} \qquad\qquad \nabla \cdot \boldsymbol{v} = \mathbf{0} \tag{2.1}$$

where

$\boldsymbol{v}$ – velocity

$\boldsymbol{\omega}$ – vorticity

ν – kinematic viscosity.

The boundary and initial conditions are as follows:

$$\begin{aligned} &\boldsymbol{v}(\boldsymbol{r}) = -\boldsymbol{v}_0 && \boldsymbol{r} \in \text{profile} + \text{its boundary} \\ &\boldsymbol{v}(\boldsymbol{r}) = \mathbf{0} && \boldsymbol{r} \to \infty \\ &\boldsymbol{v}(\boldsymbol{r}, t=0) = \mathbf{0} && \boldsymbol{r} \in R \end{aligned}$$

The above conditions state that the submerged profile moves at a constant velocity $-\boldsymbol{v}_0$ after an impulsive start from rest in a fluid remaining at rest at infinity and that the no-pass and no-slip conditions are imposed on its surface.

Part and parcel of the cloud vortex method is discrete representation of the vorticity field. High-Reynolds-number flows are distinct by virtue of the presence of regions of concentrated vorticity, namely the boundary layer, regions of vortex structure formation behind the separation point and wake. Elsewhere the flow is potential. At each instant the vorticity will be represented as a finite set of computational elements, each element described by its circulation and location. In our method we will have vortex sheet elements as the vorticity newly generated at the boundary and discrete vortices as the vorticity already existing in the flow.

The essence of the method is a sequential approach to the phenomena of generation, diffusion and convection which in nature act simultaneously. The process of solving the set of governing equations together with the boundary conditions will be resolved into three substeps.

The first step is generation. This process takes place at the boundary. The physical mechanisms of vorticity generation are still a matter of scientific interest. Sarpkaya (1990) says about vorticity generation due to the wall pressure gradient and/or body acceleration. The necessity for generation follows from the imposed boundary conditions. In the present method the vorticity generation is based on the concept of Lewis (see Lewis (1986), Lewis and Porthouse (1983)) who extended the Martensen method (see Martensen (1959), Martensen and Sengbush (1960)) of vortex sheet singularity distributed over the profile surface, the method applied originally to potential flow calculations. Unlike Martensen, Lewis does not consider the vortex sheet to be bound to the profile surface and lets it have convective and diffusive properties, owing to which vortex sheet elements become free to convect and diffuse into the bulk of fluid. The lost no-pass and no-slip conditions at the surface when the profile is set in motion or due to the action of diffusion or convection at the previous time step must be re-established then, which is done by means of generation of a new vortex sheet. As the velocity field of potential flow calculated from the Martensen method fulfils the no-pass condition and the no-slip condition is fulfilled automatically by virtue of a velocity jump across the vortex sheet, equal to the potential flow velocity, we will think that solving the potential flow problem that is finding the intensity of a new vortex sheet means generation of vorticity at the profile surface.

The vortex sheet intensity can be evaluated from a set of two Fredholm-type integral equations of the second kind, see Martensen (1959)

$$\begin{aligned} -\frac{1}{2}\gamma(\xi) + \frac{1}{2\pi}\int\limits_L \gamma(\vartheta)K_t(\xi,\vartheta)\,ds(\vartheta) &= -[v_z(\xi)+v_0(\xi)]_t \\ \frac{1}{2\pi}\int\limits_L \gamma(\vartheta)\,ds(\vartheta) &= -\sum_i \Gamma_i \end{aligned} \tag{2.2}$$

The first equation is the no-pass and no-slip condition whereas the second substantiates the requirements of conservation of circulation in the flow. The symbol γ stands for the vortex sheet intensity. ξ, θ are points along the profile surface L, v_z is the velocity induced by the vorticity in the flow, $\sum_i \Gamma_i$ is the sum of discrete vortices circulations, the symbol $[\cdot]_t$ denotes the tangential component and the kernel K_t is given by the following formulas

$$\begin{aligned} K_t(\xi,\vartheta) &= \frac{-y'_\xi(x_\vartheta - x_\xi) + x'_\xi(y_\vartheta - y_\xi)}{(x_\vartheta - x_\xi)^2 + (y_\vartheta - y_\xi)^2} \frac{1}{\sqrt{x'^2_\xi + y'^2_\xi}} \qquad \xi,\vartheta \in L \quad \xi \neq \vartheta \\ K_t(\xi,\xi) &= \frac{-y'_\xi x''_\xi + x'_\xi y''_\xi}{2\sqrt{(x'^2_\xi + y'^2_\xi)^2}} \end{aligned} \tag{2.3}$$

where x_ξ, y_ξ are Cartesian coordinates of a vector (point) ξ; $x'_\xi, y'_\xi, x''_\xi, y''_\xi$ are their derivatives with respect to a profile parameter.

The system of Eqs (2.2) has a unique solution which is found in a numerical way by solving a corresponding system of linear algebraic equations with respect to an unknown array of vortex sheet intensities in selected approximation nodes distributed evenly along the profile

$$\begin{aligned} -\frac{1}{2}\gamma(\xi_j) + \frac{1}{2\pi}\sum_{i=1}^{m} \gamma(\vartheta_i)K_t(\xi_j,\vartheta_i)h_i\Delta s(\vartheta_i) &= -[v_z(\xi_j) + v_0(\xi_j)]_t \\ \frac{1}{2\pi}\sum_{i=1}^{m} \gamma(\vartheta_i)h_i\Delta s(\vartheta_i) &= -\sum_i \Gamma_i \end{aligned} \tag{2.4}$$

The trapezoid rule was applied here to transform the integrals into sums. The weight coefficients are $h_i = 1$ for $i = 2, m-1$, $h_1 = h_m = 1/2$. It is supposed that the profile can be given in an analytical form (a parametric function) or interpolated with parametric spleen functions – either a spline function with distinguishable ends where the second derivative is imposed or a closed/cyclic spline function without distinguishable ends. The appropriate formulas can be found in Yamaguchi (1988).

The second step is diffusion. The following equation will be solved

$$\frac{\partial \omega}{\partial t} = \nu \nabla^2 \omega \tag{2.5}$$

The method is based on the analytical solutions of this equation for a single point vortex and vortex sheet:

- Point vortex of circulation Γ at the origin of the coordinate system

$$\omega(r,\vartheta,t) = \frac{\Gamma}{4\pi\nu t}\exp\Big(-\frac{r^2}{4\nu t}\Big) \qquad r \in (0,\infty) \quad \vartheta \in (0,2\pi) \tag{2.6}$$

- Vortex sheet of constant intensity γ lying in the axis x

$$\omega(y,t) = \frac{\gamma}{\sqrt{\pi\nu t}} \exp\Big(-\frac{y^2}{4\nu t}\Big) \qquad y \in (0,\infty) \tag{2.7}$$

The above solutions show certain similarities - in both cases at any instant the flow vorticity decreases with the distance from the origin exponentially as in the Gaussian distribution. It is also seen from Eqs (2.6) and (2.7) that the share of the initial vorticity of the point vortex or vortex sheet found at longer distances increases with time. In the case of the diffusing vortex sheet its 'gravity centre' drifts away from the wall in the course of time. For a large number of computational elements Eq (2.5) can be solved by means of random displacements, see Cheer (1989), Chorin (1973), Sherman (1990). For the point vortices we have

$$\Delta x_i = \eta_1 \qquad \Delta y_i = \eta_2 \tag{2.8}$$

where η_1, η_2 are independent random variables of Gaussian distribution with zero mean and the standard deviation of $\sigma = \sqrt{2\nu t}$. The vortex sheet elements can also undergo random displacements normal to the wall

$$\Delta\xi_i = |\eta| \tag{2.9}$$

where η is also an independent random variable of Gaussian distribution with zero mean and the standard deviation of $\sigma = \sqrt{2\nu t}$. The symbol $|\cdot|$ denotes the absolute value. However, the number of vortex sheet elements generated at the surface is relatively small, compared to the number of discrete vortices in the flow and it seems reasonable not to attempt the stochastic way of modelling the diffusion of the vortex sheet. It is decided in the present method that all vortex sheet elements will be given equal displacements corresponding to the distance between the wall and the 'gravity centre' of the diffusing vortex sheet

$$E = \int\limits_0^\infty y\frac{\omega}{\gamma}\, dy = \int\limits_0^\infty y\frac{2}{\sigma\sqrt{2\pi}} \exp\Big(-\frac{y^2}{2\sigma^2}\Big)\, dy = \sigma\sqrt{\frac{2}{\pi}} = \sqrt{\frac{4\nu t}{\pi}} \tag{2.10}$$

The third step is convection

$$\frac{\partial\omega}{\partial t} + \boldsymbol{v}\cdot\nabla\omega = 0 \tag{2.11}$$

This equation has the following solution

$$d\boldsymbol{r} = \boldsymbol{v}dt \tag{2.12}$$

All computational elements (vortex sheet elements and discrete vortices) undergo displacements with the local field velocities

$$\Delta\xi_j = v(\xi_j)\Delta t \qquad \Delta r_j = v(r_j)\Delta t \tag{2.13}$$

The convective velocity of a vortex sheet element can be found from the properties of the Cauchy-type integral in Eq (2.16). This velocity is tangent to the surface and is half the potential flow velocity at the surface in the coordinate system attached to the body

$$v(\xi_j) = -\frac{1}{2}\gamma(\xi_j)t(\xi_j) \tag{2.14}$$

The convective velocity of a discrete vortex is as follows

$$v(r_j) = \frac{1}{2\pi}\int\limits_L \nabla \ln \frac{1}{|r_j - r'|} \times k\gamma(r')\,ds' + \frac{1}{2\pi}\sum_{k\neq j} \nabla \ln \frac{1}{|r_j - r_k|} \times k\Gamma_k \tag{2.15}$$

where k is a unit vector perpendicular to the plane of the flow. In order to eliminate difficulties with point vortex singularities, the cut-off radius is introduced, see Chorin (1973). The circumferential velocity of a vortex with a cut-off radius is

$$v(r) = \begin{cases} \dfrac{\Gamma_j}{2\pi}\dfrac{1}{|r - r_j|} & |r - r_j| > \sigma \\ \dfrac{\Gamma_j}{2\pi}\dfrac{1}{\sigma} & |r - r_j| \leq \sigma \end{cases} \tag{2.16}$$

and remains constant within the circle of radius σ. The cut-off radius has a smoothing effect on the calculations, however it is a source of artificial viscosity. Its value is usually selected by the trial-and-error method.

Vortex sheet elements and point vortices have different properties. However, they are sometimes combined into one method of calculations, see for example Cheer (1989), where vortex sheet elements are used to simulate the boundary layer and outside the boundary layer they are replaced by point vortices. In the present method vortex sheet elements will be replaced after they are generated then diffused and convected once only. The point vortices will enter the flow at the gravity centres of the replaced vortex sheet elements and will carry their circulations.

The accuracy and convergence of viscous splitting with random walk were the subject of many papers, only to mention Baele and Majda (1981), Goodman (1987), Marchioro and Pulvirenti (1982). It was found that the rate of convergence of viscous splitting for the case of an unbounded domain (without generation) increases with the increasing number of computational elements and increasing Reynolds number. The accuracy of smoothing techniques including cut-off was investigated by Hald (1979), Hald and del Prete (1978), Baele and Majda (1982).

3. Aerodynamic force and the moment of force

A compact 2D/3D theory of force and moment of force on solid bodies, submerged in a viscous flow, executing a prescribed motion after an impulsive start from rest in the fluid remaining at rest at infinity, is presented in the paper of Wu (1981). Wu does not employ any simplifying assumptions and stands on the grounds of the Navier-Stokes equations, making use of the principle of conservation of momentum and moment of momentum in a large region bounded externally by a large circle. The final formulas express the force and its moment in terms of time-evolution of the first and second moments of vorticity. The applicability of the theory hinges on knowledge of the vorticity field development.

In a similar way the formulas can be derived in the case of the model with point vortices and vortex sheets, what was accomplished in Lampart (1993), (1995), Burka and Lampart (1996). The force and its moment about the gravity centre of the profile have the following form

$$\boldsymbol{F} = -\rho \frac{d\boldsymbol{\alpha}}{dt} \tag{3.1}$$

$$\boldsymbol{c} \times \boldsymbol{F} = \boldsymbol{r} \times \boldsymbol{F} + \boldsymbol{v}_0 \times \boldsymbol{F} = -\frac{\rho}{2}\frac{d\boldsymbol{\beta}}{dt} + \rho \boldsymbol{v}_0 \times \boldsymbol{\alpha} \tag{3.2}$$

where $\boldsymbol{\alpha}$ is the first moment of vorticity

$$\boldsymbol{\alpha} = \int_L \boldsymbol{r} \times \boldsymbol{k}\gamma \, ds + \sum_k \boldsymbol{r}_k \times \boldsymbol{k}\Gamma_k \tag{3.3}$$

and $\boldsymbol{\beta}$ is the second moment of vorticity with respect to the gravity centre of the profile

$$\boldsymbol{\beta}_c = \int_L \boldsymbol{c} \times (\boldsymbol{c} \times \boldsymbol{k})\gamma \, ds + \sum_k \boldsymbol{c}_k \times (\boldsymbol{c}_k \times \boldsymbol{k})\Gamma_k \tag{3.4}$$

where the vectors $\boldsymbol{r}$, $\boldsymbol{c}$ are defined in the coordinate systems fixed and moving with the profile, respectively.

4. Results of calculations

4.1. Flow past a circular cylinder

Numerical calculations were performed in a wide range of Reynolds numbers $\mathrm{Re} = 10^3 \div 10^5$, where $\mathrm{Re} = 2Rv_\infty/\nu$. The results presented in the paper concern $\mathrm{Re} = 1000$.

It was assumed that the time step was $\Delta t = 0.2$, cylinder radius $R = 1$, number of vortex sheet elements $n = 50$. The cut-off radius for the discrete vortices was $\sigma = 0.06$. Whenever the vortices crossed the boundary they were put back into the flow region. Random walk as a rule breaks the symmetry of the flow, however in order to speed up the process of alternate vortex shedding an asymmetric disturbance was introduced. Between $t = 4 \div 7$ ($20 \div 35$ time step) all vortices shed from the upper surface of the cylinder were slightly moved by

$$\Delta x = 0.1\frac{0.35 - i}{35} \qquad i = 20, ..., 35 \qquad (i \text{ -- time step}) \tag{4.1}$$

As a result, for $t = 4$ the flow is symmetric, at $t = 10$ the first vortex structure (a cluster of discrete vortices shed from the upper surface) is formed. The shedding of subsequent vortex clusters is two times faster. Configurations of vortex clusters in selected instants, velocity vectors and streamline patterns in the same instants are shown in Fig.1. The obtained flow patterns resemble those of van Dyke (1982), the velocity vectors agree well with the flow visualisation carried out by Cantwell and Coles (1983) ($\mathrm{Re} = 140000$). The convective velocity of vortex clusters at distances $8 \div 16$ from the cylinder is 0.77. The length of a single stretch of the wake is $8.5 \div 8.6$, which also agrees well with the experiments; for a review of experimental records see Frączak (1993). The Strouhal number $\mathrm{St} = 2Rf/v_\infty$, where f is the frequency of vortex shedding, is 0.2 – the same value is observed experimentally. A separated flow past a cylinder was solved without prior knowledge of where the separation occurs. One can roughly estimate the locations of separation points if one applies an appropriate image magnification in order to find the stations at which the thickness of a layer near the cylinder taken by vortex elements rapidly increases or streamlines next to the cylinder upstream of the separation points start breaking away from it (in the coordinate system fixed to the cylinder).

Drag and lift characteristics are also presented in Fig.1. The calculated average drag is 1.54 and exceeds by about 50% that of the experiment. It is highly probable that if vortex sheet elements had been allowed to convect and

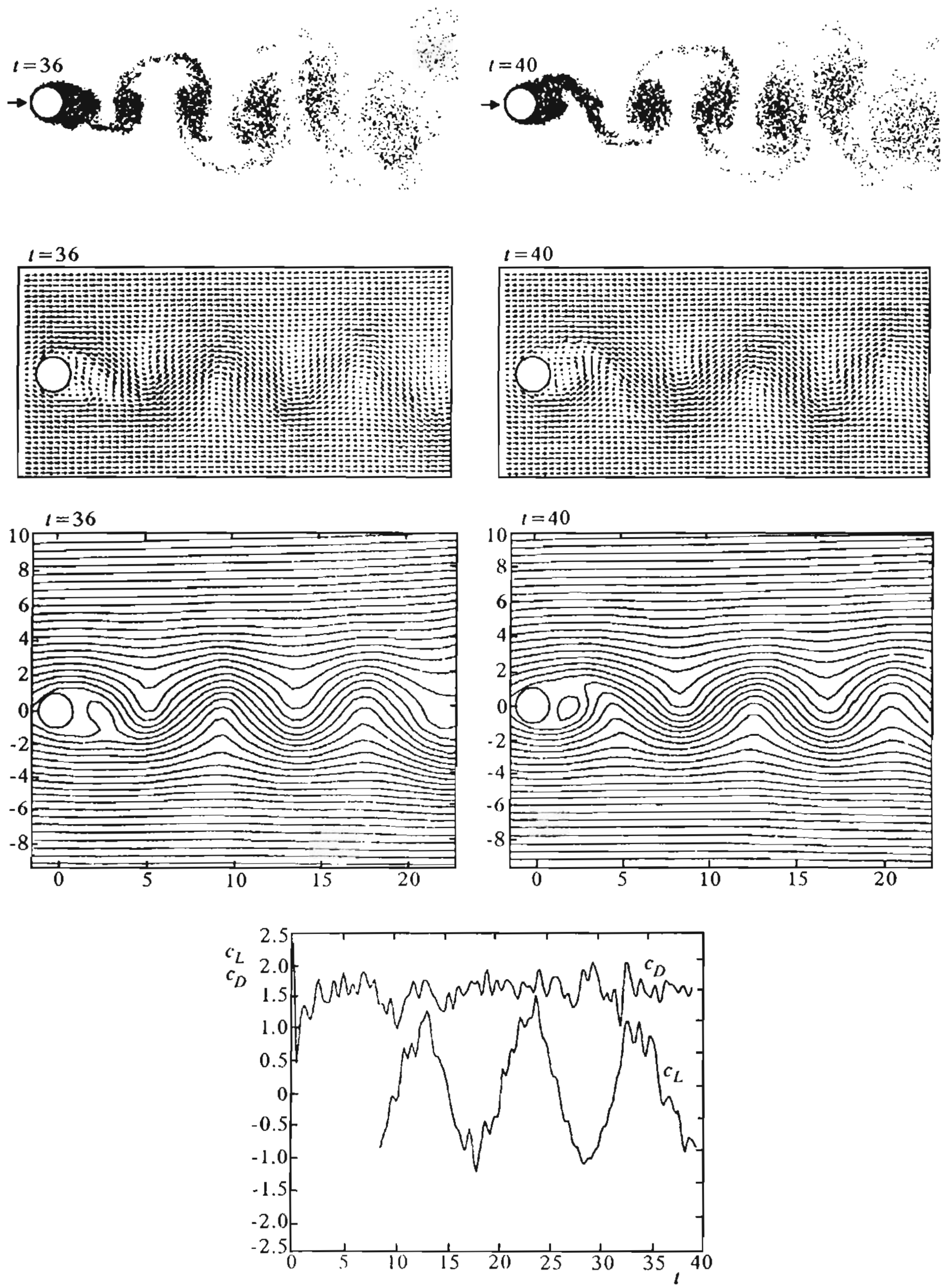

Fig. 1. Flow past a circular cylinder

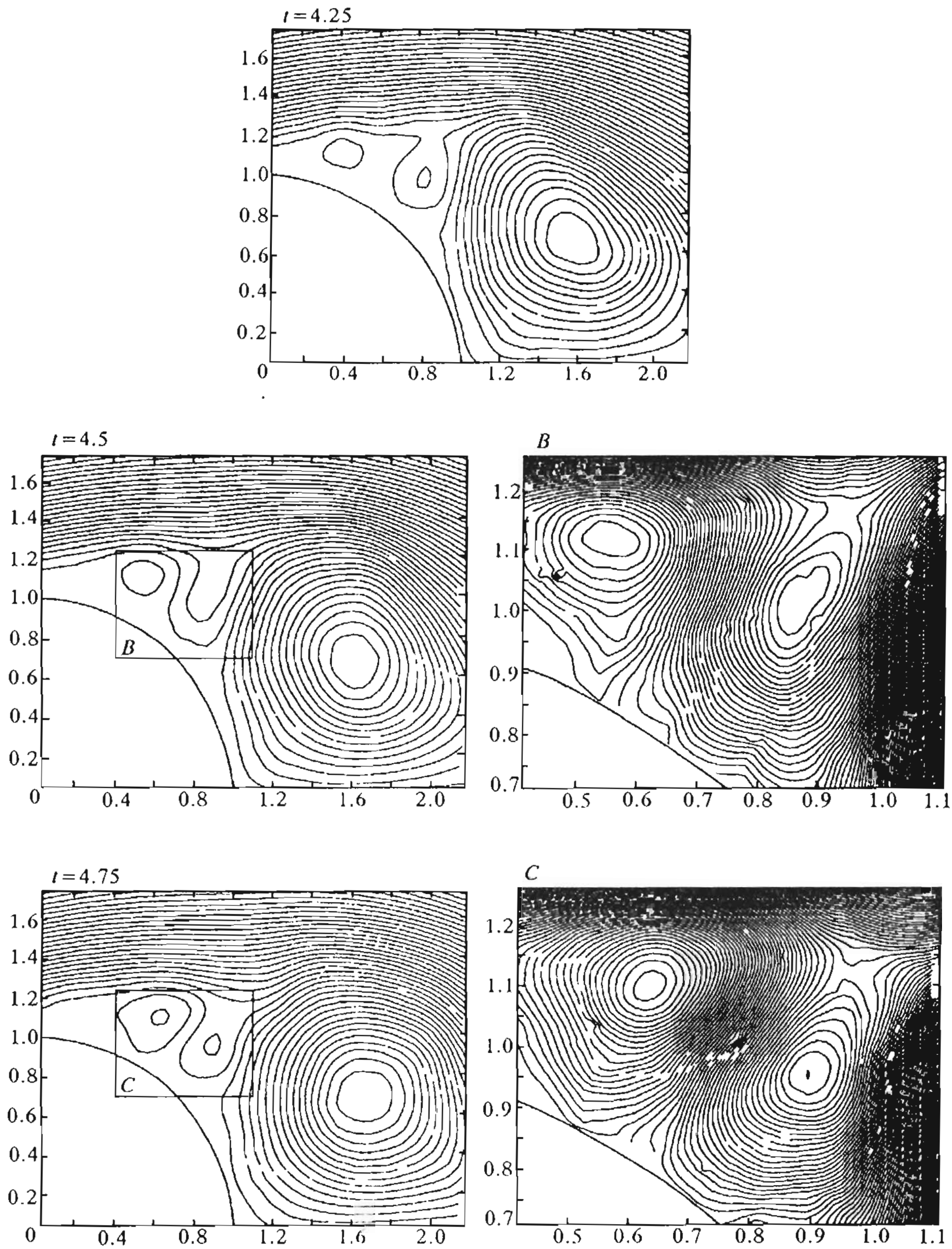

Fig. 2. Secondary vortex formation in a symmetric flow past a circular cylinder

diffuse longer within the boundary layer, as in Chorin (1978), Cheer (1989), instead of their early replacement with point vortices, or computational elements had been split whenever they induced too large velocities, as suggested by Szumbarski (1993), then the drag characteristics might have been closer to those of the experiment. Perhaps, increasing the number of vortex sheet elements at the surface by one order of magnitude, for example up to 500 and decreasing the time step can be beneficial. However, these ideas were relinquished in view of increasing computational cost.

A note is required to what extent the results of calculations depend on the Reynolds number, suppose that the range of Reynolds number is $10^3 \div 10^5$. In calculations the Reynolds number, or more so the kinematic viscosity, has an effect on the average length of random displacements. On the other hand the cut-off radius is a source of artificial viscosity, the amount of which is difficult to evaluate. It is also difficult to recognise which of these effects outweighs the other. In fact, the calculated flow patterns and aerodynamic characteristics undergo minor variations with the Reynolds number and the same can be learned about flow patterns, Strouhal number and aerodynamic characteristics from experimental data (van Dyke (1982), Frączak (1993)). Therefore, it is not surprising that calculated and experimental results referring to different Reynolds numbers are compared, provided they do not fall beyond the considered range. Perhaps, one important parameter that considerably changes over this range of Reynolds numbers is the length of the recirculation zone. In the experiment of Bloor (1964) it decreases from $3.5 \div 4.0R$ behind the cylinder for $\mathrm{Re} = 1000$ down to $1.0R$ for $\mathrm{Re} = 100000$. A somewhat decreasing tendency is also found in numerical calculations, however without quantitative agreement with the experiment. The calculated length of the recirculation zone changes from $1.5R$ for $\mathrm{Re} = 1000$ to $1.0R$ for $\mathrm{Re} = 100000$.

The process of formation of vortex structures behind the cylinder is accompanied by secondary effects. Tracing secondary vortices in periodic flow is very expensive in terms of CPU time. Therefore, secondary flow calculations were limited only to the case of symmetric flow for short times, which refers to the situation before the real flow impulsively set in motion loses its stability and turns periodic. The number of vortex sheet elements was increased to 100, time step decreased to $\Delta t = 0.05$, the Reynolds number was assumed $\mathrm{Re} = 5000$. The streamline pattern bearing resemblance to the so-called α phenomenon observed experimentally for $\mathrm{Re} = 800 \div 5000$, see Bouard and Coutanceau (1980), is depicted in Fig.2. The phenomenon occurs when a large main vortex is already formed and its stream separates from the rear surface of the cylinder, giving rise to a couple of counter rotating secondary vortices which fill the space between the primary separation and the back of the main vortex itself. For the most recent results of numerical calculations, using vortex methods, concerning impulsively started symmetrical flow past a circular cylinder one should refer to the paper of Koumoutsakos and Leonard (1995).

4.2. Flow past a NACA 23012 profile

The first group of results presents the flow past a NACA 23012 profile inclined at $45°$. It was assumed that $\mathrm{Re} = 10000$, $\Delta t = 0.2$, $n = 50$, $\sigma = 0.06$. Configurations of cloud vortices, velocity vectors and streamline patterns at selected instants in an impulsively started flow are presented in Fig.3. The obtained average drag can be estimated at 1.1, the average lift 0.75, the Strouhal number $\mathrm{St} = cf/v_\infty = 0.25$, where c – chord length.

The second group of results presents a NACA 23012 profile inclined at $30°$ for $\mathrm{Re} = 1000$. Configurations of cloud vortices, velocity vectors and streamline patterns at selected instants in an impulsively started flow are presented in Fig.4. The calculated average drag is 0.75, lift is 0.6, the Strouhal number $\mathrm{St} = 0.3$.

The authors do not know the experimental data concerning lift/drag characteristics for NACA profiles under separating conditions. However, one can have the flat plate measurements of Fage and Johanson (1927) as a reference. Let us recall that at $\mathrm{Re} = 150000$ for a flat plate inclined at $45°$ $c_D = 1.2$, $c_L = 1.2$, $\mathrm{St} = lf/v_\infty = 0.25$, whereas at $30°$ attack $c_D = 0.65$, $c_L = 1.1$, $\mathrm{St} = 0.3$. A NACA 23012 profile which has certainly better aerodynamic characteristics should have lower drag and higher lift than the flat plate even for the conditions under discussion. However, this is clearly not the case of our calculations. Once again, bearing in mind the calculations of a circular cylinder, the method presented in the paper predicts accurately the Strouhal number, or frequency of vortex shedding and flow patterns but overestimates the drag and underestimates the lift on the profile.

5. Conclusions

2D separated flow has been solved with the help of viscous splitting of the Navier-Stokes equations, including convection (of point vortices and vortex sheet elements), diffusion (by means of the random walk technique) and generation (by means of Martensen's concept of vortex sheet extended by Lewis). The method does not assume prior knowledge of where the flow separates. The

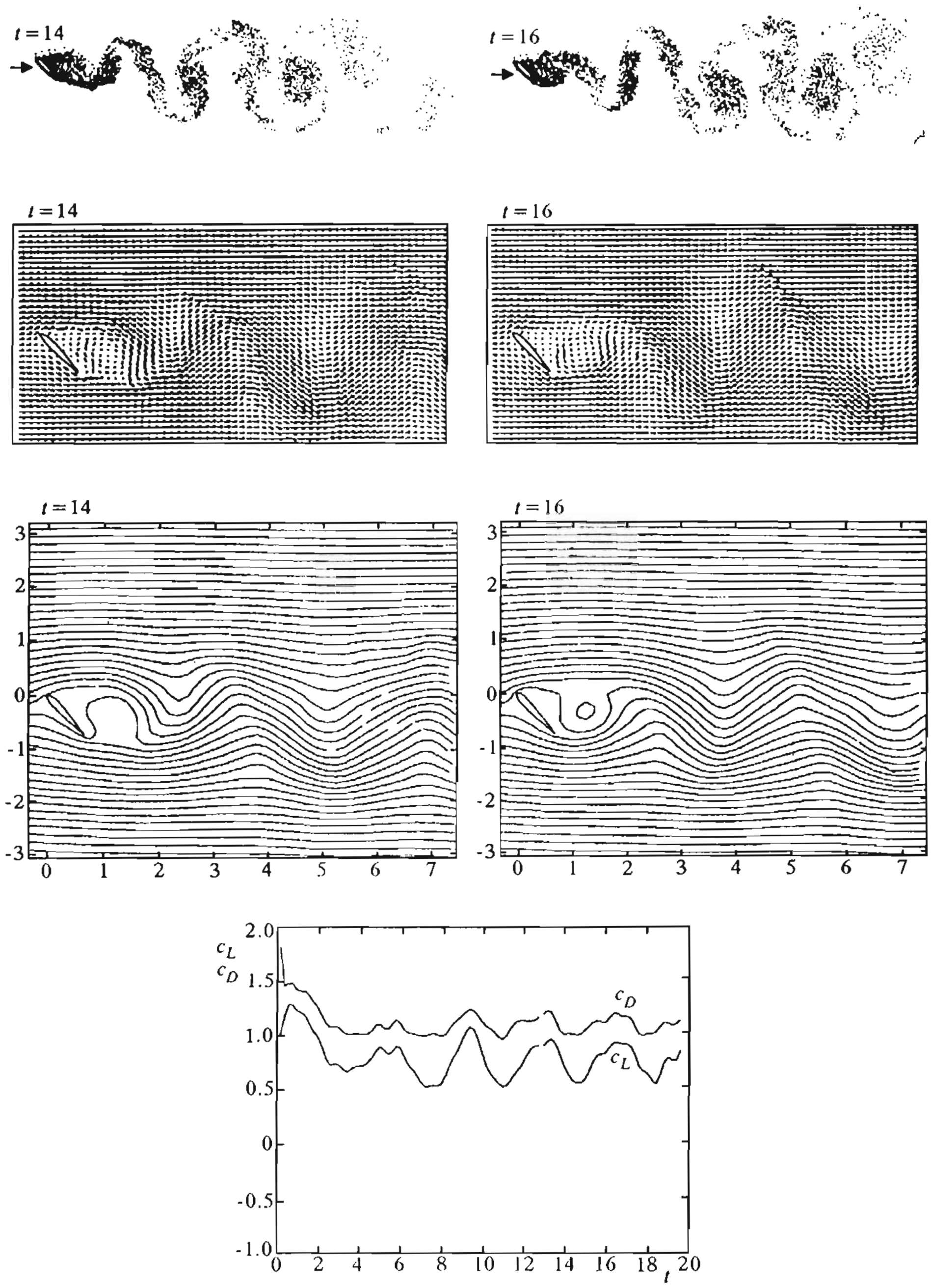

Fig. 3. Flow past a NACA 23012 profile at 45° angle of attack

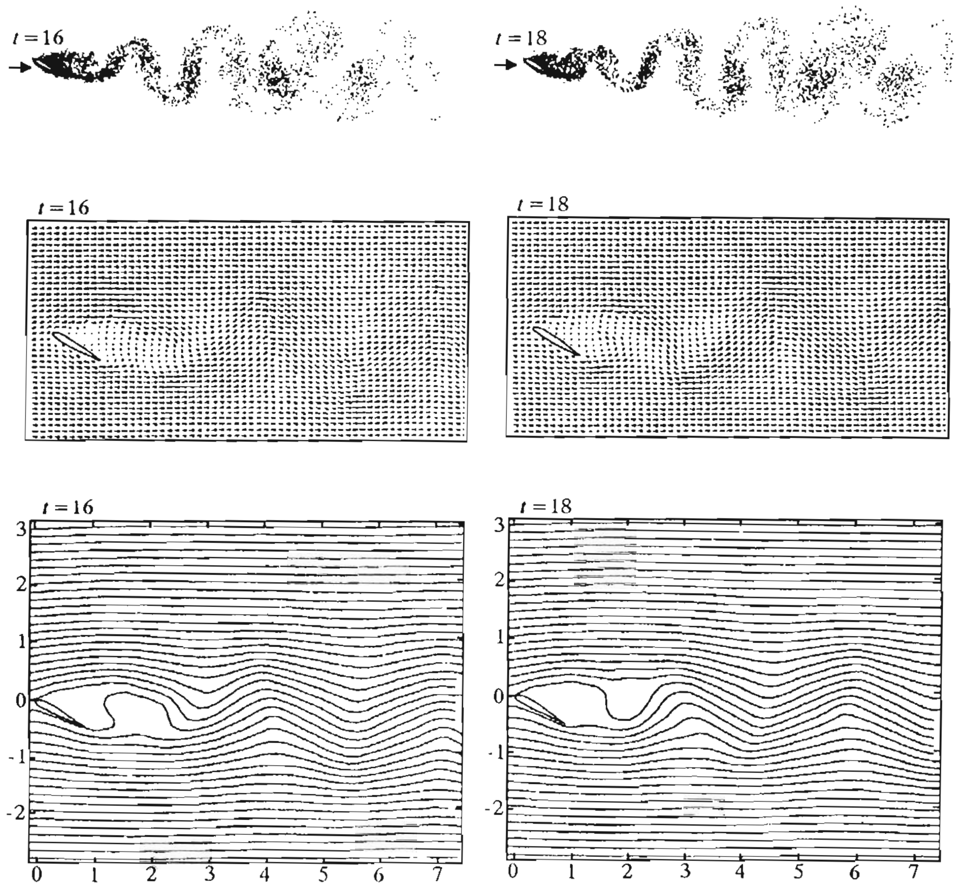

Fig. 4. Flow past a NACA 23012 profile at 30° angle of attack

aerodynamic force and its moment are calculated from the balance of momentum and moment of momentum in the flow domain. The method yields correct flow pictures, velocity vectors, streamline patterns. The frequency of vortex shedding, the Strouhal number, geometrical parameters of the wake remain in agreement with the experiments. However, the method overpredicts the drag and underpredicts the lift on the profile. More thought and computational cost is required before calculated drag/lift characteristics get anywhere near those of the experiment.

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Modelowanie przepływów z oderwaniem metodą chmury wirowej

Streszczenie

Praca zawiera opis metody obliczeniowej dla numerycznego rozwiązywania dwuwymiarowych przepływów nieściśliwych laminarnych o dużych liczbach Reynoldsa z oderwaniem z dużymi strefami recyrkulacji – obszarami formowania się struktur wirowych i śladu spływowego. Praca dotyczy oderwania z gładkich powierzchni. Metoda obliczeniowa opiera się na sekwencyjnym rozwiązaniu równania transportu wirowości. W każdym kroku czasowym rozwiązywane są oddzielnie trzy zagadnienia – konwekcja (konwekcja wirów punktowych i elementów warstwy wirowej), dyfuzja (za pomocą metody stochastycznej) i generacja wirowości (za pomocą rozszerzonej przez Lewisa koncepcji Martensena warstwy wirowej na powierzchni profilu). Metoda nie wymaga stosowania kryterium dla wyznaczenia położeń punktów oderwania. Przybliżone położenia punktów oderwania stanowią element rozwiązania, a otrzymuje się je za pomocą komputerowej wizualizacji przepływu jako miejsca, w których elementy obliczeniowe – cząstki płynu opuszczają sąsiedztwo profilu i zaczyna się strefa recyrkulacji. Powyższa metoda rozwiązywania przepływu została nazwana metodą chmury wirowej. Obciążenia aerodynamiczne profilu (siła reakcji i jej moment) wyznaczane są na podstawie bilansu pędu i momentu pędu obszaru, w którym odbywa się przepływ. Jako przykłady obliczeniowe zaprezentowano opływ cylindra kołowego i profilu NACA pracującego w oderwaniu. Podjęto także problem zjawisk wtórnych w przepływie z oderwaniem.

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