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# CHAOTIC VIBRATION AFFECTED BY INITIAL CONDITIONS IN A PARAMETRICALLY AND EXTERNALLY EXCITED SYSTEM WITH BACKLASH

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In the present paper, the influence of initial conditions on the occurrence of regular and chaotic vibrations of the system with backlash, described by a non–linear differential equation with a periodic coefficient, was investigated. For specified parameters, using the Lyapunov exponent, intervals of the initial conditions leading to chaotic motion were determined. The effect of external excitation amplitude on the character of motion was analysed. For the initial conditions determining regular and chaotic motion, Poincaré maps, bifurcation diagrams, phase trajectories and time histories were compared, respectively.

*Key words:* chaotic vibration, gear system

## 1. Introduction

In problems concerning mechanical vibrations, it can be isolated a wide class of systems with parameters periodically changing, which one simultaneously subjected to external excitation. These are; e.g., gear transmissions, rotating shafts with different principal intersection moments of inertia and piston machines. Such vibrations one can meet in power transmission systems with belt transmissions, universal couplings and suspension of vehicles using variable radial rigidity of solid type and in many other mechanical systems Szabelski and Samodulski (1985).

Influence of the self-excitation in this class of systems, was examined by Szabelski and Warmiński (1995). As mechanical systems are exploited, constructional backlashes increase, which may lead not only to quantitative changes, but also to qualitative ones in vibrations description. The influence of backlash on chaotic vibrations occurrence in the system with one degree of freedom and external excitation was presented by Kleczka et al. (1990). In Dyk et al. (1994), Hongler and Streit (1988), Litak et al. (1995a), Sato et al. (1991), the influence of different parameters (e.g. interteeth backlash of single-stage gear transmission) on the possibility of chaotic vibrations occurrence was analysed.

The Lyapunov exponent is one of factors characterizing chaotic motion. It represents sensitiveness of the system to the influence of different factors affecting regular or chaotic motion. Chaotic vibrations may affect correct operation and durability of the mechanical system.

The present paper aims at investigating the influence of initial conditions on chaotic vibrations occurrence in mechanical system with backlash, being simultaneously excited parametrically and externally.

## 2. Mathematical model

In the present paper, a vibrations of the system are represented by a non-linear differential equation with a periodically changing coefficient and external excitation. In the considered model, non-linearity in the form of backlash has been assumed. The following differential equation of vibrations has been analysed

$$\frac{d^2x}{d\tau^2} + \frac{2\zeta}{\omega}\frac{dx}{d\tau} + \frac{k(\tau)}{\omega^2}g(x,\eta) = \frac{B}{\omega^2}\cos(\tau + \Theta) + \frac{B_0}{\omega^2} \tag{2.1}$$

where:

| | | |
|---|---|---|
| $\omega$ | – | frequency of external excitation |
| $B$ | – | amplitude of external excitation |
| $B_0$ | – | constant component of excitation |
| $\zeta$ | – | damping ratio |
| $\Theta$ | – | phase angle |
| $k(\tau)$ | – | periodic function |
| $\tau$ | – | dimensionless time, $\tau = \omega t$. |

Function $g(x,\eta)$ is defined as follows

$$g(x,\eta) = \begin{cases} x & x \geq 0 \\ 0 & -\eta < x < 0 \\ x+\eta & x \leq -\eta \end{cases} \tag{2.2}$$

where $\eta$ denotes the backlash.

Eq (2.1) represents, e.g., the single-stage gear transmission vibrations (cf Sato et al. (1991)). This model was derived basing on two non-homogeneous differential equations, describing motions of the system wheels at a periodic change of meshing rigidity and backlash existence also under other input and output moments. These equations were reduced to Eq (2.1) by variables changing. Eq (2.1) is a general one, it describes vibrations of parametric systems with backlash external excitations. Let us assume, that the periodic function $k(\tau)$ generates polyharmonic parametric excitation. In order to attain that we assume that the function $k(\tau)$ is expandable in the Fourier series and takes the following form

$$k(\tau) = \begin{cases} 0.6+\tau/(0.3\pi) & 0 \leq \tau \leq 0.12\pi \\ 1 & 0.12\pi \leq \tau \leq 1.08\pi \\ 4.6-\tau/(0.3\pi) & 1.08\pi \leq \tau \leq 1.2\pi \\ 0.6 & 1.2\pi \leq \tau \leq 2\pi \end{cases} \tag{2.3}$$

## 3. Analysis of the result obtained

Eq (2.1) has been solved numerically using the Runge-Kutta-Gill method of the fourth order. For assumed system parameters the values of Lyapunov exponents were calculated using the method by Wolf et al. (1988).

For the two values of initial conditions $x_0 = -12.0$, $v_0 = -0.5$, $x_0 = -12.0$, $v_0 = -3.5$ and the assumed parameters of the system $\omega = 1.5$, $n = 0.08$, $B_0 = 1$ $B = 8$, $\eta = 10$, $\Theta = 0$ the maximal Lyapunov exponents $\lambda_1$ apart from the one which is always equal to zero versus time specified by the number $N$ of excitation cycles is presented in Fig.1.

After about $N = 600$ forced cycles the system reaches a steady state. In Fig.2 the exponent $\lambda_1$ is show schematically for different initial values $x_0$, $v_0$, where $x_0 \in (-15.5, 0.5)$, $v_0 \in (-15.5, 0.5)$, and for the two values of external excitation amplitude $B = 4$ (Fig.2a) and $B = 8$ (Fig.2b).

The distribution of $\lambda_1$ for regular and chaotic solutions has complicated structure. The structure is richer for larger value of parameter $B$. For

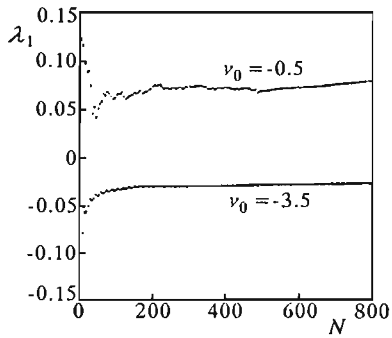


Fig. 1. Time dependence of the maximal Lyapunov exponents $\lambda_1$ ($N$ denotes the number of forced cycles) for the system parameters $\omega = 1.5$, $n = 0.08$, $B_0 = 1$, $B = 8$, $\eta = 10$, $\Theta = 0$ and different initial conditions: $x_0 = -12.0$, $v_0 = -0.5$ (the upper curve) and $x_0 = -12.0$, $v_0 = -3.5$ (the lower curve)

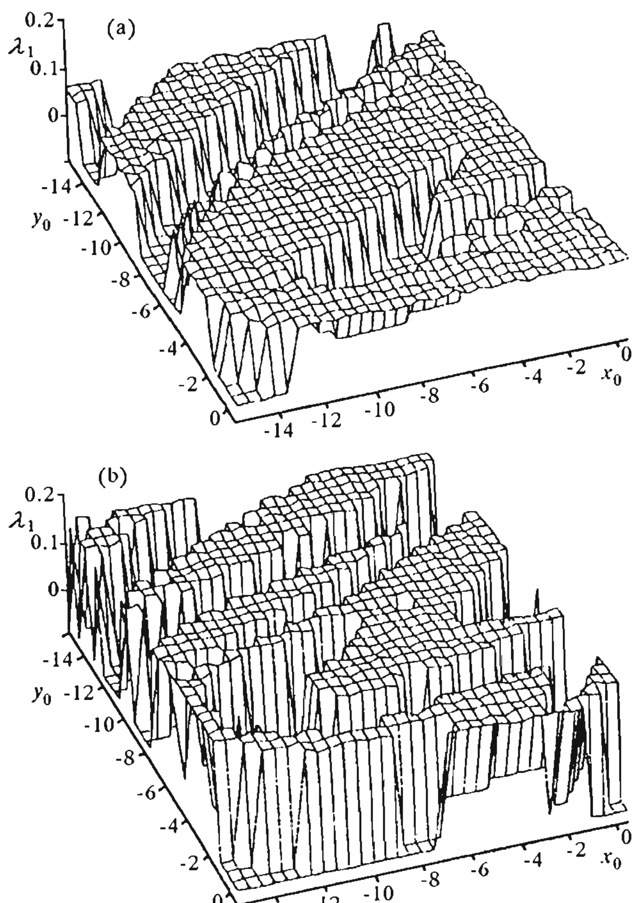


Fig. 2. Lyapunov exponent $\lambda_1$ as a function of the initial conditions for $B = 4$ (Fig.2a) and $B = 8$ (Fig.2b). Other parameters were taken as in Fig.1

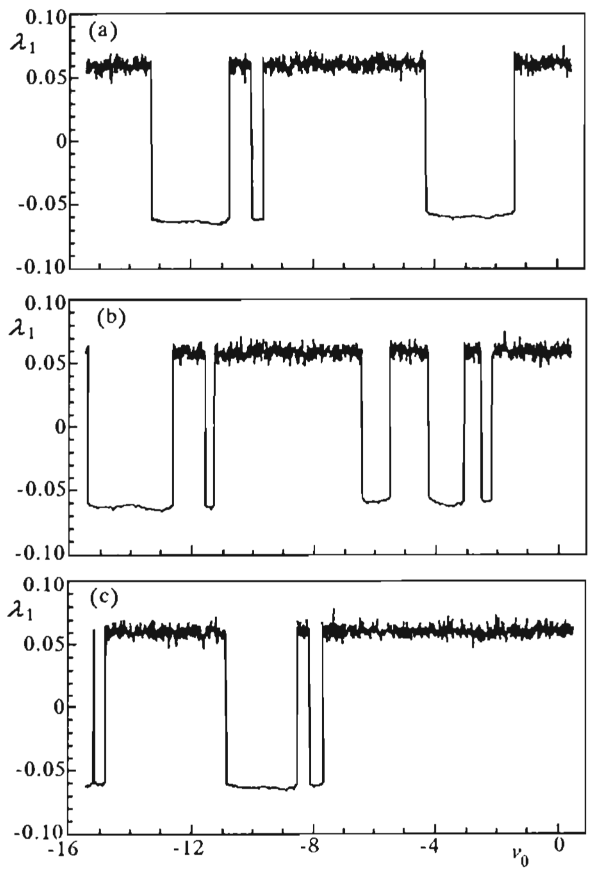


Fig. 3. Cross-sections of the plot 2a ($B = 4$) for $x_0 = -4, -8, -12$, respectively

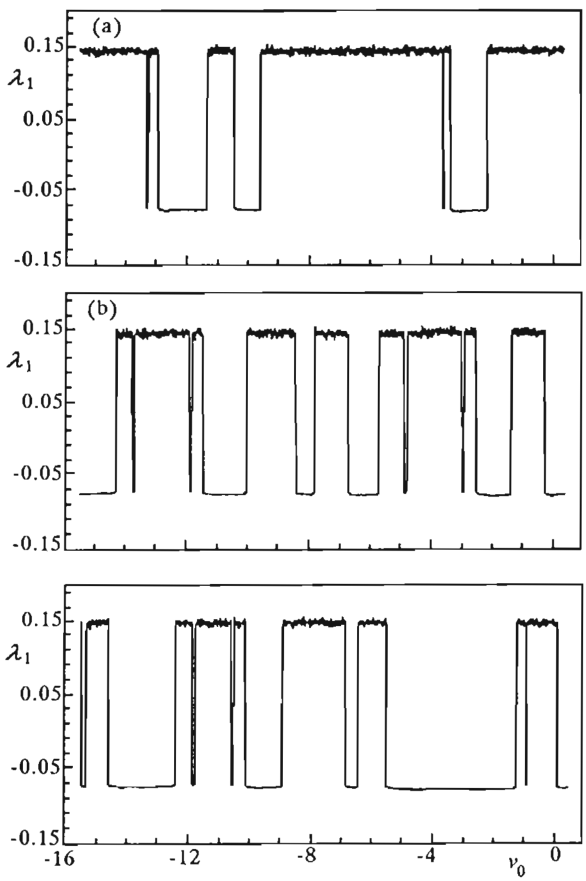


Fig. 4. Cross-sections of the plot 2b $(B = 8)$ for $x_0 = -4, -8, -12$, respectively

better illustration, the intersections of those figures by parallel planes, at $x_0 = -4, -8, -12$ (Fig.3) for $B = 4$ and (Fig.4) for $B = 8$, respectively, have been presented.

In Fig.3 and Fig.4 one can notice stronger oscillations of the Lyapunov exponent $\lambda_1$ versus the initial velocity when it is positive, i.e. in the cases of chaotic vibrations.

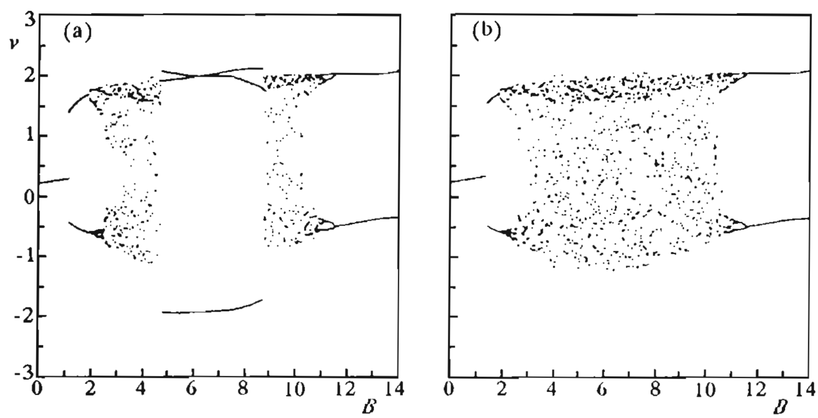


Fig. 5. Bifurcation diagrams for the same system parameters and initial conditions as in Fig.1. In Fig.5a $v_0 = -3.5$. In Fig.5b $v_0 = -0.5$

For the same values of $\eta$, $\omega$, $n$, $B_0$, $\Theta$ like in Fig.1, Fig.2 and for two different initial conditions assumed, i.e., $x_0 = -12.0$, $v_0 = -0.5$ and $x_0 = -12.0$, $v_0 = -3.5$, (see Fig.1), the bifurcation diagrams are shown in Fig.5a. For the initial value $v_0 = -0.5$ we observe additionally the interval of regular motion, which does not occur for the velocity $v_0 = -3.5$.

Fig.6a,b show the Poincaré maps for the same, initial conditions $x_0 = -12.0, v_0 = -3.5$ (Fig.6a) and $x_0 = -12.0, v_0 = -0.5$ (Fig.6b). The assumed amplitude of external excitation equals $B = 8$. Fig.6a shows, on the Poincaré map, only three points, proving that the motion is regular. Fig.6b presents chaotic attractor. Fig.6b,c,d exhibit the fractal structure of chaotic attractor, what clearly testifies to chaotic motion of the transmission. Fig.6c is the enlargement of the subdomain $A_1$ marked in Fig.6b, whereas Fig.6d presents the subdomain $A_2$ marked in Fig.6c. Taking Fig.6b,c,d into account one can draw the conclusion that the self-similarity coefficient of strange attractor amounts approximately 10.

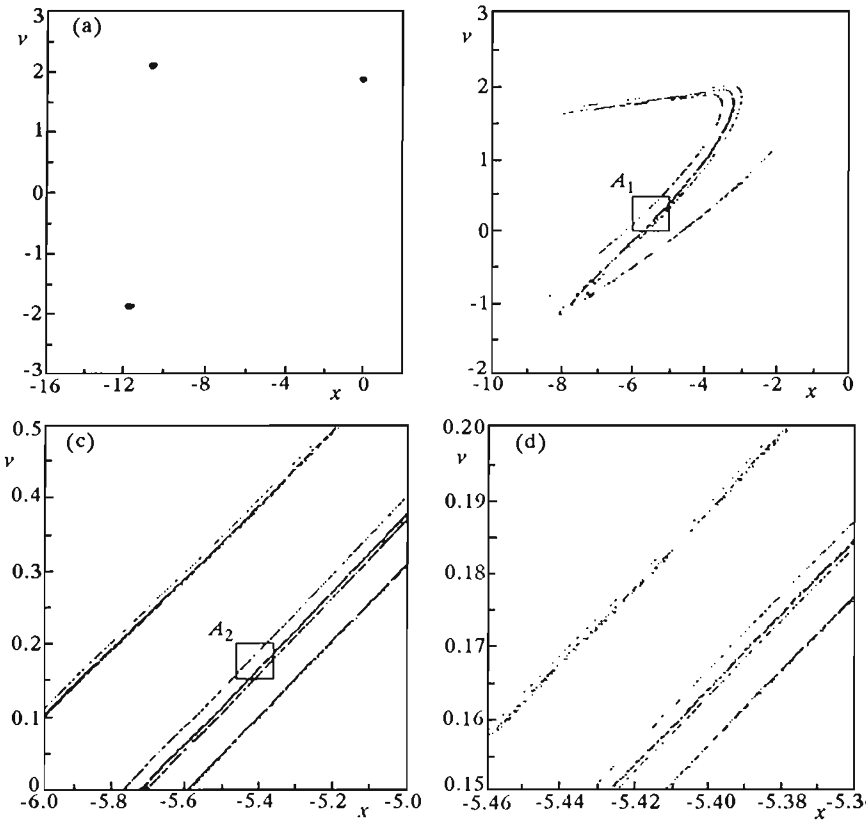


Fig. 6. Poincaré sections for the same initial conditions and system parameters (but $B = 8$), as in Fig.5. Fig.6a shows the regular motion, while Fig.6b the chaotic one

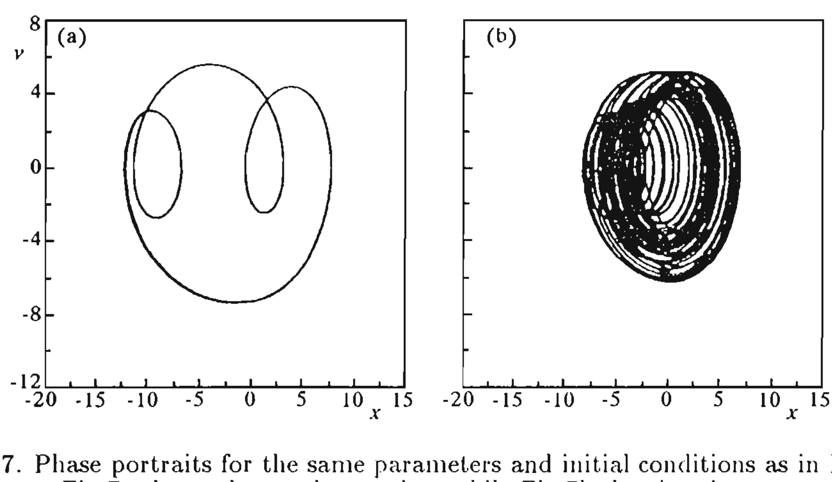


Fig. 7. Phase portraits for the same parameters and initial conditions as in Fig.6. Fig.7a shows the regular motion, while Fig.7b the chaotic one

For the same parameters of the system, like in Fig.7, the phase portraits are presented. For the initial conditions $x_0 = -12.0$, $v_0 = -3.5$ (Fig.7a), the closed trajectory of regular motion with threefold period has been obtained, which agrees with the results presented in Fig.6a. For the initial conditions $x_0 = -12.0$, $v_0 = -0.5$ (Fig.7b) the trajectory of chaotic motion has been obtained.

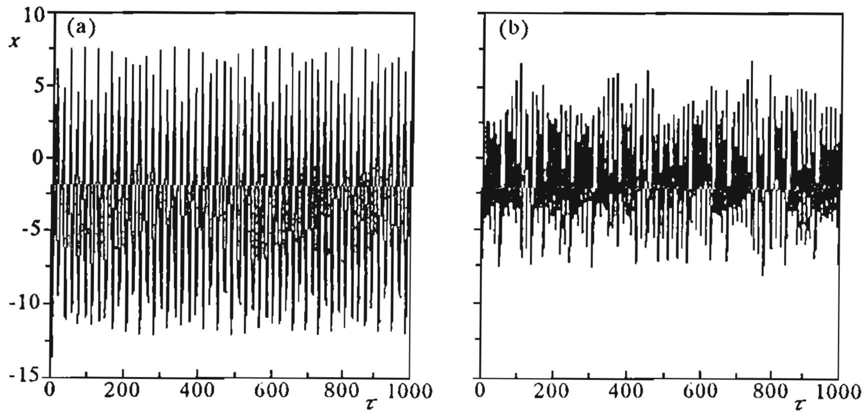


Fig. 8. Time histories for the same system parameters and initial conditions as in Fig.6 and Fig.7. Fig.8a shows the regular motion, while Fig.8b the chaotic one

Time histories of those two motions are shown in Fig.8 plotted for the values of the system parameters and the initial conditions discussed earlier. Fig.8a illustrates the regular vibrations, conformable to the results presented in Fig.6a and Fig.7a, whereas Fig.8b shows the chaotic vibrations.

## 4. Remarks and conclusions

In the present paper, the results of numerical simulation of the system vibrations with backlash, under parametric, external excitation, were described. Applying the Lyapunov exponents to assumed system parameters, the initial conditions leading to chaotic solutions were determined. The initial conditions in the considered system appeared to the additional bifurcation parameters. For the assumed system parameters, occurrence of regular and chaotic motion was checked using Poincaré maps, phase portraits and time histories. The influence of external excitation and initial conditions on the character of vibrations was illustrated in the bifurcation diagram.

Though the analysis shows the essential influence of the initial value intervals leading to regular and chaotic motion, in the actual systems characterized by the noise occurrence (cf Kunert and Pfeiffer (1990); Litak et al. (1995b)), the structure of regular and chaotic states appearance (Fig.2) may change fundamentally.

Estimating the values of model parameters and the values of initial conditions at which chaos was detected, one should state unreality of chaotic vibrations occurrence in the case of a real gear system.

Agreement of the investigation results, concerning the occurrence of regular and chaotic motion, with application of different diagnostic methods should be emphasized. It was found that the Lyapunov exponent, according to the initial conditions, was described by discontinuous function. At the discontinuity point, vibrations of the system are characterized by a sudden transition from a regular motion to the chaotic one and inversely. Such a property of the system may have important practical meaning.

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## Wpływ warunków początkowych na występowanie drgań chaotycznych układu z luzem, pobudzanego parametrycznie i zewnętrznie

Streszczenie

W pracy zbadano wpływ warunków początkowych na występowanie drgań regularnych i chaotycznych układu nieliniowego z luzem, opisanego równaniem różniczkowym z okresowo zmiennym współczynnikiem. Dla określonych parametrów, stosując wykładniki Lapunowa, wyznaczono przedziały warunków początkowych prowadzących do ruchu chaotycznego. Przeprowadzono analizę wpływu amplitudy wymuszenia zewnętrznego na charakter ruchu przekładni. Dla warunków początkowych, determinujących regularny i chaotyczny ruch układu, porównano mapy Poincarégo, diagramy bifurkacyjne, trajektorie fazowe i przebiegi czasowe.

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