MECHANIKA TEORETYCZNA
I STOSOWANA
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INFLUENCE OF INERTIAL FORCES ON THE MAGNETIC FLUID FLOW IN A CLEARANCE BETWEEN CURVILINEAR SURFACES OF REVOLUTION

JERZY SAWICKI

Department of Mechanics
Technical-Agricultural Academy

This paper treats the steady laminar flow of the magnetic fluid through a clearance between curvilinear surfaces of revolution having a common axis of symetry. The boundary layer equations are expressed in terms of the intrinsic curvilinear coordinate system x, θ, y. The method of perturbation is used to solve the boundary layer equations. As a result, the formulae for parameters of the flow i.e. the velocity components $\bar{V}_x$, $\bar{V}_\theta$, $\bar{V}_y$ and the pressure $\bar{p}$ are obtained.

Nomenclature

$\boldsymbol{H}$	–	vector of magnetic field intensity, $\boldsymbol{H} = [H_x, H_\theta, H_y]$
$2h(x)$	–	slot thickness
$\boldsymbol{M}$	–	vektor of magnetization, $\boldsymbol{M} = [M_x, M_\theta, M_y]$
POS, POK	–	nondimensional parameters coresponding to the effect of centrifugal inertial forces
PWS, PWK	–	nondimensional parameters coresponding to the effect of longitudinal inertial forces
p	–	pressure
p_i	–	inlet pressure (relative to clearance)
p_o	–	outlet pressure (relative to clearance)
$R(x)$	–	radius of a central surface in the clearance

Re	–	Reynolds number
R_F	–	magnetic pressure number
$\boldsymbol{V}$	–	velocity vector, $V = [V_x, V_\theta, V_y]$
μ	–	fluid dynamic viscosity
μ_0	–	magnetic constant
ρ	–	fluid density
ω_1, ω_2	–	angular velocities of the top and lower surfaces of revolution, respecxtively.

1. Introduction

Laminar flow of a magnetic, viscous fluid in clearances between rotating surfaces of revolution in the magnetic field presence are becoming more and more important in technology (cf Ezekiel (1975), Moscowitz (1975)).

Within the last twenty years it can be seen from the literature on the subject, that the problems of dynamic phenomena in a magnetic fluid are a point of interest. The research, started by Neuringen (1964) and Rosenzweig (1971) as – among others – Szliomis (1974) has causes an appearance of a new branch of knowlege, called "ferrohydrodynamics".

The magnetic fluids, among so called "ferrofluids" as a colloidal suspension of the dissipative agent (mainly hydrocarbons, fluorocarbons, water, esters, organo-metalic compounds) and the disperse phase (mainly – iron oxide Fe_3O_4) create after extra possibilities for solving of different constructional and maintenance tasks on a scale a friction, wearing and lubrication as well as with a theory of slide bearings, clutches and sealing glands.

The aim of this study is to analyze the inertia phenomena that is an effect of longitudinal and centrifugal inertial forces caused by the flow of ferromagnetic fluid in the clearance between curvilinear surfaces of revolution.

2. Theoretical analysis

The studied motion of the ferromagnetic liquid in the clearance as shown in Fig.1 is laminar, stationary and isothermal. The flow takes place in the external, steady and heterogenous magnetic field $(H_x, 0, 0)$. However, it has been assumed, that the ferromagnetic fluid is electrically non-conducting and

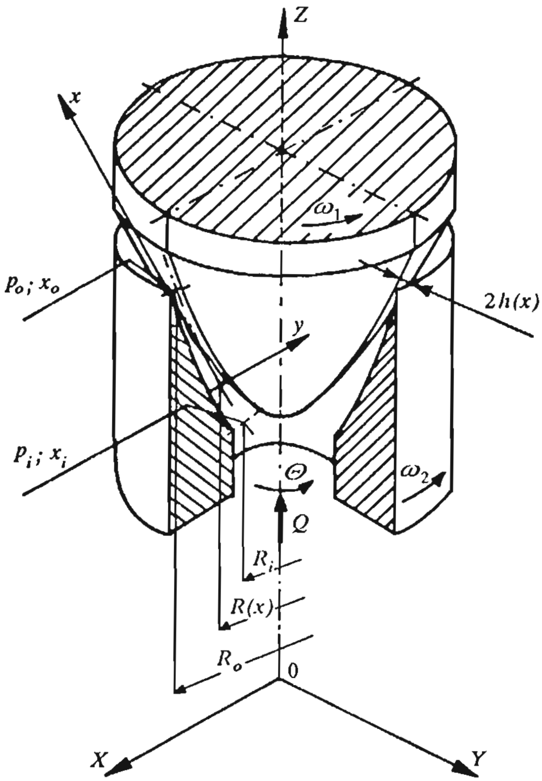

Fig. 1. Flow area of magnetic fluid

the vectors of magnetic field strenght and magnetization, respectively, are parallel to each other.

According to the principles of mass and momentum conservation, the equations of motion are (cf Wierzcholski and Janiszewski (1980))

$$\nabla V = 0 \tag{2.1}$$

$$\rho(V\nabla)V = -\nabla p + \mu\Delta V + \mu_0(M\nabla)H \tag{2.2}$$

Eqs (2.1) and (2.2) for its "completing" demand for some additional equations describing the magnetic field i.e. the equations of magnetostatic

$$\nabla \times H = 0 \tag{2.3}$$

$$\nabla B = 0 \tag{2.4}$$

$$B = \mu_0(H + M) \tag{2.5}$$

The equations for motion are formulated by Walicki (1977), with the assumption $h \ll R(x)$, describing a general case of the flow in the curvilinear coordinates system x, θ, y. After calculations necessary to make for the flows in thin layers of fluids, we will get

$$\frac{1}{R}\frac{\partial(RV_x)}{\partial x} + \frac{\partial V_y}{\partial y} = 0 \tag{2.6}$$

$$\rho\Big(V_x\frac{\partial V_x}{\partial x} + V_y\frac{\partial V_x}{\partial y} - V_\theta^2\frac{R'}{R}\Big) = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 V_x}{\partial y^2} + \mu_0 M_x\frac{\partial H_x}{\partial x} \tag{2.7}$$

$$\rho\Big(V_x\frac{\partial V_y}{\partial x} + V_y\frac{\partial V_y}{\partial y} + V_x V_\theta\frac{R'}{R}\Big) = \mu\frac{\partial^2 V_\theta}{\partial y^2} \tag{2.8}$$

$$0 = \frac{\partial p}{\partial y} \tag{2.9}$$

$$B_x = \mu_0(H_x + M_x) \tag{2.10}$$

$$M_x = M_0 \qquad\qquad H_x = \frac{R_0}{R}H_0 \tag{2.11}$$

Thus, from Eq (2.9) we have

$$p = p(x) \tag{2.12}$$

The boundary conditions for velocity components are

$$\begin{array}{lll} V_x = 0 & \text{for} & y = \mp h \\ V_y = 0 & \text{for} & y = \mp h \\ V_\theta = \omega_1 R(x) & \text{for} & y = h \\ V_\theta = \omega_2 R(x) & \text{for} & y = -h \end{array} \tag{2.13}$$

Also, at the inlet and outlet of the clearance the following boundary condition for a pressure are

$$\begin{array}{lll} p = p_i & \text{for} & x = x_i \\ p = p_o & \text{for} & x = x_0 \end{array} \tag{2.14}$$

3. Solution to the equations of motion

Introducing the following dimensionalcss quantities

$$\bar{x} = \frac{x}{R_0} \qquad\qquad \bar{R} = \frac{R}{R_0} \qquad\qquad \bar{y} = \frac{y}{H_0}$$

$$\bar{V}_x = \frac{V_x}{V_0} \qquad \bar{V}_\theta = \frac{V_\theta}{V_0} \qquad \bar{V}_y = \frac{V_y}{V_0}\frac{R_0}{h_0} \tag{3.1}$$
$$\bar{p} = \frac{ph_0}{\mu V_0}\frac{h_0}{R_0}$$

we can present the equations of motion (2.6) ÷ (2.11) in the form

$$\frac{1}{R}\frac{\partial(\bar{R}\bar{V}_x)}{\partial x} + \frac{\partial \bar{V}_y}{\partial y} = 0 \tag{3.2}$$

$$\lambda\Big(\bar{V}_x\frac{\partial \bar{V}_x}{\partial x} + \bar{V}_y\frac{\partial \bar{V}_x}{\partial y} - \bar{V}_\theta^2\frac{\bar{R}'}{\bar{R}}\Big) = -\frac{\partial \bar{p}}{\partial x} + \frac{\partial^2 \bar{V}_x}{\partial y^2} + R_F\bar{M}_x\frac{\partial \bar{H}_x}{\partial x} \tag{3.3}$$

$$\lambda\Big(\bar{V}_x\frac{\partial \bar{V}_\theta}{\partial x} + \bar{V}_y\frac{\partial \bar{V}_\theta}{\partial y} + \bar{V}_x\bar{V}_\theta\frac{\bar{R}'}{\bar{R}}\Big) = \frac{\partial^2 \bar{V}_\theta}{\partial y^2} \tag{3.4}$$

$$0 = \frac{\partial \bar{p}}{\partial y} \tag{3.5}$$

where

$$R_F = \frac{\mu_0 M_0 H_0 h_0^2}{\mu V_0 R_0} \qquad \bar{M}_x = \frac{M_x}{M_0}$$
$$\bar{H}_x = \frac{H_x}{H_0} \qquad \lambda = \mathrm{Re}\frac{h_0}{R_0}$$

The quantities marked by the subscript "zero" are average values within a discussed flow domain. λ – the modyfied Reynolds number which satisfies the condition

$$\lambda < 1 \tag{3.6}$$

From Eqs (3.2) ÷ (3.5) it follows, that for the motion of the ferromagnetic fluid – if condition (3.6) is satisfied – λ is a small parameter in Eqs (3.3) and (3.4). Thus, the solution can be sought to in the form of power series with respect to λ

$$\bar{V}_x = \sum_{i=0}^{\infty}\lambda^i\bar{V}_x^i \qquad \bar{V}_\theta = \sum_{i=0}^{\infty}\lambda^i\bar{V}_\theta^i$$
$$\bar{V}_y = \sum_{i=0}^{\infty}\lambda^i\bar{V}_y^i \qquad \bar{p} = \sum_{i=0}^{\infty}\lambda^i\bar{p}^i \tag{3.7}$$

Introducing the series (3.7) into Eqs (3.2) ÷ (3.5), after necessary transformations terms with regard to the same power of λ, if we restrict ourselves to the linear approximation and return to the previous, dimensional form, we get the equations

$$\frac{1}{R}\frac{\partial(RV_x^0)}{\partial x}+\frac{\partial V_y^0}{\partial y}=0 \tag{3.8}$$

$$0=-\frac{\partial p^0}{\partial x}+\mu\frac{\partial^2 V_x^0}{\partial y^2}+\mu_0 M_x\frac{\partial H_x}{\partial x} \tag{3.9}$$

$$0=\mu\frac{\partial^2 V_\theta^0}{\partial y^2} \tag{3.10}$$

$$0=\frac{\partial p^0}{\partial y} \tag{3.11}$$

$$\frac{1}{R}\frac{\partial(RV_x^1)}{\partial x}+\frac{\partial V_y^1}{\partial y}=0 \tag{3.12}$$

$$\rho\Big(V_x^0\frac{\partial V_x^0}{\partial x}+V_y^0\frac{\partial V_x^0}{\partial y}-V_\theta^0\frac{R'}{R}\Big)=-\frac{\partial p^1}{\partial x}+\mu\frac{\partial^2 V_x^1}{\partial y^2}+\mu_0 M_x\frac{\partial H_x}{\partial x} \tag{3.13}$$

$$\rho\Big(V_x^0\frac{\partial V_y^0}{\partial x}+V_y^0\frac{\partial V_y^0}{\partial y}+V_x^0V_\theta^0\frac{R'}{R}\Big)=\mu\frac{\partial^2 V_\theta^1}{\partial y^2} \tag{3.14}$$

$$0=\frac{\partial p^1}{\partial y} \tag{3.15}$$

The boundary conditions – in accordance with Eqs (2.13) and (2.14) – have the form

— for $y=+h$

$$\begin{array}{lll} V_x^0=0 & V_y^0=0 & V_\theta^0=\omega_1 R(x) \\ V_x^1=0 & V_y^1=0 & V_\theta^1=0 \end{array} \tag{3.16a}$$

— for $y=-h$

$$\begin{array}{lll} V_x^0=0 & V_y^0=0 & V_\theta^0=\omega_2 R(x) \\ V_x^1=0 & V_y^1=0 & V_\theta^1=0 \end{array} \tag{3.16b}$$

— for $x=x_i$

$$p^0=p_i \qquad p^1=0 \tag{3.16c}$$

— for $x=x_0$

$$p^0=p_o \qquad p^1=0 \tag{3.16d}$$

Integrating of Eqs (3.8) ÷ (3.15) and making the boundary conditions (3.16) are in force we have

$$V_x^0 = \frac{1}{2\mu Rh^3}\frac{p_i - p_o - (B_i - B - 0)}{A_i - A_0}(y^2 - h^2) \tag{3.17}$$

$$V_y^0 = \frac{h'}{2\mu Rh^4}\frac{p_i - p_o - (B_i - B - 0)}{A_i - A_0}(h^2 y - y^3) \tag{3.18}$$

$$V_\theta^0 = \frac{R}{2}\Big[(\omega_1 - \omega_2)\frac{y}{h} + (\omega_1 + \omega_2)\Big] \tag{3.19}$$

$$p^0 = B(x) + \frac{[A(x) - A_0](p_i - B_i) - [A(x) - A_i](p_o - B_0)}{A_i - A_0} \tag{3.20}$$

$$\begin{aligned} V_x^1 &= \frac{\rho C^2 (Rh)'}{840\mu^3 R^3 h^7}(35h^2y^4 - 7y^6 + 5h^6 - 33h^4y^2) + \\ &- \frac{\rho RR'}{240\mu h^2}[(\omega_1 - \omega_2)(5y^4 - 6h^2y^2 + h^4) + 20(\omega_1^2 - \omega_2^2)(y^3h - h^3y)] \end{aligned} \tag{3.21}$$

$$\begin{aligned} V_y^1 &= \frac{\rho C^2}{840\mu^3 h^7}\Big[\Big(\frac{(Rh)'}{R^2h^7}\Big)'(y^7 - 7h^2y^5 - 5h^6y + 11h^4y^3) + \\ &+ \frac{(Rh)'h'}{R^2h^7}(44h^3y^3 - 14hy^5 - 30h^5y)\Big] + \frac{\rho}{240\mu R}\Big\{\Big(\frac{R^2R'}{h^2}\Big)' \cdot \\ &\cdot [(\omega_1 - \omega_2)^2(y^5 - 2h^2y^3 + h^4y) + 5(\omega_1^2 - \omega_2^2)(hy^4 - 2h^3y^2 + h^5)] \\ &+ \frac{R^2R'}{h^2}h'[4(\omega_1 - \omega_2)^2(h^3y - hy^3) + 5(\omega_1^2 - \omega_2^2)(y^4 - 6h^2y^2 + 5h^4)]\Big\} \end{aligned} \tag{3.22}$$

$$\begin{aligned} V_\theta^1 &= \frac{\rho CR'}{120\mu^2 Rh^4}[(\omega_1 - \omega_2)(3y^5 - 10h^2y^3 + 7h^4y) + \\ &+ (\omega_1 + \omega_2)(5hy^4 - 30h^3y^2 + 25h^5)] \end{aligned} \tag{3.23}$$

$$p^1 = D(x) - \frac{[A(x) - A_0]D_i - [A(x) - A_i]D_0}{A_i - A_0} \tag{3.24}$$

here

$$R' = \frac{dR}{dx} \qquad h' = \frac{dh}{dx} \qquad C = \frac{p_i - B_i - (p_o - B_0)}{A_i - A_0}$$

$$A(x) = \int \frac{dx}{R(x)h^3} \qquad A_i = A(x_i) \qquad A_0 = A(x_0)$$

$$B(x)=\frac{\mu_0M_0H_0R_0}{R}\qquad\qquad B_i=B(x_i)\qquad\qquad B_0=B(x_0)$$

$$D(x)=\int\Big\{\frac{6\rho C^2(Rh)'}{35\mu^2R^3h^3}+\frac{\rho RR'}{20}[(\omega_1-\omega_2)^2+5(\omega_1+\omega_2)^2]\Big\}\,dx$$

$$D_i=D(x_i)\qquad\qquad D_0=D(x_0)$$

The complete solution to ferromagnetic fluid flow problem inside a clearance between (in general) curvilinear surfaces consists of the sum of partial solutions.

4. Motion of the ferromagnetic fluid between rotating conical surfaces

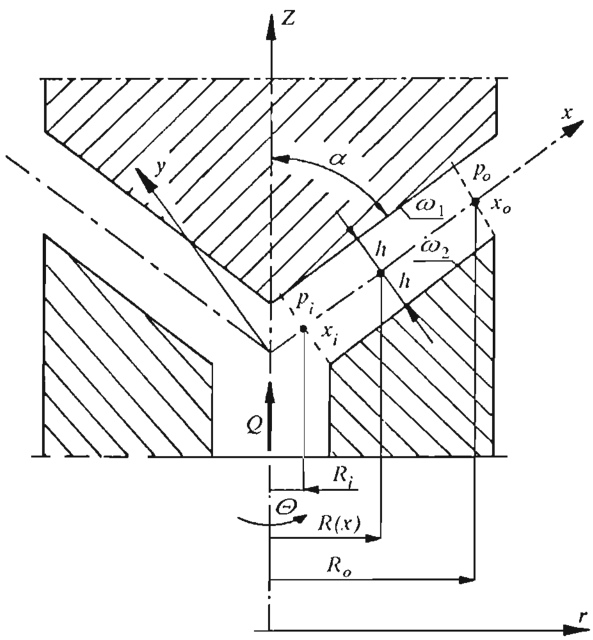

Fig. 2. Conical clearance geometry

After introducing the functions used for describing geometry the flow area (Fig.2) into Eqs (3.16) ÷ (3.24)

$$\begin{aligned} R &= x\sin\alpha & R_i &= x_i\sin\alpha \\ R_0 &= x_0\sin\alpha & R' &= \sin\alpha \end{aligned} \tag{4.1}$$

and introducing the dimensionless quantities

$$\bar{y} = \frac{y}{h} \qquad \bar{R} = \frac{R}{R_0} = \bar{x} \qquad \bar{R}' = 1$$

$$\bar{V}_x = \frac{V_x}{V^0_{x_{max}}} \qquad \bar{V}_y = \frac{V_y}{V^0_{x_{max}}}\frac{x_0\sin\alpha}{h} \qquad \bar{V}_\theta = \frac{2V_\theta}{x\omega_1 x_0\sin\alpha}$$

here

$$V^0_{x_{max}} = -\frac{p_o h^2(\bar{p}_i - 1)}{2x\mu x_0(a_i - a_0)\sin\alpha}$$

we get the following dimensionless formulas representing the velocity and pressure field in the clearance

— "zero" approximation

$$\bar{V}^0_x = \Big(1 - \frac{\bar{B}_i - \bar{B}_0}{\bar{p}_i - 1}\Big)(1 - \bar{y}^2) \tag{4.2}$$

$$\bar{V}^0_y = 0 \tag{4.3}$$

$$\bar{V}^0_\theta = [(1-K)\bar{y} + (1+K)] \tag{4.4}$$

— linear approximation (sum of partial solutions V^0 and V^1)

$$\begin{aligned} \bar{V}_x &= \bar{V}^0_x - \frac{1}{420}PWSBa\frac{1}{x^2}(5 - 7\bar{y}^6 + 35\bar{y}^4 - 33\bar{y}^2) + \\ &+ \frac{1}{120}POSAp\bar{x}^2[(1-K)^2(5\bar{y}^4 - 6\bar{y}^2 + 1) + 20(1-K^2)(\bar{y}^3 - \bar{y})] \end{aligned} \tag{4.5}$$

$$\begin{aligned} \bar{V}_y &= \frac{1}{240}PWSBa\frac{1}{x^3}(\bar{y}^7 - 7\bar{y}^5 + 11\bar{y}^3 - 5\bar{y}) + \\ &+ \frac{1}{60}POSAp\bar{x}[(1-K)^2(\bar{y}^5 - 2\bar{y}^3 + \bar{y}) + 5(1-K^2)(\bar{y}^4 - 2\bar{y}^2 + 1)] \end{aligned} \tag{4.6}$$

$$\begin{aligned} \bar{V}_\theta &= [(1-K)\bar{y} + (1+K)] + \frac{1}{60}PWS(a_i - a_0)[\bar{p}_i - 1 - (\bar{B}_i - \bar{B}_0)] \cdot \\ &\cdot \frac{1}{\bar{x}^2}[(1-K)(3\bar{y}^5 - 10\bar{y}^3 + 7\bar{y}) + (1+K)(5\bar{y}^4 - 30\bar{y}^2 + 25)] \end{aligned} \tag{4.7}$$

$$\bar{p} = \bar{B}(\bar{x}) + \bar{D}(\bar{x}) + \frac{[a(\bar{x}) - a_0](\bar{p}_i - \bar{B}_i - \bar{D}_i) - [a(\bar{x}) - a_i](1 - \bar{B}_0 - \bar{D}_0)}{a_i - a_0} \tag{4.8}$$

where

$$Ap = \frac{a_i - a_0}{\bar{p}_i - 1} \qquad Ba = \frac{a_i - a_0}{\bar{p}_i - 1}[\bar{p}_i - 1 - (\bar{B}_i - \bar{B}_0)]^2$$

$$a(\bar{x}) = \ln \bar{x} \qquad a_i = a(\bar{x}_i) \qquad a_0 = a(\bar{x}_0)$$

$$\bar{B}(\bar{x}) = R_F \frac{1}{\bar{x}} \qquad \bar{B}_i = \bar{B}(\bar{x}_i) \qquad \bar{B}_0 = \bar{B}(\bar{x}_0)$$

$$R_F = \frac{\mu_0 M_0 H_0}{p_o} \qquad \bar{D}(\bar{x}) = PWSfw\frac{1}{\bar{x}^2} + POSfo\bar{x}^2$$

$$\bar{D}_i = \bar{D}(\bar{x}_i) \qquad \bar{D}_0 = \bar{D}(\bar{x}_0) \qquad K = \frac{\omega_2}{\omega_1}$$

$$fw = -\frac{3}{35}[\bar{p}_i - 1 - (\bar{B}_i - \bar{B}_0)]^2 \qquad fo = \frac{1}{40}[(1 - K)^2 + 5(1 + K)^2]$$

$$PWS = \frac{\rho p_o h^4}{\mu^2 x_0^2 \sin^2\alpha(a_i - a_0)^2} \qquad POS = \frac{\rho\omega_1^2 x_0^2 \sin^2\alpha}{p_o}$$

The above formulas have been illustrated in Fig.3 ÷ Fig.9 while the influence of longitudinal inertial forces on the longitudinal velocity component profile are ilustrated in Table 1.

Table 1

$\bar{y}$	$\bar{x} = 0.3$		$\bar{x} = 0.9$	
	$PWS = 0$, $POS = 0$	$PWS = 0.023$, $POS = 0$	$PWS = 0$, $POS = 0$	$PWS = 0.023$, $POS = 0$
−1.0	0	0	0	0
−0.8	0.2250	0.2202	0.2250	0.2245
−0.6	0.4000	0.3965	0.4000	0.3956
−0.4	0.5250	0.5258	0.5250	0.5251
−0.2	0.6000	0.6049	0.6000	0.6005
−0.0	0.6250	0.6316	0.6250	0.6257
0.2	0.6000	0.6049	0.6000	0.6005
0.4	0.5250	0.5258	0.5250	0.5251
0.6	0.4000	0.3965	0.4000	0.3956
0.8	0.2250	0.2202	0.2250	0.2245
1.0	0	0	0	0

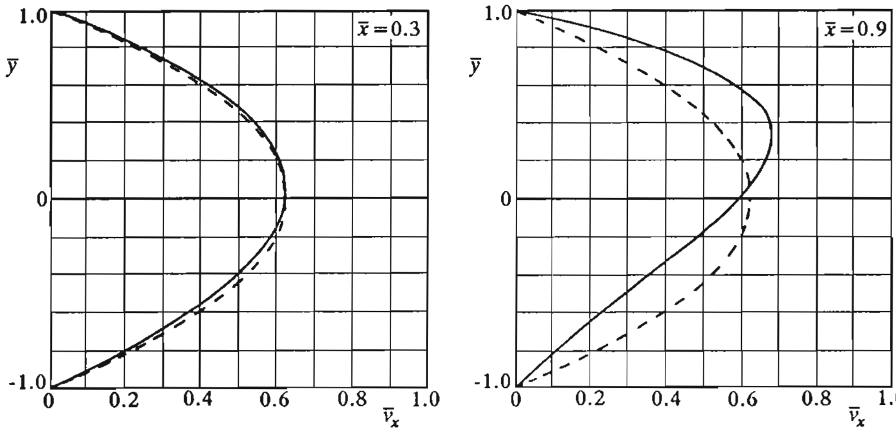

Fig. 3. Effect of centrifugal inertial forces on the velocity profile $\bar{V}_x$; $PWS = 0$, $POS = 10$, $K = 0$, $R_F = 0.5$; – – – – $PWS = POS = 0$

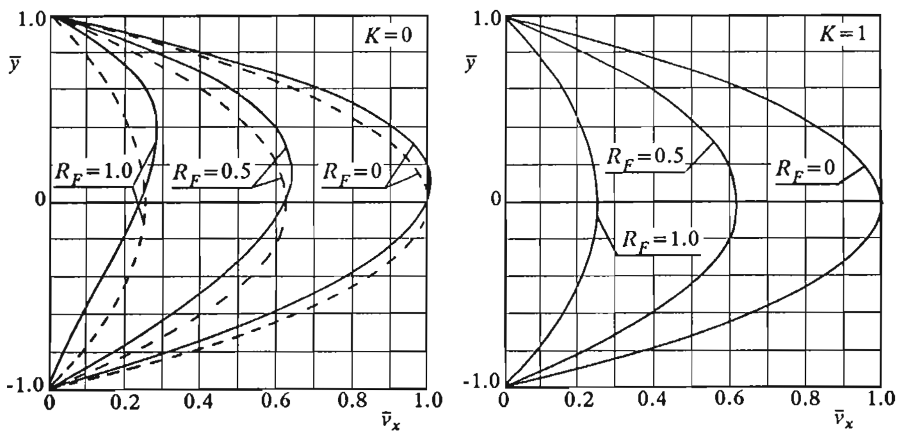

Fig. 4. Effect of a magnetic pressure number R_F on the velocity profile $\bar{V}_x$; $PWS = 0.023$, $POS = 10$, $\bar{x} = 0.6$; – – – – $PWS = POS = 0$

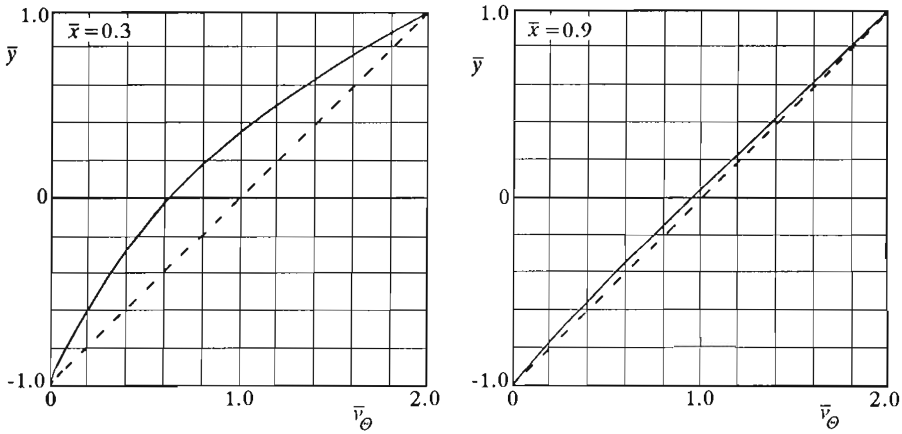

Fig. 5. Effect of longitudinal, inertial forces on the velocity $\bar{V}_\theta$ component profile; $PWS = 0.023$, $POS = 0$, $K = 0$, $R_F = 0.5$; – – – – $PWS = POS = 0$

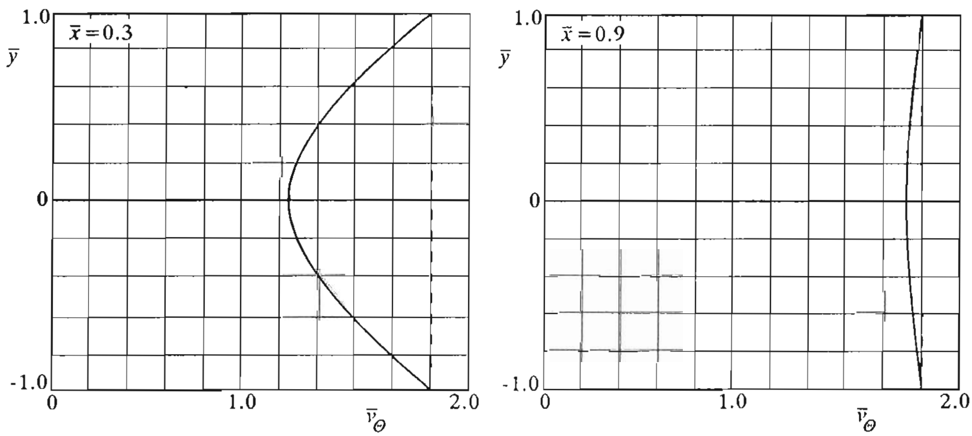

Fig. 6. Effect of longitudinal, inertial forces on the velocity $\bar{V}_\theta$ component profile; $PWS = 0.023$, $POS = 0$, $K = 1$, $R_F = 0.5$; – – – – $PWS = POS = 0$

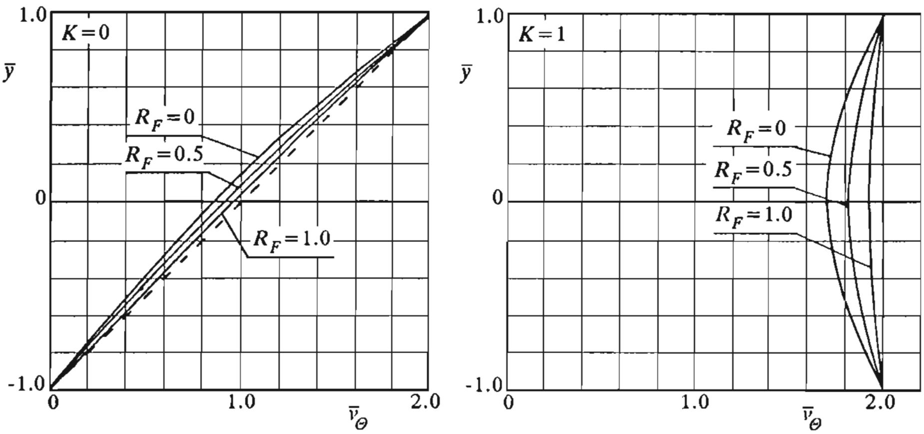

Fig. 7. Effect of a magnetic pressure number R_F on the velocity profile $\bar{V}_\theta$; $PWS = 0.023$, $POS = 10$, $\bar{x} = 0.6$; – – – – $PWS = POS = 0$

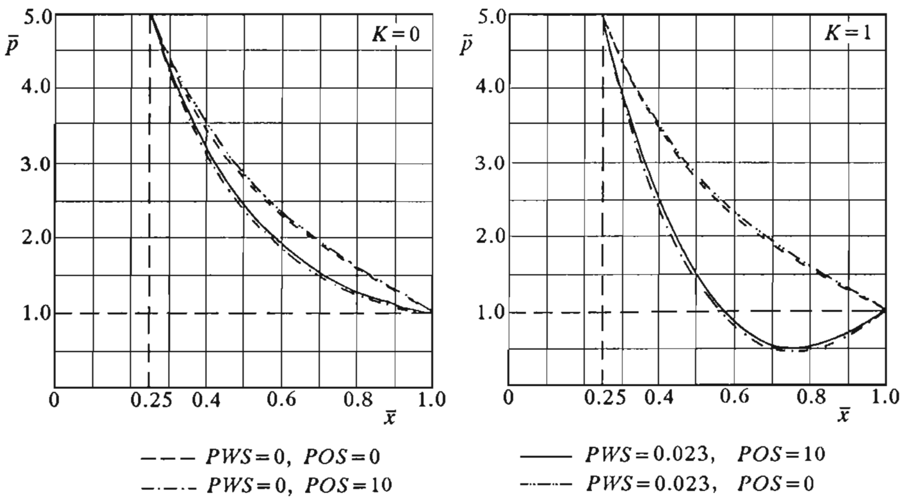

Fig. 8. Effect of the inertial forces on the pressure $\bar{p}$ profile; $R_F = 0.5$

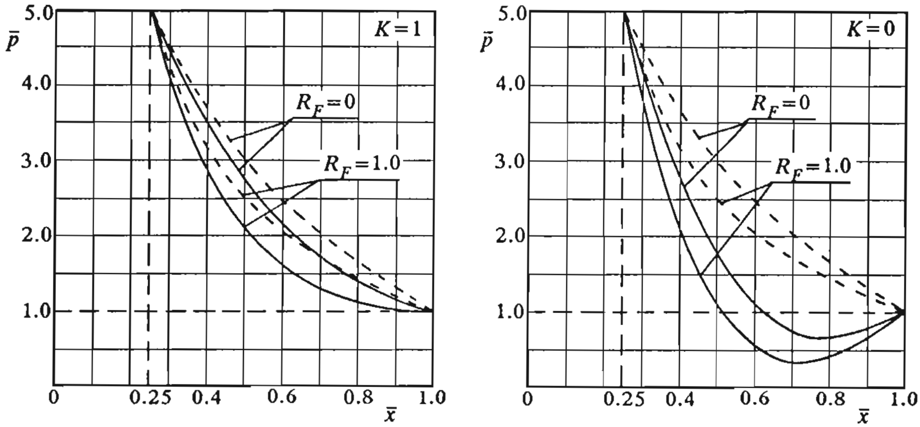

Fig. 9. Effect of a magnetic pressure number R_F on the pressure $\bar{p}$ profile; $PWS = 0.023$, $POS = 10$; – – – – $PWS = POS = 0$

5. Motion of the ferromagnetic fluid between rotating, spherical surfaces

The parameters useful to describe the geometry of a flow area can be written as follows (Fig.10)

$$R = R_k \sin\varphi \qquad R_i = R_k \sin\varphi_i \qquad R_0 = R_k \sin(\varphi_0) \tag{5.1}$$

$$\varphi = \frac{x}{R_k} \qquad R' = \cos\varphi$$

After introducing Eqs (5.1) into the solutions (3.17) ÷ (3.24) in trrms of the following dimensionless quantities

$$\bar{y} = \frac{y}{h} \qquad \bar{R} = \frac{R}{R_k}\sin\varphi$$

$$\bar{R}' = \cos\varphi \qquad \bar{V}_x = \frac{V_x}{V^0_{x_{max}}}$$

$$\bar{V}_y = \frac{V_y}{V^0_{x_{max}}}\frac{R_k\sin\varphi_0}{h} \qquad \bar{V}_\theta = \frac{2V_\theta}{R_k\omega_1\sin\varphi_0\sin\varphi}$$

where

$$V^0_{x_{max}} = -\frac{p_o h^2(\bar{p}_i - 1)}{2\mu x_0(a_i - a_0)\sin\varphi_0\sin\varphi}$$

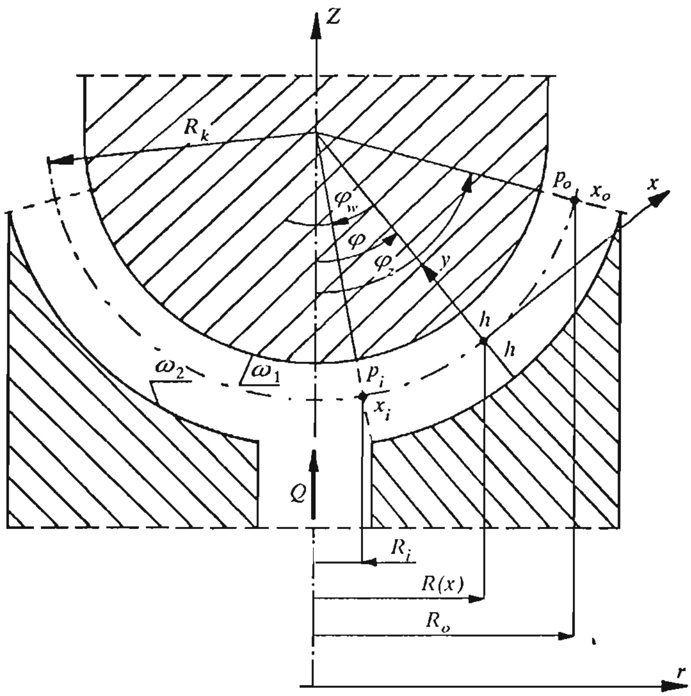

Fig. 10. Spherical clearance geometry

we obtain the formulas representing the motion of the ferromagnetic fluid inside the clearance between the rotating spherical surfaces

— "zero" approximation

$$\bar{V}_x^0 = \Big(1 - \frac{\bar{B}_i - \bar{B}_0}{\bar{p}_i - 1}\Big)(1 - \bar{y}^2) \tag{5.2}$$

$$\bar{V}_y^0 = 0 \tag{5.3}$$

$$\bar{V}_\theta^0 = [(1 - K)\bar{y} + (1 + K)] \tag{5.4}$$

— linear approximation (sum of partial solutions V^0 and V^1)

$$\begin{aligned} \bar{V}_x \;&=\; \bar{V}_x^0 - \frac{1}{420}PWKBa\frac{\cos\varphi}{\sin^2\varphi}(5 - 7\bar{y}^6 + 35\bar{y}^4 - 33\bar{y}^2) + \frac{1}{120}POK\cdot \\ &\quad Ap\sin^2\varphi\cos\varphi[(1-K)^2(5\bar{y}^4 - 6\bar{y}^2 + 1) + 20(1-K^2)(\bar{y}^3 - \bar{y})] \end{aligned} \tag{5.5}$$

$$\begin{aligned}
\bar{V}_y &= \frac{1}{240}PWKBa\frac{1+\cos^2\varphi}{3}(\bar{y}^7 - 7\bar{y}^5 + 11\bar{y}^3 - 5\bar{y}) + \\
&+ \frac{1}{60}POKAp(2\sin\varphi\cos^2\varphi - \sin^3\varphi)[(1-K)^2(\bar{y}^5 - 2\bar{y}^3 + \bar{y}) + \qquad (5.6)\\
&+ 5(1-K^2)(\bar{y}^4 - 2\bar{y}^2 + 1)]
\end{aligned}$$

$$\begin{aligned}
\bar{V}_\theta &= [(1-K)\bar{y} + (1+K)] + \frac{1}{60}PWK(a_i - a_0)[\bar{p}_i - 1 - (\bar{B}_i - \bar{B}_0)] \cdot \\
&\cdot \frac{\cos\varphi}{\sin^2\varphi}[(1-K)(3\bar{y}^5 - 10\bar{y}^3 + 7\bar{y}) + (1+K)(5\bar{y}^4 - 30\bar{y}^2 + 25)]
\end{aligned} \tag{5.7}$$

$$\begin{aligned}
\bar{p} &= \bar{B}(\varphi) + \bar{D}(\varphi) + \\
&+ \frac{[a(\varphi) - a_0](\bar{p}_i - \bar{B}_i - \bar{D}_i) - [a(\varphi) - a_i](1 - \bar{B}_0 - \bar{D}_0)}{a_i - a_0}
\end{aligned} \tag{5.8}$$

where

$$Ap = \frac{a_i - a_0}{\bar{p}_i - 1} \qquad Ba = \frac{a_i - a_0}{\bar{p}_i - 1}[\bar{p}_i - 1 - (\bar{B}_i - \bar{B}_0)]^2$$

$$a(\varphi) = \ln\left|\tan\frac{\varphi}{2}\right| \qquad a_i = a(\varphi_i) \qquad a_0 = a(\varphi_0)$$

$$\bar{B}(\varphi) = R_F\frac{1}{\sin\varphi} \qquad \bar{B}_i = \bar{B}(\varphi_i) \qquad \bar{B}_0 = \bar{B}(\varphi_0)$$

$$R_F = \frac{\mu_0 M_0 H_0}{p_o} \qquad \bar{D}(\varphi) = PWKfw\frac{1}{\sin\varphi^2} + POKfo\sin^2\varphi$$

$$\bar{D}_i = \bar{D}(\varphi_i) \qquad \bar{D}_0 = \bar{D}(\varphi_0) \qquad K = \frac{\omega_2}{\omega_1}$$

$$fw = -\frac{3}{35}[\bar{p}_i - 1 - (\bar{B}_i - \bar{B}_0)]^2 \qquad fo = \frac{1}{40}[(1-K)^2 + 5(1+K)^2]$$

$$PWK = \frac{\rho p_o h^4}{\mu^2 R_k^2 \sin^2\varphi_0 (a_i - a_0)^2} \qquad POK = \frac{\rho\omega_1^2 R_k^2 \sin^2\varphi_0}{p_o}$$

Since diagrams of the ferromagnetic fluid flow velocity distribution along the clearance between the rotating spherical surfaces show no substantial differences from the ferro-fluid flow velocity distribution in the clearance between the rotating conical surfaces, only the formulas for pressure profiles are illustrated in the presented diagrams (Fig.11 and Fig.12)

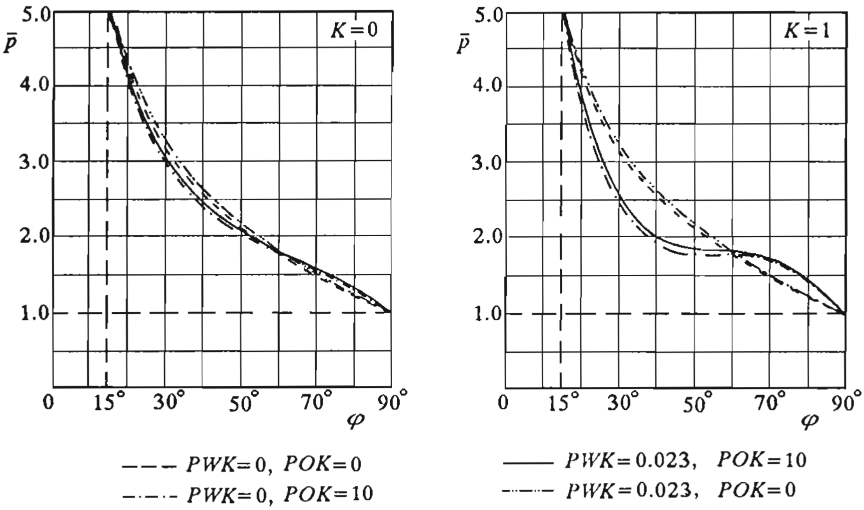

Fig. 11. Effect of inertial forces on the presure $\bar{p}$ profile; $R_F = 0.5$

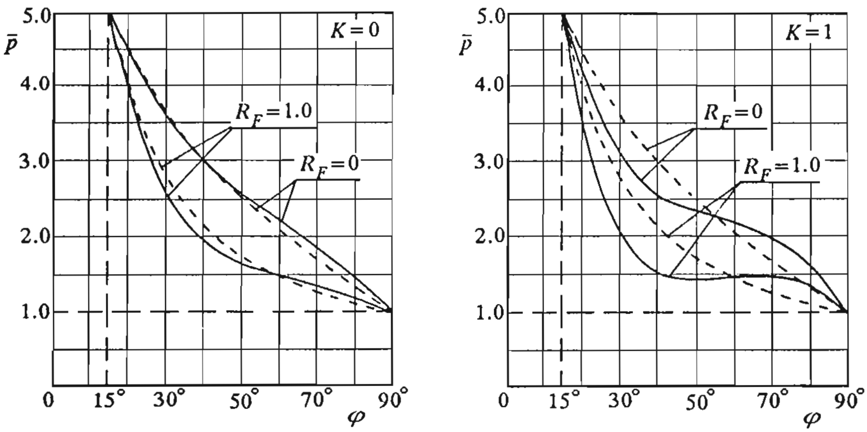

Fig. 12. Effect of a magnetic pressure number R_F on the pressure $\bar{p}$ profile; $PWS = 0.023$, $POS = 10$, – – – – $PWS = POS = 0$

6. Discussion of the results

From the diagrams and forms of formulas used to describe the pressure and velocity components, the following conclusions can be drawn:

- For the longitudinal velocity component $\bar{V}_x$:
  - the effect of longitudinal inertial forces $(PWS \neq 0,\ POS = 0)$ is negligible and can be observed in the vicinity of the fluid inlet into the clearance,
  - the effect of centrifugal inertial forces $(PWS = 0,\ POS \neq 0)$ can be observed mainly in the area situated in th evicinity of the fluid outlet from the clearance. It is related to the case when only one of the surfaces is rotating $(K = 0)$. However, the reported effect declines when both the surfaces are rotating $(K = 1)$.
- For the tangential velocity component $\bar{V}_\theta$:
  - the effect of longitudinal inertial forces is important in the vicinity the fluid inlet into a clearance.
- For the pressure:
  - longitudinal inertial forces slightly increase the pressure value along the clearance,
  - centrifugal inertial forces cause a significant drop in the pressure value inside a clearance in particular, when both surfaces are rotating $(K = 1)$.
- The flows of fluid between curvilinear surfaces are less open to influence of inertial forces.
- Increment of the magnetic field strenght (increasing of the magnetic pressure number R_F) will cause:
  - breaking of a longitudinal velocity component V_x,
  - increase of a tangential velocity component $\bar{V}_\theta$,
  - drop in the pressure value along the clearance.

It should be noted, that the obtained results had been tested for convergance criteria. Thus, the PWS, PWK, POS, POK parameters, respectively, were determined.

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Przepływ inercyjny cieczy magnetycznej w szczelinie między krzywoliniowymi powierzchniami obrotowymi

Streszczenie

W pracy rozpatrzono stacjonarny, laminarny, przeplyw cieczy magnetycznej w szczelinie między krzywoliniowymi powierzchniami obrotowymi o wspólnej osi symetrii. Rozwiązanie problemu przedstawiono w oparciu o równania warstwy przyściennej zapisane w krzywoliniowym układzie współrzędnych x, θ, y. Równania warstwy przyściennej rozwiązano metodą małego parametru. Otrzymano formuły, określające pole przeplywu tj. składowe prędkości $\bar{V}_x$, $\bar{V}_\theta$, $\bar{V}_y$ oraz ciśnienie $\bar{p}$.

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