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MODELLING OF A DISCRETE-CONTINUOUS SYSTEM WITH A SHAPE MEMORY ALLOY SUPPORT

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This paper presents the hysteretic damping effect caused by a shape memory alloy spring which is used as a support of a cantilever beam. Free vibration diagrams of the discrete-continuous system are compared with amplitude envelopes obtained for two equivalent one-degree-of-freedom systems with the nonlinear pseudoelastic spring. Also the logarithmic decrement to amplitude relations have been calculated. The problems of modelling of the considered discrete-continuous structure by means of the proposed simplified one-degree-of-freedom systems, and some remarks concerning inaccuracy of the applied asymptotic method have been presented and discussed.

1. Introduction

Shape Memory Alloys (SMA) are materials that reveal unique characteristics which do not appear in conventional alloy materials. The most remarkable characteristic is the Shape Memory Effect (SME) which is associated with a reverse transformation of the martensitic phase to the higher temperature austenite phase. Shape memory alloys have also ability to change their material properties: stiffness and internal friction during a temperature activated phase transformation.

If a temperature is much higher than the austenitic finish point, shape memory alloys show the pseudoelastic effect with hysteresis during loading and unloading cycles. This phenomenon is the result of the reverse phase transformation induced by the change in stress. The pseudoelastic behaviour of SMA is observed in the working temperature range so it can be exploited in passive/adaptive vibration control (cf Liang and Rogers (1991)).

The analysis of free vibrations of a cantilever beam with a shape memory alloy support has been presented by Pietrzakowski and Galkowski (1993). The system response was calculated using a simulation method and was compared with results obtained for the equivalent one-degree-of-freedom system applying an asymptotic method.

In this paper the authors focused on the damping effect due to the pseudoelastic hysteresis of the SMA support, and some problems of modelling of discrete-continuous structures in terms of equivalent one-degree-of-freedom systems.

2. Free vibrations of a discrete-continuous system

The system presented herein is the same as analyzed by Pietrzakowski and Gałkowski (1993). The dynamic model is composed of a cantilever beam with a concentrated mass, m, and a pseudoelastic SMA spring as a support on the other end of the beam (Fig.1).

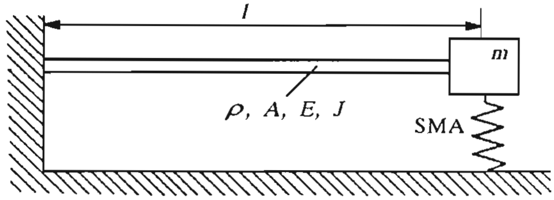

Fig. 1. Sheme of the investigated discrete-continuous system

According to the idea described by Liang and Rogers (1991) it was assumed that the pseudoelastic support reveals a nonlinear behaviour which may be represented by the force-displacement relation with hysteresis as shown in Fig.2.

The limiting forces F_1 and F_2 relating to the martensitic and austenitic transformations, respectively, and the spring stiffness, K_s, can be assumed to be constant parameters within the testing temperature range.

To determine free vibrations of the structure the equations of motion representing all sections of the pseudoelastic characteristic are solved. The problem has been described in details by Pietrzakowski and Gałkowski (1993). In this

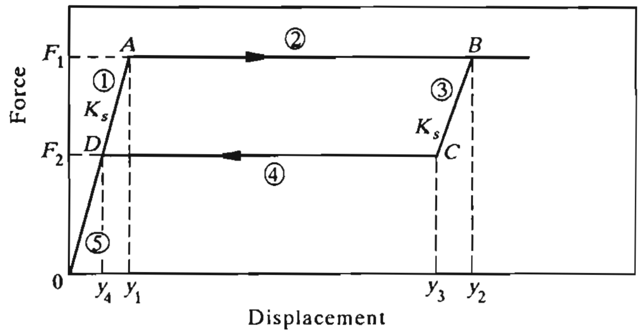

Fig. 2. Simplified pseudoelastic force-displacement relation

paper only the governing equations and basic relations used in the algorithm of the numerical simulation are presented.

The equation of motion corresponding to the elastic range of the support characteristic ($0A$ section) is

$$EJ\frac{\partial^4 y}{\partial x^4} + \rho A\frac{\partial^2 y}{\partial t^2} = 0 \tag{2.1}$$

where

E – Young moduls
ρ – mass density
J – moment of inertia of the beam
A – cross-section area of the beam.

The boundary conditions are given by

$$\begin{aligned} &y(0,t) = 0 &&\frac{\partial y}{\partial x}\Big|_{x=0} = 0 \\ &\frac{\partial^2 y}{\partial x^2}\Big|_{x=l} = 0 &&\frac{\partial^3 y}{\partial x^3}\Big|_{x=l} = \frac{K_s}{EJ}y(l,t) + \frac{m}{EJ}\frac{\partial^2 y}{\partial t^2}\Big|_{x=l} \end{aligned} \tag{2.2}$$

The shape of the nth vibration mode for the investigated structure has the following form

$$\begin{aligned} X_n(x) &= (\sin k_n x - \sinh k_n x)(\cos k_n l + \cosh k_n l) + \\ &- (\cos k_n x - \cosh k_n x)(\sin k_n l + \sinh k_n l) \end{aligned} \tag{2.3}$$

where the infinite series of eigenvalues k_n, ($n = 1,2,...$) is found in a well known manner from the characteristic equation which is obtained employing the boundary conditions, Eqs (2.2).

The solution to Eq (2.1) free vibrations is determined using the separation of variables technique

$$y(x,t) = \sum_{n=1}^{\infty} (K_n \cos\omega_n t + L_n \sin\omega_n t) X_n(x) \tag{2.4}$$

where ω_n is a natural frequency which satisfy the relation

$$\omega_n = k_n^2 \sqrt{\frac{EJ}{\rho A}} \tag{2.5}$$

Assuming initial conditions

$$\begin{aligned} y(x,0) &= y_0(x) \\ \dot{y}(x,0) &= v_0(x) \end{aligned} \tag{2.6}$$

and after the orthogonalization, the modal coefficients K_n and L_n are obtained

$$\begin{aligned} K_n &= \frac{1}{\gamma_n^2}\Big[\int\limits_0^l y_0(x)X_n(x)\;dx + \mu l y_0(l)X_n(l)\Big] \\ L_n &= \frac{1}{\gamma_n^2\omega_n}\Big[\int\limits_0^l v_0(x)X_n(x)\;dx + \mu l v_0(l)X_n(l)\Big] \end{aligned} \tag{2.7}$$

where

$$\gamma_n^2 = \int\limits_0^l X_n^2(x)\;dx + \mu l X_n^2(l) \tag{2.8}$$

and $\mu = \dfrac{m}{\rho A l}$ – is the ratio of the concentrated mass to the mass of beam.

When the displacement of the end of the beam, $y(l,t)$, exceeds the elastic range of the SMA spring, $(y(l,t) > y_1)$, the support reaction is equal to the force F_1 acting against the displacement. The movement of the system can be expressed as a result of free vibrations and vibrations induced by a constant force loading the end-point of the beam.

The equation of motion becomes

$$EJ\frac{\partial^4 y}{\partial x^4} + \rho A\frac{\partial^2 y}{\partial t^2} = -F_1\delta(x-l) \tag{2.9}$$

where the symbol $\delta(x)$ indicates the Dirac delta-function.

Now, in the boundary conditions the shearing force caused by the support response is not involved

$$
\begin{aligned}
&y(0,t) = 0 && \frac{\partial y}{\partial x}\Big|_{x=0} = 0\\
&\frac{\partial^2 y}{\partial x^2}\Big|_{x=l} = 0 && \frac{\partial^3 y}{\partial x^3}\Big|_{x=l} = \frac{m}{EJ}\frac{\partial^2 y}{\partial t^2}\Big|_{x=l}
\end{aligned}
\tag{2.10}
$$

The Equation (2.9) fulfils the initial conditions which are obtained from the solution (2.4) at an instant t_A, when the displacement of the SMA spring reaches the limit of elastic deformations, i.e., when $y(l,t_A) = y_1$

$$
\begin{aligned}
y(x,t_A) &= y_A(x)\\
\dot{y}(x,t_A) &= v_A(x)
\end{aligned}
\tag{2.11}
$$

Free vibrations can be solved from the homogeneous equation, Eq (2.1), with the boundary conditions, Eqs (2.10), by applying the initial conditions, Eqs (2.11). The solution has the form given in Eq (2.4), where the modal coefficients, K_n and L_n, depend on displacements and velocites of the system at the time t_A which represents the change of the path of the pseudoelastic characteristic

$$
\begin{aligned}
K_n &= \frac{1}{\gamma_n^2}\Big[\int\limits_0^l y_A(x)X_n(x)\;dx + \mu l y_A(l)X_n(l)\Big]\\
L_n &= \frac{1}{\gamma_n^2\omega_n}\Big[\int\limits_0^l v_A(x)X_n(x)\;dx + \mu l v_A(l)X_n(l)\Big]
\end{aligned}
\tag{2.12}
$$

Since there is a concentrated mass in the analysed structure, forced vibrations for zero-value initial conditions have been obtained using the Lagrange's equation. Finally, the general solution to Eq (2.9) can be written as

$$
y(x,t) = \sum_{n=1}^{\infty}\Big[(K_n\cos\omega_n t + L_n\sin\omega_n t) - \frac{F_1}{\rho A}\frac{X_n(l)}{\gamma_n^2\omega_n^2}(1-\cos\omega_n t)\Big]X_n(x) \tag{2.13}
$$

At the considered stage of motion, the amplitude, y_2, of the beam end-point is unknown. In the worked out computer programme it is assumed that the ratio of the concentrated mass to the mass of the beam, μ, has a value, which allows to apply a change of the velocity sign as a criterion for entering the next section of the pseudoelastic characteristic.

At the instant t_B when the end of the beam begins to move towards the equlibrium state, the displacements and velocities of the system are calculated. Therefore, in the range of the elastic response of the SMA support determined by the BC section of the characteristic the initial conditions are

$$\begin{aligned} y(x,t_B) &= y_B(x) \\ \dot{y}(x,t_B) &= v_B(x) \end{aligned} \tag{2.14}$$

The equation of motion is

$$EJ\frac{\partial^4 y}{\partial x^4} + \rho A\frac{\partial^2 y}{\partial t^2} = (y_2 - y_1)K_s\delta(x-l) \tag{2.15}$$

The above equation fulfils the boundary conditions (Eqs (2.2)) for the cantilever beam supported on the other end. The solution of the initial-value problem can be represented in terms of the Fourier series (Eq (2.4)), where the modal coefficients K_n and L_n depend on the initial conditions, Eqs (2.14)

$$\begin{aligned} K_n &= \frac{1}{\gamma_n^2}\Big[\int\limits_0^l y_B(x)X_n(x)\,dx + \mu l y_B(l)X_n(l)\Big] \\ L_n &= \frac{1}{\gamma_n^2\omega_n}\Big[\int\limits_0^l v_B(x)X_n(x)\,dx + \mu l v_B(l)X_n(l)\Big] \end{aligned} \tag{2.16}$$

Employing the forced vibrations for zero-value initial conditions, the general solution to Eq (2.15) has the following form

$$\begin{aligned} y(x,t) &= \sum_{n=1}^{\infty}\Big[(K_n\cos\omega_n t + L_n\sin\omega_n t) + \\ &+ \frac{(y_2-y_1)K_s}{\rho A}\frac{X_n(l)}{\gamma_n^2\omega_n^2}(1-\cos\omega_n t)\Big]X_n(x) \end{aligned} \tag{2.17}$$

The movement described by Eq (2.17) occurs up to the time t_C, when the displacement of the beam end-point decreases to the value of y_3 which can be found assuming the same slope of the elastic sections of the characteristic.

The next stage of motion represents the CD section of the pseudoelastic characteristic, so, the response of the SMA support can be replaced by the constant force F_2. The governing equation of motion is

$$EJ\frac{\partial^4 y}{\partial x^4} + \rho A\frac{\partial^2 y}{\partial t^2} = -F_2\delta(x-l) \tag{2.18}$$

The initial conditions at given time t_C are obtained using Eq (2.17)

$$\begin{aligned} y(x,t_C) &= y_C(x) \\ \dot{y}(x,t_C) &= v_C(x) \end{aligned} \tag{2.19}$$

The displacements of the structure for the CD section of the SMA spring characteristic can be determined applying the same procedure as at the second stage of motion (AB section). It is clear that the new initial conditions appear in the formulae for the modal coefficients, K_n and L_n

$$\begin{aligned} K_n &= \frac{1}{\gamma_n^2}\Big[\int\limits_0^l y_C(x)X_n(x)\,dx + \mu l y_C(l)X_n(l)\Big] \\ L_n &= \frac{1}{\gamma_n^2\omega_n}\Big[\int\limits_0^l v_C(x)X_n(x)\,dx + \mu l v_C(l)X_n(l)\Big] \end{aligned} \tag{2.20}$$

The general solution to Eq (2.18) has the form analogous to the displacement response, Eq (2.13)

$$y(x,t) = \sum_{n=1}^{\infty}\Big[(K_n\cos\omega_n t + L_n\sin\omega_n t) - \frac{F_2}{\rho A}\frac{X_n(l)}{\gamma_n^2\omega_n^2}(1-\cos\omega_n t)\Big]X_n(x) \tag{2.21}$$

The movement defined by Eq (2.21) occurs until the time t_D, when the displacement of the beam end reaches the value of y_4.

Furthermore, the movement of the structure is given by Eq (2.4) representing the linear elastic behaviour of the SMA support. The initial conditions at the instant t_D can be calculated using the equation (2.21)

$$\begin{aligned} y(x,t_D) &= y_D(x) \\ \dot{y}(x,t_D) &= v_D(x) \end{aligned} \tag{2.22}$$

Following the described procedure, the vibrations of the system during next cycles can be determined. In every cycle, in which the range of elastic displacements of the SMA spring is exceeded the energy dissipation caused by the hysteresis appears. Therefore, the system finally stabilizes to undamped free vibrations.

3. Discrete systems with pseudoelastic spring

For dynamic analysis it is useful to replace a continuous or a discrete-continuous system with a discrete model which is much easier to solve. The general condition of this simplification is to keep the main dynamic properties of the modeled systems.

In the paper the damping effect caused by the SMA support in the considered discrete-continuous structure is compared with the energy dissipation in two models of one-degree of freedom. This comparison is possible because the movement of the end of the cantilever beam is analyzed, and it was also assumed that the mass concentrated on the beam end is significantly greater then the mass of the beam, so the first mode of the system is mainly expected.

The first proposed model is composed of a mass and a pseudoleastic SMA spring, and has been investigated by Pietrzakowski and Gałkowski (1993).The second model differs from the first one in adding another linear spring, which referes to the beam stiffness. For the sake of simplicity, in the both cases the pseudoelastic force-displacement relation is modified by a separation of the linear elastic component and the nonlinear component which describes hysteretic damping in the SMA spring (Fig.3).

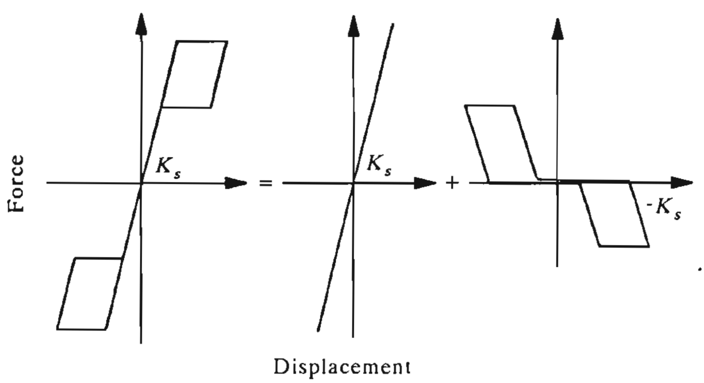

Fig. 3. Modified pseudoelastic force-displacement relation

The modified characteristic can be expressed in analitical form

$$\varepsilon F(y) = \operatorname{sign}\dot{y} \begin{cases} K_s(y_3 - y_4) & \text{for} \quad y_3 < y < y_2 \quad \text{and} \quad 0 < \beta < \beta_3 \\ K_s(y - y_4) & \text{for} \quad y_4 < y < y_3 \quad \text{and} \quad \beta_3 < \beta < \beta_4 \\ 0 & \text{for} \quad y_5 < y < y_4 \quad \text{and} \quad \beta_4 < \beta < \beta_5 \\ K_s(y - y_5) & \text{for} \quad y < y_5 \quad \text{and} \quad \beta_5 < \beta < \pi \end{cases} \tag{3.1}$$

where ε is a small parameter concerning with the applied method for analysis.

3.1. One-spring model

This model is simply a mass with a SMA spring (Fig.4).

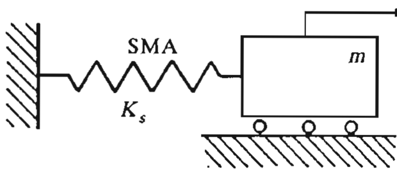

Fig. 4. Scheme of the one-spring model

The equation of motion of the system can be written as

$$m_z\ddot{y} + K_sy + \varepsilon F(y) = 0 \tag{3.2}$$

The above equation describes a harmonic oscillator with a disturbance given by Eqs (3.1).

To obtain the system which, in the same way, is equivalent to the investigated discrete-continuous structure, the mass, m_z, have to the relation

$$m_z = \frac{K_s}{\omega_1^2} \tag{3.3}$$

where ω_1 is the first natural frequency of the discrete-continuous system.

The asymptotic Bogolubov-Krylov-Mitropolski method described by Mitropolskiĭ (1971) has been employed to solve the equation of motion (3.2). Applying the first approximation leads to the following equations in the amplitude derivative and the phase derivative (see Pietrzakowski and Gałkowski (1993))

$$\frac{dy_2}{dt} = \frac{\omega_0}{\pi\alpha}y_2\Big[\cos\beta_3 - \cos\beta_4 + \cos\beta_1 - \frac{1}{2}(\cos^2\beta_3 - \cos^2\beta_4 + \cos^2\beta_1 + 1)\Big] \tag{3.4}$$

and

$$\frac{d\xi}{dt} = \alpha\omega_0 + \frac{\omega_0}{2\pi\alpha}\Big[\frac{1}{2}(\sin 2\beta_3 - \sin 2\beta_4 - \sin 2\beta_1) + (\beta_3 - \beta_4 - \beta_1)\Big] \tag{3.5}$$

where

$\omega_0 = \sqrt{\frac{K_s}{m_r}}$ – natural frequency of the linear vibrations

$\cos\beta_i = \frac{y_i}{y_2}$ – ratio of the limiting displacements

α – non-dimensional coefficient, in the considered case $\alpha = 1$.

The equivalent damping factor and the equivalent natural frequency are given by

$$h_{eq}(y_2) = \frac{\omega_0}{2\pi\alpha}\Big[\cos\beta_3 - \cos\beta_4 + \cos\beta_1 - \frac{1}{2}(\cos^2\beta_3 - \cos^2\beta_4 + \cos^2\beta_1 + 1)\Big] \tag{3.6}$$

and

$$\omega_{eq}^2(y_2) = \alpha^2\omega_0^2 + \frac{\omega_0^2}{\pi}\Big[\frac{1}{2}(\sin 2\beta_3 - \sin 2\beta_4 - \sin 2\beta_1) - (\beta_3 - \beta_4 - \beta_1)\Big] \tag{3.7}$$

Employing Eqs (3.6) and (3.7), the logarithmic decrement, δ, can be determined

$$\delta = h_{eq}T_{eq} = 2\pi\frac{h_{eq}}{\omega_{eq}} \tag{3.8}$$

3.2. Two-spring model

The model considered herein after is composed of a mass and two springs. The first coil is the pseudoelastic SMA spring, and the second one has the constant stiffness which replaces the stiffness of the cantilever beam (Fig.5).

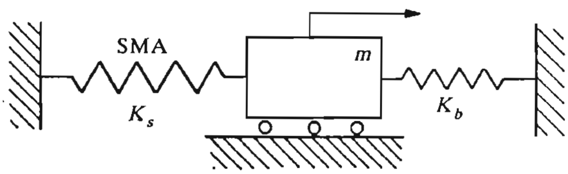

Fig. 5. Scheme of the two-spring model

Applying the modified characteristic, Eqs (3.1), free vibrations of the system are described by

$$m_r\ddot{y} + (K_s + K_b)y + \varepsilon F(y) = 0 \tag{3.9}$$

where

K_b – reduced stiffness of the beam, $K_b = 3EJ/l^3$

m_r – reduced mass.

The reduced mass is obtained following the standard Reyleigh procedure, and for the cantilever beam with the additional mass concentrated on its end can be expressed as

$$m_r = m + \frac{31}{140}m_b \tag{3.10}$$

where m_b indicates mass of the beam $(m_b = \rho Al)$.

The solution to Eq (3.9), assuming the first approximation, is given in Eqs (3.5) and (3.6). Therefore, the equivalent damping factor and equivalent natural frequency as well as the logarithmic decrement are of the same forms as relations given in Eqs (3.6), (3.7) and (3.8), respectively. But for the two-spring model the natural frequency, ω_0, refers to the reduced mass, m_r, and the non-dimensional coefficient, α, becomes

$$\alpha = \sqrt{1 + \frac{K_b}{K_s}} = \sqrt{1 + \frac{3}{\kappa_s}} \tag{3.11}$$

where κ_s – is the ratio of the support stiffness to the bending stiffness of the beam

$$\kappa_s = \frac{K_s l^3}{EJ} \tag{3.12}$$

4. Results and discussion

In order to obtain the dynamic response of the considered discrete-continuous system it were assumed non-zero initial displacements due to the static force P loading the end of the cantilever beam. Therefore, the initial conditions can be expressed as

$$\begin{aligned} y_0(x) &= \frac{Px^2}{6EJ}(3l - x) \\ v_0(x) &= 0 \end{aligned} \tag{4.1}$$

The dimensions of the beam and properties of material and parameters of the SMA support as follows

$l = 1.0$ m	$A = 1.0 \cdot 10^{-4}$ m^2	$J = 5.0 \cdot 10^{-9}$ m^4
$E = 2.0 \cdot 10^{11}$ N/m^2	$\rho = 7.8 \cdot 10^3$ kg/m^3	$\mu = 3.0$
$F_1 = 20.0$ N	$F_2 = 5.0$ N	

The influence of the slope of the pseudoelastic characteristic of the SMA spring on dissipation of energy is investigated.

In Fig.6, Fig.7 and Fig.8 free vibrations of the end of the cantilever beam are shown. They were simulated for three different values of the non-dimensional stiffness coefficient κ_s, but the same initial displacements.

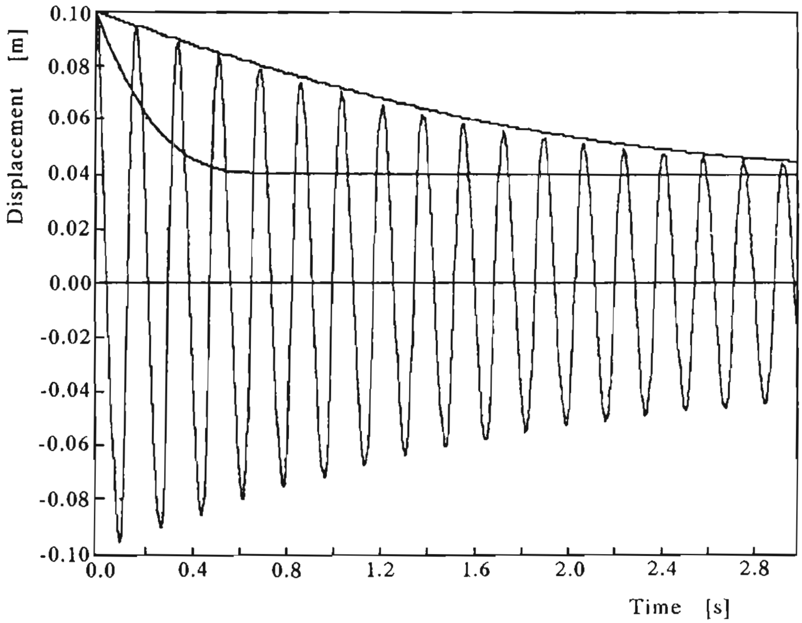

Fig. 6. Free vibrations of the cantilever beam with the SMA support ($\kappa_s = 0.5$) and the amplitude envelopes of the one-spring model (lower curve) and the two-spring model (upper curve)

It can be observed that the rate of the energy loss (per cycle) increases for the high stiffness of the pseudoelastic spring. This effect is caused by the greater initial area of the force-displacement hysteresis. With time, vibrations of the system stabilize and continue with the amplitude determined by the maximum elastic deformation of the SMA support.

Calculations were also performed for the two discrete models equivalent to

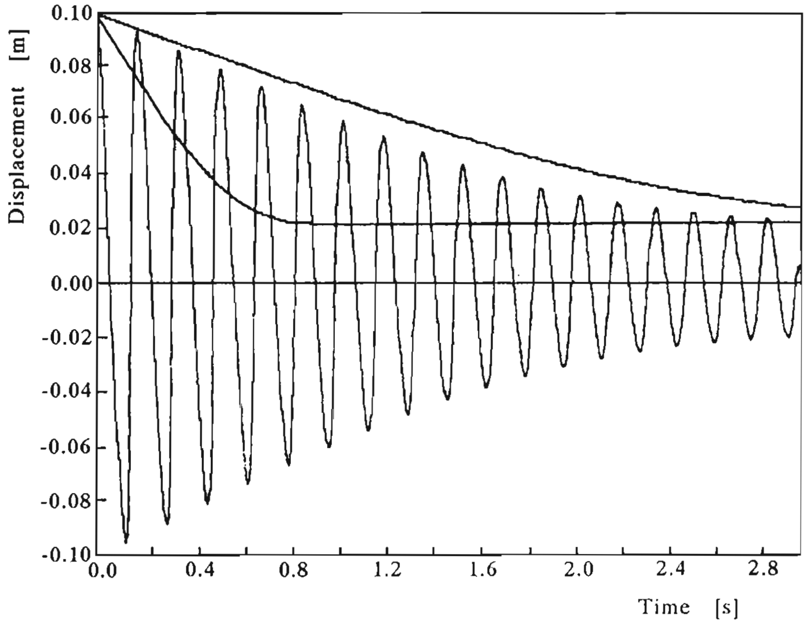

Fig. 7. Free vibrations of the cantilever beam with the SMA support ($\kappa_s = 1.0$) and the amplitude envelopes of the one-spring model (lower curve) and the two-spring model (upper curve)

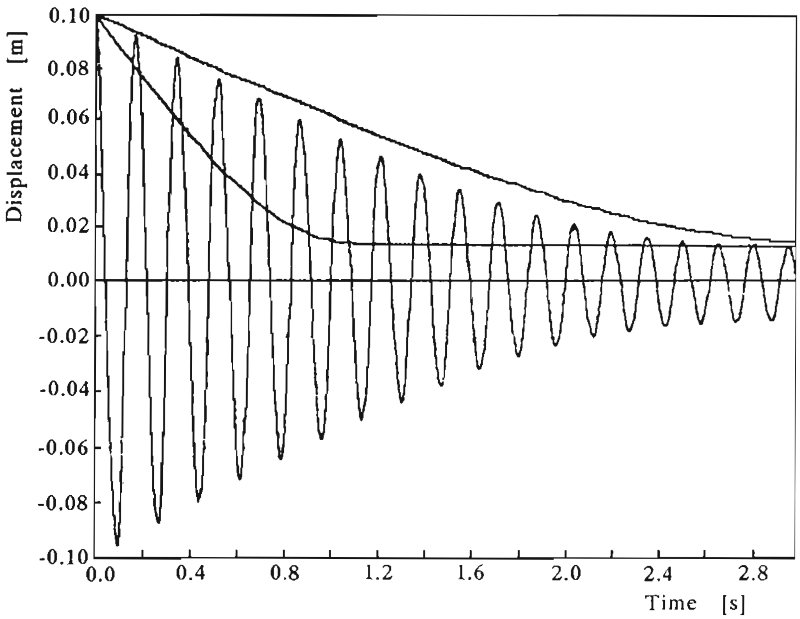

Fig. 8. Free vibrations of the cantilever beam with the SMA support ($\kappa_s = 1.5$) and the amplitude envelopes of the one-spring model (lower curve) and the two-spring model (upper curve)

the discrete-continuous system. Their parameters were chosen according to the governing equations, Eqs (3.2) and (3.9), respectively, and to the explanations given in Section 3. On the presented diagrams the amplitude envelopes for free vibrations of the proposed discrete systems are shown. The lower curves represent the varying in time amplitude obtained for the one-spring model. In this case the amplitude stabilizes much faster in comparison with vibrations of the beam end-point. The damping is so intensive because the elastic beam energy is neglected in the one-spring model. The better results are obtained using the two-spring model. The amplitude envelope (the upper curve) is close to the amplitude of the beam system particularly for a low stiffness ratio ($\kappa_s = 0.5 \div 1.0$). When the stiffness of the SMA spring increases the effect of diminishing hysteretic damping is observed in the both discrete models. This unexpected behaviour can be explain by an inaccuracy of the employed asymptotic method.

The above observations are confirmed by the analysis of the logarithmic decrement.

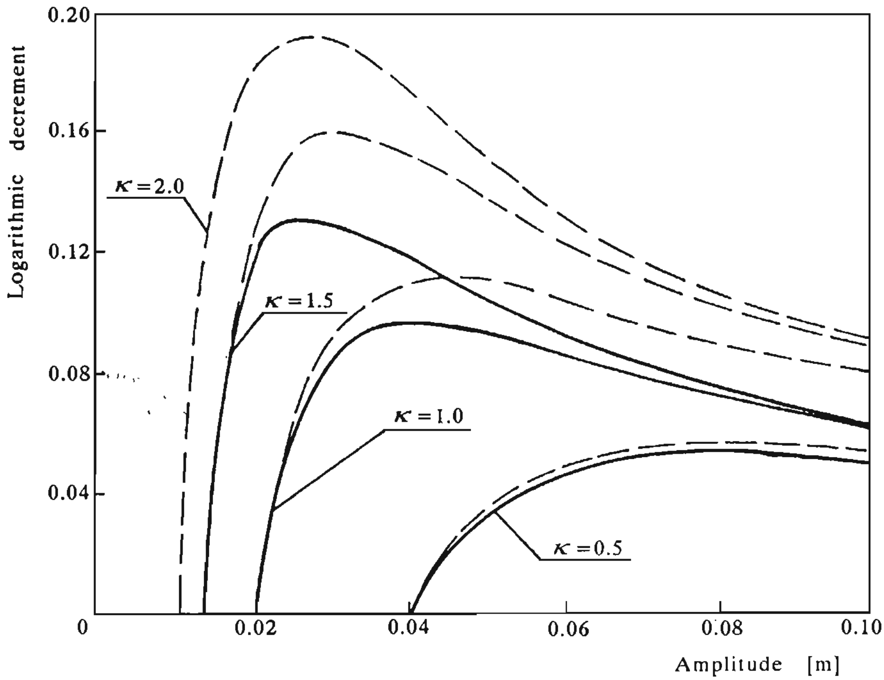

Fig. 9. Logarithmic decrement vs. amplitude courses

Figure 9 illustrates the logarithmic decrement vs. displacement amplitude course. The dashed lines represent the decrement of free vibrations of the end-point of the cantilever beam while the solid lines correspond to vibrations of the two-spring discrete model. The curves were obtained for a few values of the SMA spring stiffness expressed by the nondimensional coefficient κ_s. Generally, the logarithmic decrement value obtained for the discrete model is smaller than that calculated for the discrete-continuous system. For relatively small stiffness ratio $\kappa_s = 0.5$ both dash and solid lines are close to each other. When the SMA spring stiffness becomes greater, the distance between the compared curves increases except for the range of the amplitude close to the amplitude of elastic undamped vibrations.

The clear evidence that applied asymptotic method is inaccurate is intersection of the curves (solid lines) obtained for $\kappa_s = 1.0$ and $\kappa_s = 1.5$. The inaccuracy is more glaring for the greater stiffness of the elastic range of the SMA spring characteristic.

5. Conclusions

The energy dissipation appearing in the investigated systems depends on the stiffness of the SMA pseudoelastic spring. Assuming the same initial conditions, it is more intensive for the high stiffness because of an increase in the force-displacement hysteresis area.

The logarithmic decrement value calculated for the discrete-continuous system as well as that obtained for the one-degree-of-freedom model shows an extremum. Therefore, there is a range of amplitude when the most effective hysteretic dumping occurs.

The two-spring model simulates vibrations of the end-point of the cantilever beam much better than the one-spring model. However, the results obtained for the high stiffness of the SMA spring differ glaring because of the inaccuracy of the applied asymptotic method which is sensitive for strongly nonlinear characteristics.

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Modelowanie układu dyskretno ciągłego z podporą ze stopu z pamięcią kształtu

Streszczenie

W pracy zbadano efekt tlumienia drgań belki wysięgnikowej wywolany przez pseudosprężyste wlasności podpory ze stopu z pamięcią kształtu. Przebieg drgań swobodnych wybranego punktu ukladu dyskretno-ciaglego porównano z obwiedniami amplitud wyznaczonymi dla dwóch zastępczych ukladów o jednym stopniu swobody. Dla porównywanych ukladów wyznaczono także zależności logarytmicznego dekrementu tlumienia od amplitudy. W pracy zwrócono uwagę na problemy związane z modelowaniem ukladu dyskretno-ciąglego przez zaproponowane uklady o jednym stopniu swobody oraz na niedokladność stosowanej metody asymptotycznej.

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