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# SYNTHESIS AND EXAMINATION OF THE VIBRATION CONTROL SYSTEMS WITH VARIABLE SPRING AND DAMPING COEFFICIENTS

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This paper deals specifically with a synthesis of the semi-active pneumo-hydraulic vibration control systems with variable structure. The concepts of the vibration control systems with a variable stiffness and a variable damping were developed. The analysis of the aforementioned models was carried out along with the verification of it in the digital simulation procedure. This investigation indicated that the vibration control systems with a variable structure provide a considerable vibration isolation efficiency in the frequency range beyond 1 Hz. The design of the semi-active vibration control system with a variable structure is discussed on the example of its application to the driver seat suspension. The research model of such a system was formulated and the experimental examination was carried out. This investigation has confirmed the possibility of the application of the developed system to the human body protection against the harmful vibration effect.

## 1. Introduction

The application of the vibration control systems offers the significant potential for overcoming the limitations of the passive systems. However they are are more costly, more complex and therefore less reliable than the conventional passive systems. For these reasons the usage is limited to the cases in which vibration control efficiency gain outweighs the disadvantages of the increased cost and complexity. These cases include vehicle suspension systems and driver seat suspensions [4,5].

It is commonly known that vibration control systems generally operate like systems which automatically control the vibrations of the isolated plant and they can be classified into two groups: active and semi-active systems. The first group involves the vibration control systems where the additional control input, being dependent on the the input and output signals, is generated according to the strategy which minimizes the force acting on the isolated system. The changes of this control input can be either continuous [3] or discrete ones, e.g. with a time–optimalcontrol [1]. In this case expensive and complex servo–systems can be replaced by inexpensive and widely available discrete flow control valves. The second group includes systems with variable spring and damping coefficients. These parameters vary either continuously or discretely depending on the state variables of the isolated system [1].

The vibration control systems with continuously varying parameters have a fixed structure with a non–varying constraints between the members. The discrete change is achieved by the rapid disconnection and restoration of the constraints, i.e. through the changes of the system structure. For this reason these type of systems are called the variable structure systems. The structure diagram of the semi–active uniaxial vibration control system is shown in Fig.1. The switching policy is developed in a control unit US which receives and converts the required information on a system state. The logic key KL provides the direct structure switching (i.e. the switching of the $\alpha$ and $\beta$ parameters).

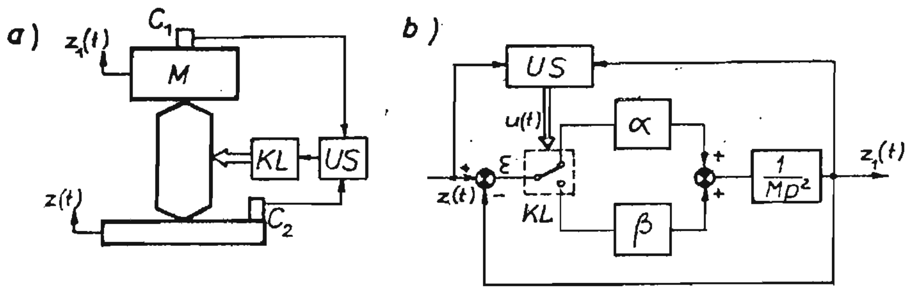



Fig. 1. Variable–structure semi–active vibration control system: a) physical model, b) structural model

This structure diagram indicates that the control input acting on the controlled system can be expressed as

$$u(t) = \Psi\varepsilon(t) \tag{1.1}$$

where $\Psi$ represents a switching function being dependent in each moment of time on the assumed control policy (for example $\Psi = \alpha$ or $\Psi = \beta$).

This paper deals with the synthesis algorithms of the free and forced vibration of the vibration control system with variable spring and damping coefficients.

The results of the examination of the pneumo–hydraulic variable–structure vibration control system applied to the driver seat suspension are also discussed in the following sections.

## 2. The concept of the system with a variable stiffness

The formulation of the problem of the synthesis of the vibration control system with variable structure will be discussed on the possibly the simplest example, i.e. the classical model of the vibration control system with one degree of freedom which is shown in Fig.2.

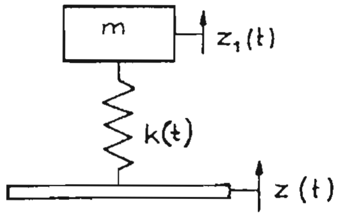



Fig. 2. Physical model of the vibration control system of a free mass being kinematically excited to a vibration

The differential equation of motion for a system with variable stiffness can be written as

$$m\ddot{z}_1 + k(t)z_1 = k(t)z(t) \tag{2.1}$$

Placing $\omega^2(t) = k(t)/m$ into the above equation yields

$$\ddot{z}_1 + \omega^2(t)z_1 = \omega^2(t)z(t) \tag{2.2}$$

Assuming the low of the stepped stiffness variation we obtain

$$\omega(t) = \begin{cases} \omega_1 \\ \omega_2 \end{cases} \tag{2.3}$$

If we consider a free motion of the system described by the Eq (2.2) then in the phase-coordinate system the equations of the phase trajectories become the equations of the ellipses [5]

$$\frac{z_1^2}{a} + \frac{\dot{z}_1^2}{a^2\omega^2(t)} = 1 \tag{2.4}$$

where $a$ denotes an arbitrary constant dependent of the initial conditions.

The semi-axis of the ellipses are equal when $\omega^2(t) = 1$ and different for either $\omega^2(t) > 1$ or $\omega^2(t) < 1$. The phase portraits of the last two structures corresponding to the natural frequencies $\omega_1$ and $\omega_2$ are presented in Fig.3.

Both structures are on the stability limit. If we switch the structure when crossing the coordinate axes then the transient process will be stable (Fig.4).

In this case the control policy will be written as follows

$$u(t) = -\Psi z_1(t) \tag{2.5}$$

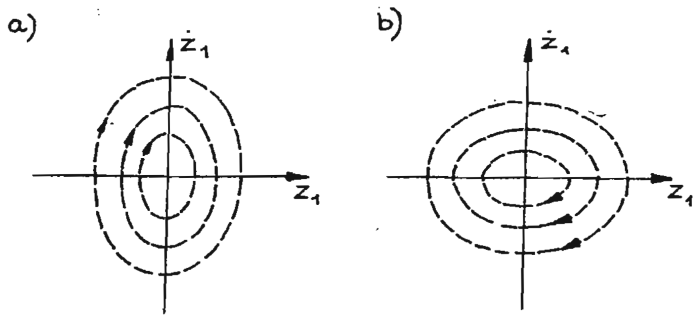



Fig. 3. Phase trajectories for the two structures of the vibration control systems of the natural frequencies: a) $\omega_1$ and b) $\omega_2$

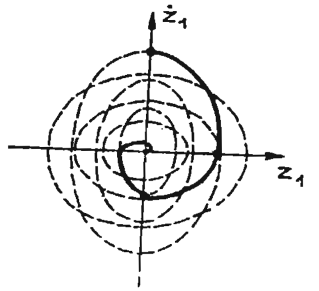



Fig. 4. Phase trajectory of the stable vibration control system with a variable structure

where

$$\Psi = \begin{cases} \omega_1^2 & \text{for } z_1\dot{z}_1 > 0 \\ \omega_2^2 & \text{for } z_1\dot{z}_1 < 0 \end{cases}$$

The above equations imply that the stiffness of the vibration control system varies discretely according to some time–dependent function $k = k(t)$.

It can be noticed that the differential equation of the transient process

$$\frac{d^2 z_1}{dt^2} + \frac{k(t)}{m} z_1 = 0 \tag{2.6}$$

with a switching rule specified as

$$u(t) = -\Psi z_1(t)$$

is the Hill Equation in which $k(t)$ is a periodical function of time with a period $T = \pi$.

It is known that the motion modeled by the Hill Equation could be unstable for some particular values of parameters. Hence it is the stability condition which should be considered as a first criterion when choosing spring parameters for a given system.

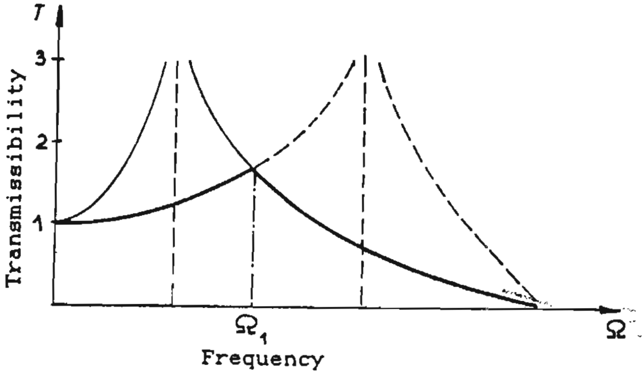


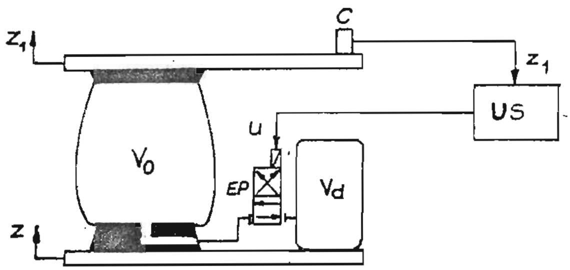

Fig. 5. Amplitude vs. frequency characteristics for a variable stiffness system

Fig.5 (curves 1 and 2) shows the amplitude vs. frequency plots for the system subject to a harmonic excitation $z(t) = Z \sin \Omega t$ where the natural frequencies of the system are denoted as $\omega_1$ and $\omega_2$. The results shown in this figure indicate that for a frequency $\Omega < \Omega_1$ the minimal value of the transmissibility $T = Z_1/Z$ is provided by a system with a natural frequency $\omega_1$ while for a frequency $\Omega > \Omega_1$ by a system with natural frequency $\omega_2$. Therefore in order to obtain the best dynamic properties of the system with given natural frequencies $\omega_1$ and $\omega_2$ the structure switching is required in the $\Omega_1$ frequency. For this case the switching policy can be written as

$$u(t) = -\Psi(z_1 - z) = -\Psi\varepsilon(t) \tag{2.7}$$

$$\Psi = \begin{cases} \omega_1^2 & \text{for } \Omega < \Omega_1 \\ \omega_2^2 & \text{for } \Omega > \Omega_1 \end{cases}$$

Technical implementation of the vibration control system with variable stiffness can be developed using electromagnetic or pneumatic springs. The connection of the pneumatic spring, such as for example the bellows of the volume $V_0$, with additional chamber of the volume $V_d$ allows one to achieve the wide range of the stiffness variation [2]. Fig.6a shows the diagram of the pneumatic vibration control system with a variable structure which consists of the pneumatic bellows (1.1) connected with the chamber (2.1) of additional volume $V_d$ through the two-position electropneumatic valve (2.2). The algorithm for a structure switching is developed in the control unit $US$ which transforms the suitable information on the system state received from the sensor $C$. The direct switching is accomplished by a)




b)

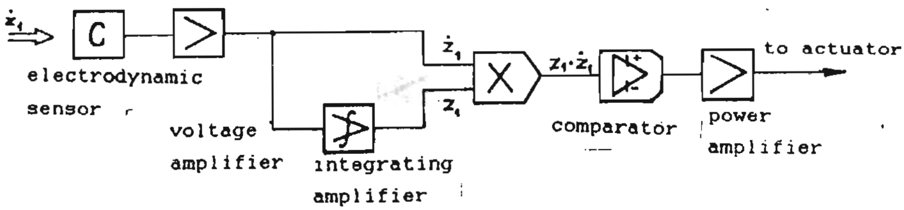



Fig. 6. a) Schematic diagram of the pneumatic vibration control system with a variable stiffness, b) Schematic diagram of the logical switching circuit

the logic key which is in this case implemented as a two-position electro-pneumatic valve $EP$. The logic switching circuit $S$ is to control the system according to the policy specified by (2.5). It enables to develop the required control low using the information on the system state coming from the sensor (2.3). The block diagram of the control unit $US$ is shown in Fig.6b.

## 3. Model of the system with a variable damping

The vibration control systems with a variable damping are far more simple in concept and realization. For this case the damper is controlled using the same measurements of the system state as in the system with a variable stiffness. In the conventional passive system the damper can only extract energy from a system. Nevertheless if the ground vibration velocity $\dot{z}(t)$ is greater than the vibration velocity of the controlled system $\dot{z}_1(t)$ (in the direction shown) then the force generated in the damper is actually accelerating the controlled system. This can be avoided by using the active damper (Fig.7).

The active concept, in its simplest form, would say that the damper should be turned off whenever such conditions exist that produce the unwanted acceleration of the controlled system. When conditions imply that the damper is on then the

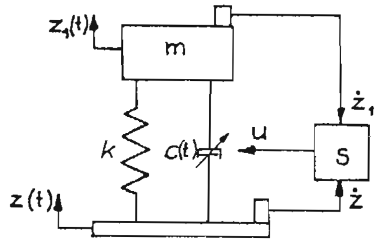



Fig. 7. Model of the vibration control system with a variable damping coefficient

damping forces are responsible for the deceleration of the sprung mass. The control low for the damping force $F_t$ can be written as

$$F_t = \begin{cases} 0 & \text{for } \dot{z}_1\dot{z} > 0 \text{ and } \dot{z}_1 > \dot{z} \\ c(\dot{z}_1 - \dot{z}) & \text{otherwise} \end{cases} \tag{3.1}$$

This concept is presented in Fig.8.

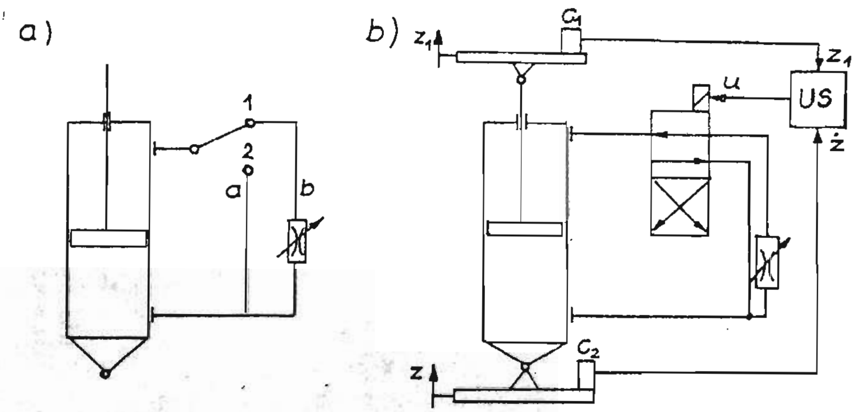



Fig. 8. Subsystem responsible for a variation of the damping coefficient a) schematic diagram, b) technical implementation

The logic key connects the upper and lower chambers of the hydraulic damper: directly using the pipe $a$ when in the position 1 or through the hydraulic gland using pipe $b$ when in the position 2. The key position is determined according to the value and direction of both the ground velocity $\dot{z}(t)$ and controlled system velocity $\dot{z}_1(t)$. The two-position four-way electrohydraulic valve was used as a logic key. The bloc diagram in the Fig.9 shows the switching circuit US which is to execute the control policy specified by the low (3.1). It consists of the two parallel measurement channels, one for the ground vibration velocity $\dot{z}(t)$ and the

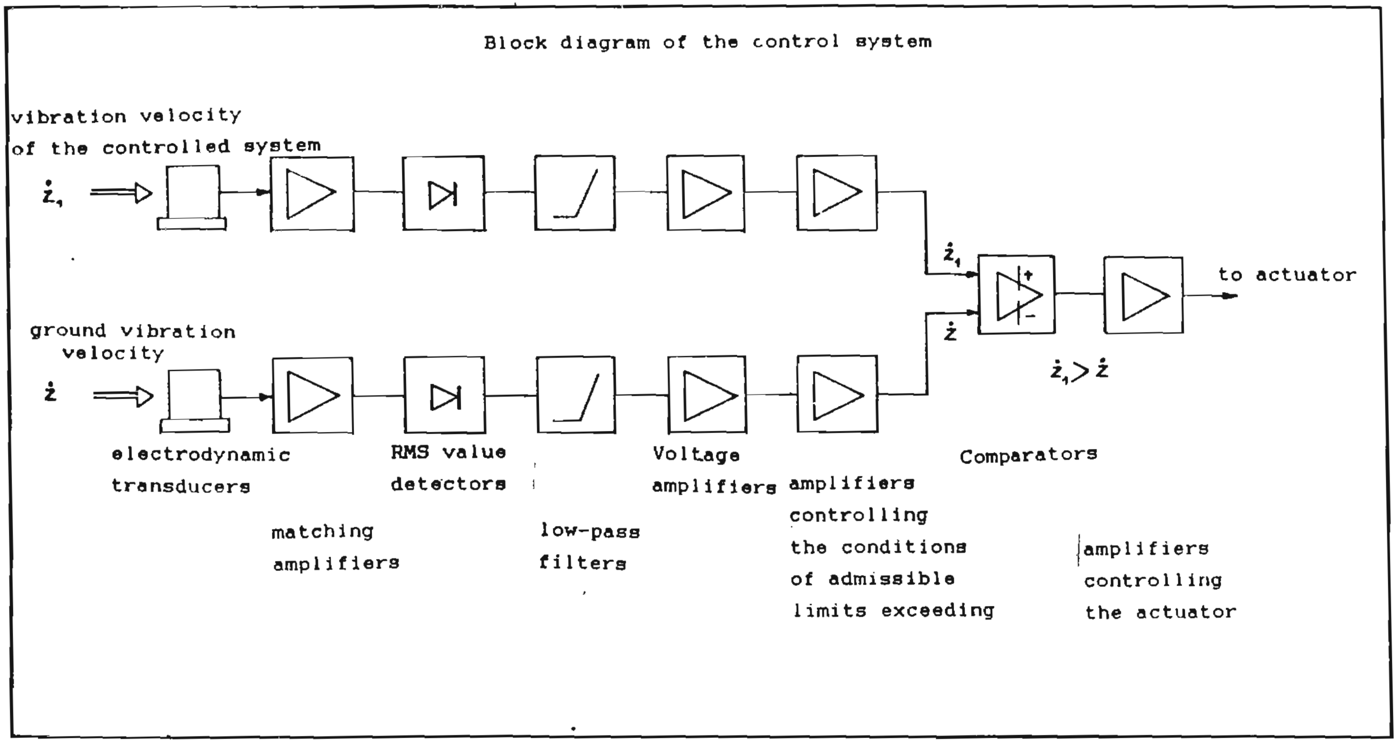



Fig. 9. Block diagram of the control system of the hydraulic damper

other for the controlled system vibration velocity $\dot{z}_1(t)$. The voltage signals from the electrodynamic sensors $C_1$ and $C_2$ are then fed to the amplifiers, root-mean-square value detectors and low-pass filters. Finally both signals are compared in the comparator to produce the control signal which, after amplified is fed to the actuator.

## 4. Simulation analysis

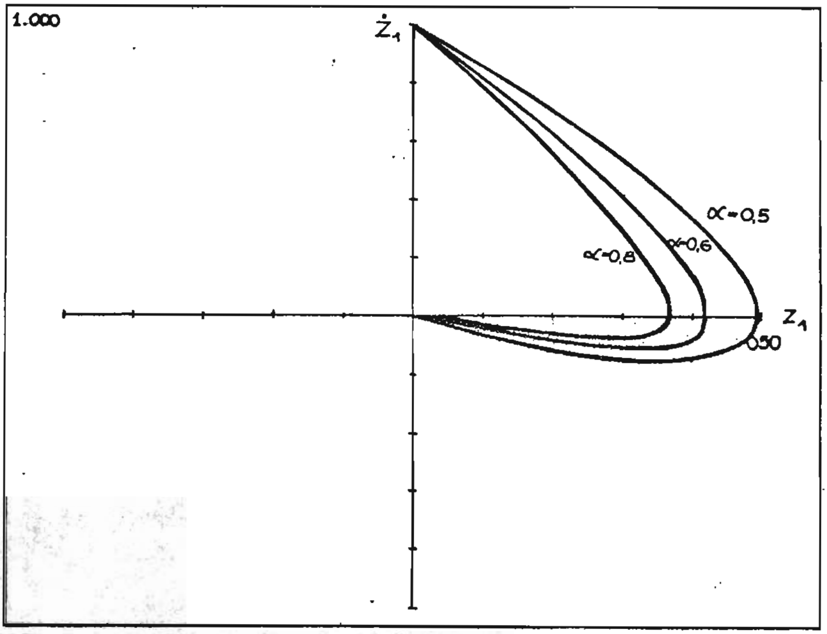



Fig. 10. Phase plots for the examined system obtained as a result of the digital simulation

The digital and analog simulation was carried out in order to verify the developed concept of the vibration control system with a variable damping. The following parameters were taken into the computation: $m = 80$ kg, $k = 10^4$ N/m and various values of the damping ratio ranged from $0.5 < \alpha = c/c_{kr} < 1.2$. The results of this analysis were compared with the computation carried out for the passive vibration control system of the same spring coefficient $k$ and the same mass of the controlled system $m$. Some obtained results are presented in Fig.10 and 11. The phase portraits for the various values of the damping ratio are shown in Fig.10. These plots indicate that the examined transient process is stable and

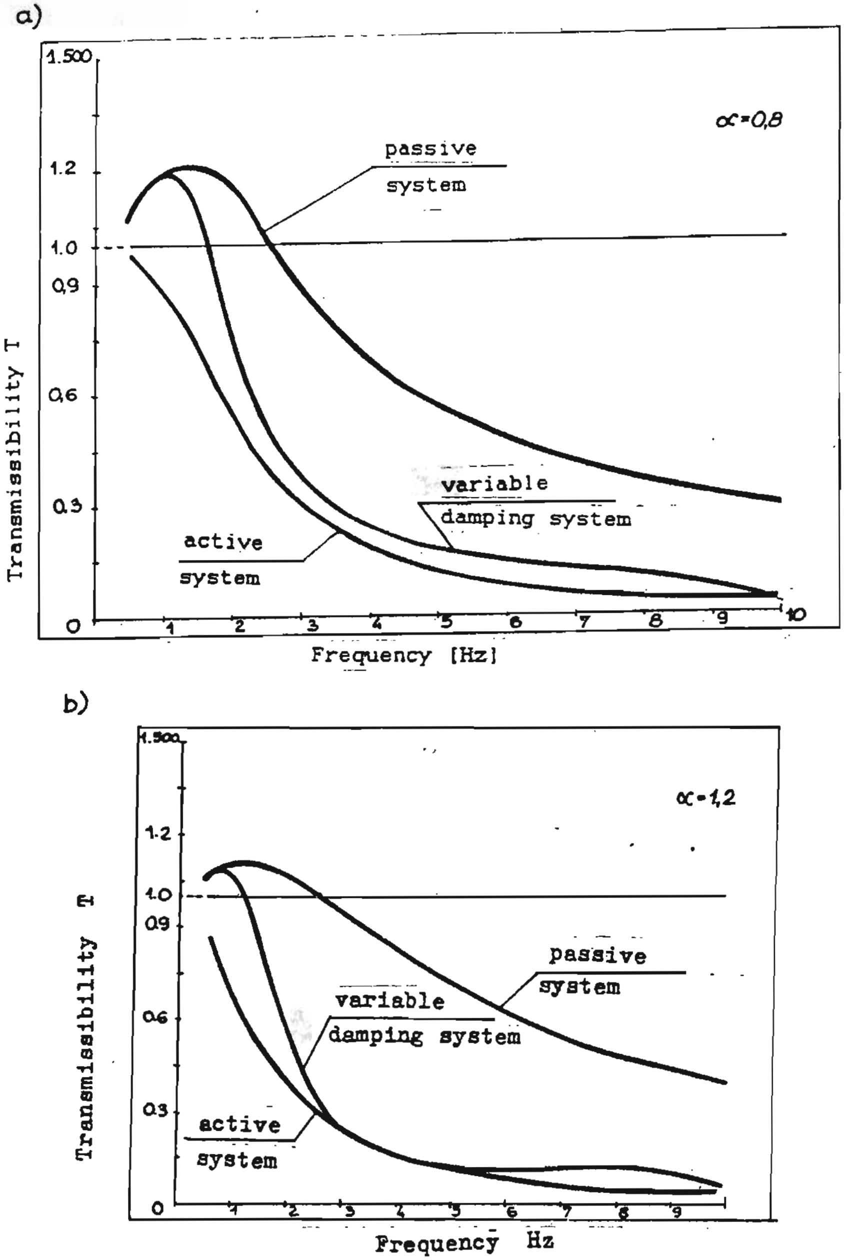



Fig. 11. Transmissibility plots: for the passive vibration control system with a constant damping coefficient, for the semi-active system with a variable damping and for the active system. a) damping ratio $\alpha = 0.8$, b) $\alpha = 1.2$

the response depends on the value of the damping ratio. The discussion of the transmissibility $T = (z_1/z)$ plots shown in Fig.11a and b, implies that our concept of the semi-active system with a variable damping provides the parameters which are very close to those of the optimal active system. The differences are mainly spotted in the range of low frequencies (lower than 2.5 Hz) while in the post-resonance range (beyond 1.8 Hz) the transmissibility is on average less by five than the corresponding value for the conventional passive spring-damper system of the same mass of the controlled plant $m = 80$ kg and the same spring coefficient $k = 10^4$ N/m. Fig.11 yields that the vibration control efficiency $\ni = (1 - T)100\%$ for the system with variable damping at the frequency 8 Hz is 75% while for the passive system it is only equal to 7%.

## 5. Example of application

The proposed vibration control system with a variable structure was applied to the driver seat suspension system presented in Fig.12. The suspension system consists of vibration control system, seat height control mechanism, seat midposition control mechanism and the cross guideway mechanism. The vibration control system consists of the pneumatic spring implemented as a membrane bellows (1) and the controlled hydraulic damper (2). The hydraulic damper is controlled through the two-position electrohydraulic valve (3). The cam seat-height control mechanism provides the seat position adjustment depending on a driver's height. The seatmidposition control mechanism consists of the leveling valve (4) and the mechanical feed-back (5). It provides the automatic variation of the seat stiffness according to the driver's weight (through the change of the initial pressure $p_0$ inside the pneumatic bellows) and is responsible for keeping the constant static deflection (seat midposition). Since the leveling valve has the zone of insensitivity and the element which delays the control signal, the efficiency of the system is reduced to the reaction on the slow changes of the seat load.

The simplified system model shown in Fig.12 was developed by setting the simplifying assumptions saying to take into account the seat and operator mass while to neglect the far less mass of another elements such as for example rolls, angle lever, guideway mechanism, etc. If we consider the small vibration around the static equilibrium state then the dynamical equation describing the seat vertical motion can be written as follows [3]

$$m\ddot{z}_1 + iS_e(z_1 - z)\frac{p_0 V_0^{\kappa}}{\left(V_0 - \int\limits_0^{z_1 - z} S_e dz\right)^{\kappa}} + F_t = 0 \tag{5.1}$$

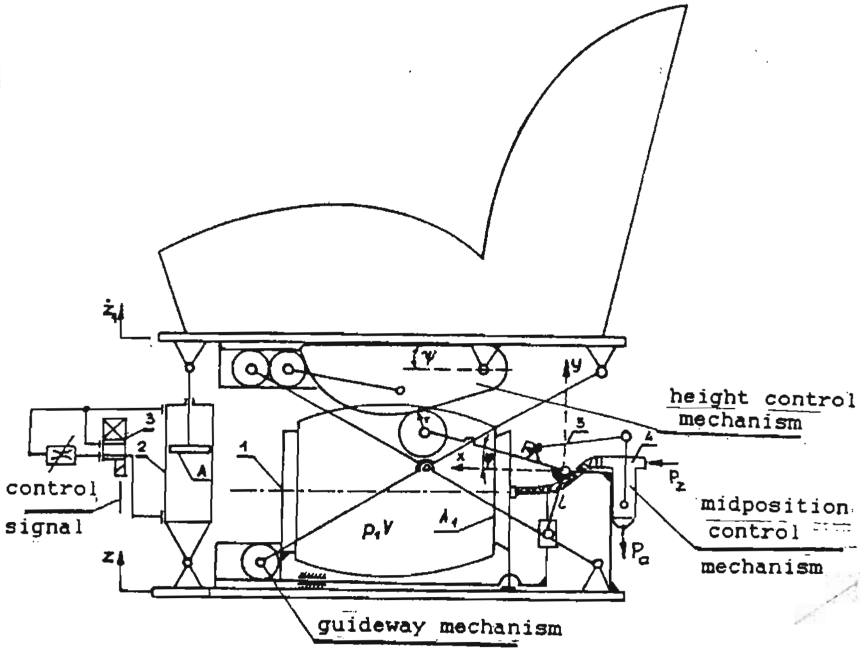



Fig. 12. Kinematic diagram of the driver seat suspension system

where $F_t$ is defined by Eq (2.7), the ratio $i = R/l$, $p_0$ and $V_0$ denote the initial pressure and volume of the bellows, respectively $S_e$ is the effective area of the bellows, $p_a$ represents the atmospheric pressure and the $\kappa$ is the polytropic exponent.

The above model was verified in both digital simulation and experimental investigation.

## 6. Experimental examination

The experimental examination of the designed and realized vibration control system for a driver seat suspension was carried out on a special laboratory stand. A fundamental part of this stand is a one–way electrohydraulic system generating the vertical vibration $z(t)$ of the base which travels in the guideways. A seat with the examined vibration control system is mounted to this base. The frequency of the generated harmonic vibration is ranged from 0 to 30 Hz. The above is possible due to the electronic board supporting the generator which controls the operation of the electrohydraulic servo–valve. The system responsible for the measurement

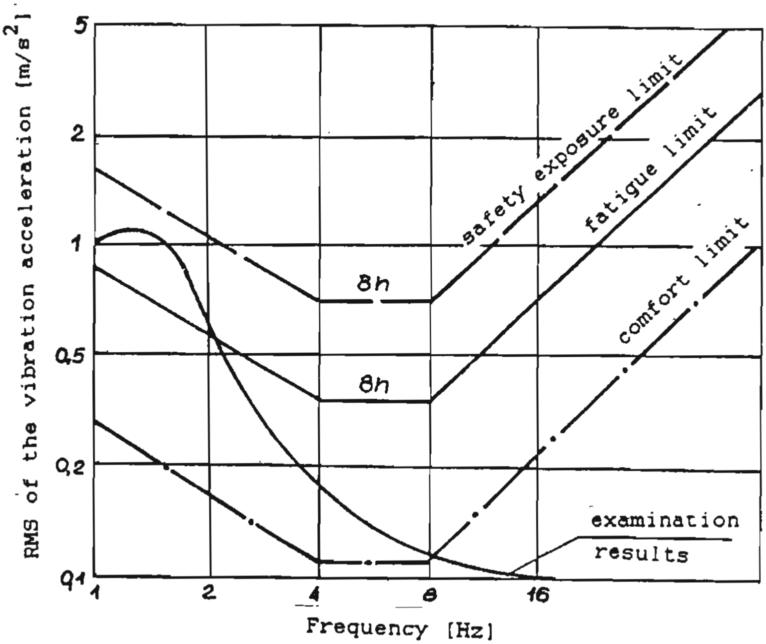



Fig. 13. Comparison of the amplitude–phase characteristics for the acceleration of the seat vibration obtained during the experimental tests while applying the limit values of the vibration acceleration recommended by ISO 2631 standard

of the seat vibration parameters consists of the accelerometer $C$, the charge amplifier, the voltage amplifier and the recorder. The experiment was carried out using the harmonic excitation of the frequency varying in the range from 0 to 20 Hz, the constant amplitude of the vibration acceleration 1 $m/s^2$ and at the seat load 75 kg. The supplied system pressure was 0.5 MPa and the volume of the pneumatic spring was $V_0 = 0.9$ $dm^3$. Fig.13 shows an example of the amplitude – frequency characteristics of the seat vibration acceleration. The admissible vibration acceleration limits proposed by the ISO 2631 standard (FDP – fatigue–decreased–proficiency) are also marked in the same figure. The above figures indicate that the investigated vibration control system with variable damping provides the efficient protection of the human body against harmful vibration effects. Nevertheless Fig.13 implies that the seat vibration acceleration exceeds the fatigue limit for 8 hours exposure at the excitation frequency 2 Hz and the comfort limit for the frequencies lower then 8 Hz.

## 7. Conclusions

The results discussed throughout this paper imply the following conclusions

- The theoretical basis for the synthesis of the vibration control systems with variable structure was developed. The systems in question provide the spring and damping variation.

- The construction of the variable–structure pneumatic and pneumo–hydraulic systems was concluded to be possible and a simplicity of its design was demonstrated.

- The properties of the vibration control system with a variable damping were discussed on the example of its application to the driver seat suspension. The simulation analysis carried out indicated that this suspension provides a wide range of the vibration isolation covering the frequencies beyond 1 Hz. Its vibration isolation efficiency is about 80%.

- By formulating the research model of the driver seat suspension system it was shown that the the variable-structure vibration control system is possible to implement. Then the experimental tests were carried out to confirm the above properties.

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**Synteza i badania układów wibroizolacji o zmiennych parametrach sprężystości i tłumienia**

Streszczenie

W pracy przedstawiono zagadnienia syntezy pneumo–hydraulicznych wibroizolatorów semiaktywnych o zmiennej strukturze. Opracowano koncepcje ukladów wibroizolacji ze zmienną sztywnością i zmiennym tłumieniem. Przeprowadzono analizę modeli takich układów, które zweryfikowano na drodze symulacji cyfrowej. Jak wykazały badania, wibroizolatory o zmiennej strukturze charakteryzują się dużą skutecznością izolacji drgań w zakresie częstotliwości powyżej 1 Hz. Konstrukcję wibroizolatora semiaktywnego o zmiennej strukturze zaprezentowano na przykładzie zastosowania go do zawieszenia fotela kierowcy.Zbudowano model badawczy takiego ukaładu oraz przeprowadzono badania doświadczalne. Potwierdziły one przydatność opracowanego układu dla celów ochrony człowieka przed szkodliwym wpływem drgań.

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