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DECOHESIVE CARRYING CAPACITY IN PERFECT AND ASYMPTOTICALLY PERFECT PLASTICITY[1] (A SURVEY)

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1. Introduction

In the theory of plasticity various models describing behaviour of the material after plastification are used. Very often it is a model of a body with some kind of hardening, according to real properties of material. If such is a case, then stress-strain curve must be limited, not admitting infinite value of stresses in material (infinite strength). Corresponding physical laws of decohesion describing such limits were proposed by many authors. Details can be found in a monograph by Życzkowski (1981) in Secs. 18.7 and 18.8, but in spite of large number of criteria of decohesion, their applications are rather scarce.

For engineering calculations frequently the model of perfect plasticity is applied. This model, as well as asymptotically perfect plasticity, does not need any limitation, as even for infinitely large strains, stresses do not exceed certain value σ_0 (yield stress). For structures made of such materials usually two values of limit external loadings are distinguished: the elastic carrying capacity (e.c.c.) connected with the onset of first plastic deformations, and the limit carrying capacity (l.c.c.) when certain mechanism of plastic collapse is reached.

However, for some structures the limit carrying capacity cannot be

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reached, due to the formation of certain inadmissible discontinuities. Some other paradoxes in perfect plasticity were pointed out e.g. by Shoemaker (1968), (1974), Del Piero (1975), and Gamer (1983).

Życzkowski (1957) anticipated, that for disks the process of elastic-plastic deformations cannot be continued over certain value of external loadings without violation of continuity of material. Życzkowski and Szuwalski (1973) proposed to call such loadings the decohesive carrying capacity (d.c.c.) and treat them as real estimation of admissible loadings for structure.

The aim of this paper is to give a review of papers dealing with problems of decohesive carrying capacity, and present the most important features of this phenomenon.

2. Bars and bars systems

A bar under uniform tension made of the perfectly or asymptotically perfectly plastic material can increase its length infinitely. Introduction of any factor causing nonhomogeneity of the stress state, such as body forces or notching, changes the problem. Only one cross-section can be plastified, and in it strains can be infinitely large, while in all others cross-sections they are limited. As a result the elongation of the whole bar cannot exceed certain maximal value, and tensile force cannot be larger than the elastic carrying capacity. Further increase of external loadings is impossible and will lead to separation of material in the cross-section with maximal tensile stress, where the derivative of axial displacement tends to infinity.

The similar results were obtained by Szuwalski and Życzkowski (1973) for some types of asymptotically perfect plasticity. The elongation of the bar then depends on the value of an integral, interpreted as a part of the complementary specific energy. For the most commonly used laws of asymptotically perfect plasticity given by Prager and Ylinen this integral is convergent resulting in limited elongation. Authors proposed another law:

$$\varepsilon = f(\sigma) = \frac{\sigma}{E\,(1 - \sigma/\sigma_0)^n}\,, \tag{1}$$

and proved that for $n \geq 1$ elongation of the bar can be infinitely large leading to the limit carrying capacity.

The results presented above were obtained within the framework of the small-strain theory, though the process ended with infinitely large strains. To avoid this internal inconsistency of theory Życzkowski and Szuwalski (1982) investigated the problem of nonhomogeneous tension of the bar, using the finite strain theory. Namely they took into account changes of cross-sections using true stresses and logarithmic strains, but retaining the assumption of uniaxial stress. The maximum of tensile force with respect to the strain was determined. The condition is identical with the condition of necking in homogeneous tension, but its meaning is quite different. If the process is to be continued (the force is decreasing) then only in one cross-section the process will be active, whereas in the rest of the bar we will have unloading. As a result we obtain again a termination of the process, but the reason changes. Instead of inadmissible discontinuities of displacements, we obtain here inadmissible discontinuities of stress field. For the maximal force the derivative of normal stresses with respect to the material coordinate tends to infinity.

The problem of limited elongation of the bar becomes of special importance when such a bar works in the bar system. This problem was discussed by Szuwalski and Życzkowski (1973) for simple perfectly elastic-plastic bar system with either tapered bars, or with their own weight taken into account. After the first decohesion, coinciding with the elastic carrying capacity, the whole system immediately collapses.

The same bar system, but made of asymptotically perfectly plastic material, was analyzed by Szuwalski (1980a). The Ylinen's material, as giving the possibility of limiting procedure to the perfect plasticity, was used. For maximal admissible loadings the maximal stress in the system equals σ_0. The corresponding d.c.c. of the system with weightless (or prismatic) bars and perfectly plastic material exhibits the property of nonuniquness and depends on the limiting procedure. Disregarding the weight of bars first, we always obtain classical l.c.c. while assuming at first the perfect plasticity, even for weightless bars the d.c.c. is reached. The results were compared by Szuwalski (1980b) with those obtained from the criterion of limited strains and condition for extended work after the first decohesion was formulated.

The concept of d.c.c. is especially useful in the case of purely thermal loadings, when thermal and plastic strains compensate each other, and no mechanism of plastic collapse can be found. In absence of l.c.c. Życzkowski and Szuwalski (1975) determined the d.c.c. of a bar system, as an estimation of admissible value of thermal loadings.

3. Statically indeterminate beams

The mechanism of plastic collapse for statically indeterminate beams usually requires formation of more than one plastic hinge. However, from the viewpoint of continuous medium, the process of elastic-plastic deformations must end with the formation of the first plastic hinge. Afterwards any finite rotation angle in the hinge is impossible, since it cannot be described by continuous displacement field. Further rotation would lead to vacancies on the tensile side and to overlapping of the material on the compressive side of the beam. External loadings causing formation of the first plastic hinge describe the d.c.c. of the beam.

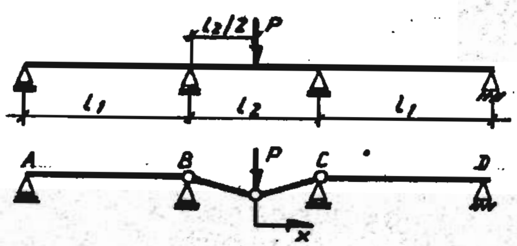

Fig 1. The Stussi-Kollbrunner paradox.

Making use of this concept Tran-Le Binh and Życzkowski (1976) clarified the well known Stüssi-Kollbrunner (1935) paradox. This paradox consists in the independence of the classic l.c.c. of the beam in Fig. 1, ($\bar{F}_1=8\bar{M}/l_2$), from the length of the beam span l_1 (the only possible mechanism of plastic collapse is shown in Fig. 1).

But for l_1 ∞ the plastic hinges at points B and C are not necessary, outer spans do not affect the middle span, and we obtain quite different value of l.c.c. $\bar{F}_2=4\bar{M}/l_2$. Admitting continuous displacement field only, Tran-Le Binh and Życzkowski obtained a continuous function for dependence of d.c.c. from l_1, varying from $\bar{F}_2$ for $l_1=0$, to $\bar{F}_1$ for l_1 ∞.

The decohesive carrying capacity for others beams was calculated by Tran-Le Binh and Szuwalski (1977), who showed that the first plastic hinge may be localized not necessary in the cross-section in which the first plastification took place.

The effect of decohesion can be observed also in bending of strongly curved bar with I-cross section. Szuwalski (1989) proved, that for sufficiently stiff flanges, the lower one separates from the web.

4. Disks and sheets

To generalize the problem of d.c.c. to the two-dimensional stress state, most commonly circularly symmetric disks and sheets were investigated.

An infinite perfectly elastic-plastic sheet with rigid circular inclusion, subject to uniform tension at infinity, was discussed by Szuwalski and Życzkowski (1973). Using H-M-H yield condition and the Hencky-Ilyushin theory of small elastic-plastic deformations, they proved that process cannot be continued over certain value of tensile loading. The termination of continuous solution is due to the infinite increase of radial strain at the point of joint of sheet with the rigid inclusion. As the radial strain is a derivative of the radial displacement, further increase of external loading leads to jump of displacement - separation of the sheet from the inclusion. The plastic zone at the moment of decohesion is rather small, and for an incompressible material vanishes.

The general criterion of decohesion of the same infinite sheet, made of asymptotically perfectly plastic material was formulated by Szuwalski (1986a). The results are similar to the mentioned above for bars under tension - decohesion can be avoided only for $n \geq 1$ in the law (1). The same results obtained Szuwalski (1985b) using the generalized power series.

Annular axially symmetric disks with finite external radius were analyzed by Szuwalski (1979). The criterion of further work of the free disk, after decohesion was formulated. Very narrow disks can work before decohesion even after full plastification. Quite different approach to the problem of decohesion of annular disk with rigid inclusion proposed Mróz and Kowalczyk (1989), by introduction of an additional constitutive relation between the rate of displacement discontinuity and respective traction rate. With its help, the process of decohesion can be analyzed under geometrical parameter control, while under loadings parameter control, jump of displacement was obtained. In this way decohesion treated in earlier mentioned papers as the rapid "brittle" phenomenon, after change of control parameter was described as continuous process.

The effect of termination of the process of elastic-plastic deformations was observed by Szuwalski (1984) also for disks with variable thickness. For hyperbolic disks the inadmissible discontinuity of displacement field may occur also at the external radius. Different type of termination was found by Szuwalski (1985a) and (1986b) for rotating disks consisting of two parts made of different materials. The Tresca-Guest yield condition was applied and therefore the various arrangements of principal stresses had to be discussed. If the external part of disc has lower yield stress than the inner one, it can be totally plastified, though the inner part remains, at least, partially elastic. The continuous solution cannot be obtained for greater angular velocity without violating the boundary conditions.

In absence of l.c.c. in the case of purely thermal loadings, d.c.c. may serve as an estimation of admissible values of such loadings.For uniformly heated infinite sheet with circular rigid inclusion the d.c.c. was evaluated by Życzkowski and Szuwalski (1975). This solution was generalized by Szuwalski and Życzkowski (1984) to case of combined loadings - uniform traction at infinity and uniform elevated temperature. The corresponding interaction curve exhibits the property of concavity, while usually the interaction curves in the theory of plasticity are convex. Moreover it depends on the elastic modulus - the Poisson's ratio ν.

Disks made of homogeneous material, but consisting of two parts of different (constant for each part) thickness, subject to temperature gradient were discussed by Skoczeń and Szuwalski (1988). For majority of such disks the continuous solution is limited by gradient of temperature, at which the radial strain in the outer (thinner) ring, at the radius of joint with the thicker inner disk, tends to infinity. Corresponding d.c.c. depends on geometrical properties of disc: ratio of thicknesses and ratio of radii of both parts. Those ratios must be sufficiently large - their critical values were determined.

Życzkowski and Szuwalski (1982) discussed the work of tensioned infinite sheet with circular rigid inclusion, using the finite strain theory. Using the Nadai-Davis theory of similarity of deviators of logarithmic strains and true stresses they found out that process will be terminated, but this time, by occurance of inadmissible discontinuities of stress field. Corresponding decohesive carrying capacity is even slightly smaller, than obtained using the small strain theory, because of decreasing thickness of the disc.

The case of combined thermal and surface loadings was also solved with help of finite strain theory by Szuwalski and Życzkowski (1984). Obtained interaction curves are concave, but only for elevated temperature. For cooled sheet the d.c.c. does not exist, due to the increasing thickness of the sheet at the point of joint with the rigid inclusion.

The effect of decohesion was observed by Szuwalski (1990) for circular sandwich plates with rigid inclusion, subject to tension and bending.

5. Shells

Investigations of d.c.c. for shells are usually more complicated and need numerical treatment. Tran-Le Binh and Życzkowski (1984) were looking for interaction curves of an infinitely long sandwich circular cylindrical shell under a ring of radial forces and axial loading at infinity. The material of shell is perfectly elastic-plastic and the small-deflection theory is employed. The process of elastic-plastic deformations cannot be extended over loadings causing infinitely large axial strain in the outer layer at the point where the ring of radial forces is applied. At that moment d.c.c. of the shell is reached. The corresponding interaction curve is concave.

The effects of termination of continuous solution of many types are observed in analysis of toroidal shells, when large plastic deformations are allowed for. Skrzypek and Hodge (1975), and Skrzypek (1978-82) pointed out some discontinuities in both stress and velocity fields, for thick, ideal sandwich incomplete toroidal shell. The material is assumed to be rigid/perfectly-plastic.

More detailed analysis of such shells was given by Skrzypek (1979). He discussed several modes of termination of the process of plastic deformation, depending on the choice of loading trajectories. Termination can be caused by bulging effect or inadmissible kinematic singularities. In the latter case d.c.c. of shell is reached. Applying the small strain theory, as well, as flow theory limit curves were determined.

Skrzypek and Życzkowski (1983) compared results for incomplete toroidal shell obtained with use of deformation theory and incremental theory of plasticity, for both small and large strains. They proposed certain classification of possible modes of termination of the process of plastic deformations. Two types of local kinematic discontinuities were distinguished: of order 0 - connected with infinitely large strains and of order 1 - infinitely large material derivative. The possibility of occurance of both types of discontinuities was discussed.

For elastic-plastic toroidal shells ways of termination of continuous process of deformations were investigated by Skrzypek and Muc (1988). Inadmissible kinematic discontinuities may appear then either at an elastic-plastic interface, leading to infinitely narrow strain localization, or within the plastic zone by formation of a plastic hinge. Further generalization of this problem is due to Bielski and Skrzypek (1989). Three possibilities of exhaustion of d.c.c. were observed. It may coincide with e.c.c. (no plastic zone), or decohesion may occur in elastic zone, but some plastic zones in other parts of shell may spread out earlier, or it may appear inside the plastic zone (plastic hinge). The influence of geometrical parameters was discussed.

8. Final remarks

The concept of d.c.c. enables formulation of purely mathematical criterion of termination of the elastic-plastic deformations process. This criterion may be treated as an upper bound of all physical criteria of decohesion.

Termination of continuous process is due to some inadmissible kinematic discontinuities. Further increase of loadings must lead to separation (decohesion) of two parts of the system. Sometimes a part of system after separation can carry even greater loadings, but system as a whole ceases to exist. Details were discussed by Życzkowski (1981).

In contrast with limit carrying capacity, d.c.c. depends on the elastic constants of material and often leads to concave interaction curves. This concept is especially useful in case of purely thermal loadings, when in absence of limit carrying capacity gives a possibility of estimation of admissible values of such loadings. The problem is of great importance, when numerical calculations are involved, as they do not bear infinities.

Application of the finite strain theory also leads to inadmissible discontinuities, at least in case of tension. For compression increase of cross-section helps to avoid decohesion.

Almost all papers on this topic were prepared in Technical University of Cracow, with participation or under supervision of Prof. M.Życzkowski.

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Summary

NOŚNOŚĆ ROZDZIELCZA W TEORII IDEALNEJ I ASYMPTOTYCZNIE IDEALNEJ PLASTYCZNOŚCI.

Dla pewnych układów z materiału idealnie, lub asymptotycznie idealnie plastycznego nie można osiągnąć nośności granicznej. Wcześniej, przy obciążeniu nazwanym nośnością rozdzielczą, pojawiają się pewne niedopuszczalne nieciągłości pola przemieszczeń lub naprężeń. W pracy dokonano przeglądu opracowań związanych z nośnością rozdzielczą i przedstawiono najważniejsze cechy tego zjawiska.